

# A SOLID TRANSPORTATION PROBLEM WITH MIXED CONSTRAINT IN DIFFERENT ENVIRONMENT

Amrit Das<sup>1,†</sup>, Uttam Kumar Bera<sup>1,†</sup> and Barun Das<sup>2</sup>

**Abstract** In this paper, we have introduced a Solid Transportation Problem where the constraints are mixed type. The model is developed under different environment like, crisp, fuzzy and intuitionistic fuzzy etc. Using the interval approximation method we defuzzify the fuzzy amount and for intuitionistic fuzzy set we use the  $(\alpha, \beta)$ -cut sets to get the corresponding crisp amount. To find the optimal transportation units a time and space based with order of convergence  $O(MN^2)$  meta-heuristic Genetic Algorithm have been proposed. Also the equivalent crisp model so obtained are solved by using LINGO 13.0. The results obtained using GA treats as the best solution by comparing with LINGO results for this present study. The proposed models and techniques are finally illustrated by providing numerical examples. Degree of efficiency have been find out for both the algorithm.

**Keywords** Solid transportation problem mixed constraint, fuzzy set, intuitionistic fuzzy set, interval analysis, genetic algorithm.

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## 1. Introduction

Transportation is about moving goods from one place to another place using a variety of vehicles across different arrangement systems. Not only the different technology (namely vehicles, energy, and infrastructure) are involved in it, but also people's time and effort are there. So basically transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., plant) to a set of destinations (e.g., warehouse) to meet the specific necessities. The goal is to satisfy the demand at destinations from the supply constraints at the minimum transportation cost possible. The transportation problem (TP) was developed by Hitchcock [10] in 1941. When in a transportation system there are mixed transportation modes are available for the shipments of goods, we might transport from sources to destinations by different transportation ways to reduce costs or to meet time schedule. In that case the solid transportation problem (STP), which considers three item properties, is very suitable. The STP was first stated by Shell [17]. Haley [8, 9] developed

<sup>†</sup>the corresponding authors. Email address: [das.amrit12@gmail.com](mailto:das.amrit12@gmail.com)(A. Das), [bera\\_uttam@yahoo.co.in](mailto:bera_uttam@yahoo.co.in)(U. K. Bera), [barundas2004@yahoo.co.in](mailto:barundas2004@yahoo.co.in)(B. Das)

<sup>1</sup>Department of Mathematics, National Institute of Technology Agartala, Jirania, PIN-799046, Tripura, India

<sup>2</sup>Department of Mathematics, Sidho-Kanho-Birsha University, Purulia-723101, West Bengal, India

the solution procedure of a STP and made a comparison between the STP and the classical TP. In literature a good number of researchers have developed solid transportation model considering the equality constraint [6, 12, 15, 21]. Pandian et al. [16] and H. Isermann [11] have studied transportation problem with mixed constraint and develop its solution technique. In spite of this development till now, there are some important issues that possibly missed by the previous researchers. Here these issues are pointed out.

- (i) In literature it found that, most of the papers related to mixed constraint are two dimensional, i.e. transportation problem without vehicle or conveyance constraint. But the vehicles are very important in transportation system. So here we consider a TP where the conveyance constraint are present.
- (ii) Most of the previous researchers have investigated TP with mixed constraint in crisp environment. But it is often observed that in transportation system basis on customer inexact demand, weather condition, bad road due to hilly area, rainy season, landslide etc. uncertain situation take place. To handle such situation we have considered fuzzy and intuitionistic fuzzy environment in our model.
- (iii) Another issue that we raised here is use of meta-heuristic algorithm. Now a days the use of such algorithms take a great interest among the researchers. Also such algorithms report the more convergency. In our model we have considered genetic algorithm (GA).

All the above mentioned issues motivate us to make this research. The paper has been organized as follows. Section 1 is introduction, section 2 is about the basic concepts that are mandatory to develop this paper. The defuzzification methods has been discussed in this section. The formulation of the three proposed models are in section 3 and the solution procedure is followed by section 4. Next in section 5 we have provided the numerical experiments, in this section we have evaluated the degree of efficiency (cf 5.3) for the two solution techniques. Last section numbered as 6 gives the conclusion and future extension.

## 2. Preliminaries and definitions

### 2.1. Fuzzy Set

Fuzzy sets were first proposed by Lofti A. Zadeh [22] in 1965. The definition of fuzzy set is as follows

**Definition 2.1.** If  $X$  is a collection of objects denoted generically by  $x$ , then a fuzzy set  $\tilde{A}$  in  $X$  is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) | x \in X)\}$$

$\mu_{\tilde{A}}(x)$  is called the membership function (generalized characteristic function) which maps  $X$  to the membership space  $M$ . Its range is the subset of nonnegative real numbers whose supremum is finite.

**Definition 2.2. Fuzzy number** (Grzegorzewski 2002)

A fuzzy subset  $\tilde{A}$  of real number  $\Re$  with membership function  $\mu_{\tilde{A}} : \Re \rightarrow [0, 1]$  is said to be a fuzzy number if

- $\mu_{\tilde{A}}(x)$  is upper semi-continuous membership function;
- $\tilde{A}$  is normal, i.e., there exists an element  $x_0$  such that  $\mu_{\tilde{A}}(x_0) = 1$ ;
- $\tilde{A}$  is fuzzy convex, i.e.  $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2) \forall x_1, x_2 \in \mathfrak{R}$  and  $\lambda \in [0, 1]$ ;
- Support of  $\tilde{A} = \{x \in \mathfrak{R} : \mu_{\tilde{A}}(x) > 0\}$  is bounded.

Fuzzy numbers are represented by two types of membership functions: (a) Linear membership functions e.g. Triangular fuzzy number (TFN), Trapezoidal fuzzy number, Piecewise Linear fuzzy number etc. (b) Non-linear membership functions e.g. Parabolic fuzzy number (PFN), Exponential fuzzy number and other non-linear fuzzy number. We used the following fuzzy numbers:

**Definition 2.3. Triangular Fuzzy Number**

Triangular Fuzzy Number (TFN) is the fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  with the membership function  $\mu_{\tilde{A}}(x)$ , a continuous mapping  $\mu_{\tilde{A}} : \mathfrak{R} \rightarrow [0, 1]$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & -\infty < x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & a_2 \leq x \leq \infty. \end{cases}$$

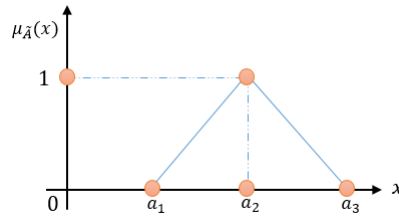


Figure 1. Membership function of Triangular Fuzzy Number

## 2.2. Defuzzification Methods

When a defuzzification method is applied to a fuzzy valued function then it converts the fuzzy amount to its corresponding crisp valued function. There are so many important defuzzification methods like as the nearest interval approximation, centroid method, graded mean and modified graded mean integration representation etc.

### 2.2.1. The Nearest Interval Approximation

Here, we like to approximate a fuzzy number by a crisp interval. Let  $\tilde{A}$  and  $\tilde{B}$  be two fuzzy numbers, with respective  $\alpha$ -cuts are  $[A_L(\alpha), A_R(\alpha)]$  and  $[B_L(\alpha), B_R(\alpha)]$ . Then according to Grzegorzewski [7] the distance between  $\tilde{A}$  and  $\tilde{B}$  can be defined as:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 \{A_L(\alpha) - B_L(\alpha)\}^2 d\alpha + \int_0^1 \{A_R(\alpha) - B_R(\alpha)\}^2 d\alpha}.$$

Let  $C_d(\tilde{A}) = [C_L, C_R]$  be the nearest crisp interval of the fuzzy number  $\tilde{A}$  with respect to the above distance metric  $d$ . Since each interval is also a fuzzy number with constant  $\alpha$ -cuts  $(C_d(\tilde{A}))_\alpha = [C_L, C_R]$ , for all  $\alpha \in [0, 1]$ . Now according to above distance metric  $d$ , distance of  $\tilde{A}$  from  $C_d(\tilde{A}), d(\tilde{A}, C_d(\tilde{A}))$  is given by

$$d(\tilde{A}, C_d(\tilde{A})) = \sqrt{\int_0^1 \{A_L(\alpha) - C_L\}^2 d\alpha + \int_0^1 \{A_R(\alpha) - C_R\}^2 d\alpha.}$$

So  $C_d(\tilde{A})$  is optimal when  $d(\tilde{A}, C_d(\tilde{A}))$  is minimum with respect to  $C_L$  and  $C_R$ . In order to minimize  $d(\tilde{A}, C_d(\tilde{A}))$ , it is sufficient to minimize the function  $D(C_L, C_R) (= d^2(\tilde{A}, C_d(\tilde{A})))$ . The first partial derivatives are

$$\frac{\partial D(C_L, C_R)}{\partial C_L} = -2 \int_0^1 A_L(\alpha) d\alpha + 2C_L.$$

$$\frac{\partial D(C_L, C_R)}{\partial C_R} = -2 \int_0^1 A_R(\alpha) d\alpha + 2C_R.$$

Therefore, solution of  $\frac{\partial D(C_L, C_R)}{\partial C_L} = 0$  and  $\frac{\partial D(C_L, C_R)}{\partial C_R} = 0$  are given by  $C_L^* = \int_0^1 A_L(\alpha) d\alpha$  and  $C_R^* = \int_0^1 A_R(\alpha) d\alpha$ . Again since

$$\frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_L^2} = 2 > 0, \quad \frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_R^2} = 2 > 0, \quad \frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_L \partial C_R} = 0$$

and

$$H(C_L^*, C_R^*) = \frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_L^2} \cdot \frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_R^2} - \left( \frac{\partial^2 D(C_L^*, C_R^*)}{\partial C_L \partial C_R} \right)^2 = 4 > 0.$$

So  $D(C_L, C_R)$  i.e.  $d(\tilde{A}, C_d(\tilde{A}))$  is global minimum. Therefore the interval

$$C_d(\tilde{A}) = \left[ \int_0^1 A_L(\alpha) d\alpha + \int_0^1 A_R(\alpha) d\alpha \right]$$

is nearest interval approximation of fuzzy number  $\tilde{A}$  with respect to metric  $d$ . Let  $\tilde{A} = (a_1, a_2, a_3)$  be a fuzzy number.. The  $\alpha$ -level interval of  $\tilde{A}$  is defined as  $(\tilde{A})_\alpha = [A_L(\alpha), A_R(\alpha)]$ . When  $\tilde{A}$  is TFN then

$$A_L(\alpha) = a_1 + \alpha(a_2 - a_1)$$

and

$$A_R(\alpha) = a_3 - \alpha(a_3 - a_2).$$

By nearest interval approximation method, lower and upper limits of the interval are respectively

$$C_L = \int_0^1 A_L(\alpha) d\alpha = \int_0^1 [a_1 - \alpha(a_2 - a_1)](\alpha) d\alpha = \frac{1}{2}(a_2 + a_1)$$

and

$$C_R = \int_0^1 A_R(\alpha) d\alpha = \int_0^1 [a_3 - \alpha(a_3 - a_2)](\alpha) d\alpha = \frac{1}{2}(a_2 + a_3).$$

Therefore, interval number considering  $\tilde{A}$  as a TFN is  $[\frac{1}{2}(a_2 + a_1), \frac{1}{2}(a_2 + a_3)]$ .

### 2.3. Intuitionistic Fuzzy Set(IFS)

According to Krassimir Atanassov [1, 2] the intuitionistic fuzzy set is characterized by the degrees of membership and non-membership of its elements. A formal definition of intuitionistic fuzzy set is given by

**Definition 2.4** ([1-3]). Let  $X \neq \phi$  be a given set. An intuitionistic fuzzy set in  $X$  is an object  $\hat{A}$  given by

$$\hat{A} = \{ \langle x, \mu_{\hat{A}}(x), \nu_{\hat{A}}(x) \rangle; x \in X \},$$

where  $\mu_{\hat{A}} : X \rightarrow [0, 1]$  and  $\nu_{\hat{A}} : X \rightarrow [0, 1]$  satisfy the condition  $0 \leq \mu_{\hat{A}}(x) + \nu_{\hat{A}}(x) \leq 1$ , for every  $x \in X$ .

Here the two functions  $\mu_{\hat{A}} : X \rightarrow [0, 1]$  and  $\nu_{\hat{A}} : X \rightarrow [0, 1]$  represents degree of membership and non-membership respectively.

**Definition 2.5** ([4-8]). An intuitionistic fuzzy set  $\hat{A} = \{ \langle x, \mu_{\hat{A}}(x), \nu_{\hat{A}}(x) \rangle; x \in \mathfrak{R} \}$  such that  $\mu_{\hat{A}}$  and  $1 - \nu_{\hat{A}}$ , where

$$(1 - \nu_{\hat{A}})(x) = 1 - \nu_{\hat{A}}(x), \forall x \in X$$

is called an intuitionistic fuzzy number.

**Definition 2.6.** ( $(\alpha, \beta)$ -cuts) A set  $(\alpha, \beta)$ -cuts, generated by IFS  $\hat{A}$ , where  $\alpha, \beta \in [0, 1]$  are fixed number such that  $\alpha + \beta \leq 1$  is defined as

$$\hat{A}_{\alpha, \beta} = \begin{cases} \langle x, \mu_{\hat{A}}(x), \nu_{\hat{A}}(x) \rangle, & x \in X, \\ \mu_{\hat{A}}(x) \geq \alpha, \nu_{\hat{A}}(x) \leq \beta, & \alpha, \beta \in [0, 1], \end{cases}$$

where  $(\alpha, \beta)$ -cut, denoted by  $\hat{A}_{\alpha, \beta}$  is defined as the crisp set of elements  $x$  which belong to at least  $\hat{A}$  at least to the degree  $\alpha$  and which does belong to  $\hat{A}$  at most to the degree  $\beta$ .

#### 2.3.1. Generalized Triangular Intuitionistic Fuzzy Number (GTIFN)

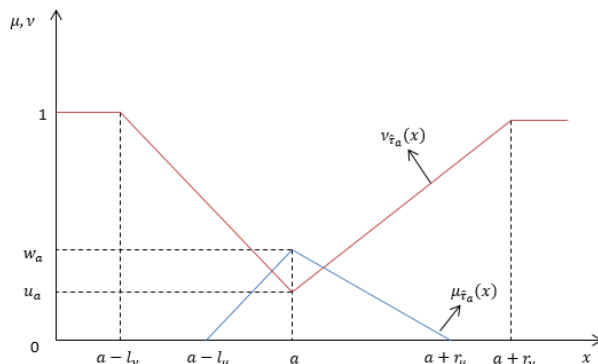
According Seikh et al. [5] the generalized triangular intuitionistic fuzzy number (GTIFN) can be define as follows:

A generalized triangular intuitionistic fuzzy number (GTIFN)  $\hat{\tau}_a = \langle (a, l_{\mu}, r_{\mu}; w_a), (a, l_{\nu}, r_{\nu}; u_a) \rangle$  is a special intuitionistic fuzzy set on a real number set  $\mathfrak{R}$  whose degree of membership and non-membership functions are given by,

$$\mu_{\hat{\tau}_a}(x) = \begin{cases} \frac{x - a + l_{\mu}}{l_{\mu}}, & a - l_{\mu} \leq x < a, \\ w_a, & x = a, \\ \frac{a + r_{\mu} - x}{r_{\mu}}, & a < x \leq a + r_{\mu}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\nu_{\hat{\tau}_a}(x) = \begin{cases} \frac{(a - x) + u_a(x - a + l_{\nu})}{l_{\nu}}, & a - l_{\nu} \leq x < a, \\ u_a, & x = a, \\ \frac{(x - a) + u_a(a + r_{\nu} - x)}{r_{\nu}}, & a < x \leq a + r_{\nu}, \\ 1, & \text{otherwise,} \end{cases}$$



**Figure 2.** Membership and Non-membership function GTIFN

where  $l_\mu, r_\mu, l_\nu, r_\nu$  are called the spreads of membership and non-membership function respectively and  $a$  is called mean value.  $w_a$  and  $u_a$  represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the conditions  $0 \leq w_a \leq 1, 0 \leq u_a \leq 1$  and  $0 \leq u_a + w_a \leq 1$ .

### 2.3.2. $(\alpha, \beta)$ -Cut set of GTIFN

**Definition 2.7.** A  $(\alpha, \beta)$ -Cut set of GTIFN  $\hat{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a), (a, l_{\nu_a}, r_{\nu_a}; u_a) \rangle$  is a crisp subset of  $\mathfrak{R}$ , which is defined as

$$\hat{\tau}_a^{\alpha, \beta} = \{x : \mu_{\hat{\tau}_a}(x) \geq \alpha, \nu_{\hat{\tau}_a}(x) \leq \beta\},$$

where  $0 \leq \alpha \leq w_a, u_a \leq \beta \leq 1$  and  $0 \leq \alpha + \beta \leq 1$ .

A  $\alpha$ -cut set of a GTIFN  $\hat{\tau}_a$  is a crisp subset of  $\mathfrak{R}$ , which is defined as

$$\hat{\tau}_a^\alpha = \{x : \mu_{\tau_a}(x) \geq \alpha\},$$

where  $0 \leq \alpha \leq w_a$ .

According to the definition of GTIFN it can be easily shown that  $\hat{\tau}_a^\alpha = \{x : \mu_{\tau_a}(x) \geq \alpha\}$  is a closed interval, defined by

$$\hat{\tau}_a^\alpha = [a_L(\alpha), a_R(\alpha)],$$

where  $a_L(\alpha) = (a - l_{\mu_a}) + \frac{\alpha l_{\mu_a}}{w_a}$  and  $a_R(\alpha) = (a - r_{\mu_a}) - \frac{\alpha r_{\mu_a}}{w_a}$ .

Same way a  $\beta$ -cut set of a GTIFN  $\hat{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a), (a, l_{\nu_a}, r_{\nu_a}; u_a) \rangle$  is a crisp subset of  $\mathfrak{R}$ , which is defined as

$$\hat{\tau}_a^\beta = \{x : \nu_{\tau_a}(x) \leq \beta\},$$

where  $u_a \leq \beta \leq 1$ .

It follows from definition that  $\hat{\tau}_a^\beta$  is a closed interval, denoted by  $\hat{\tau}_a^\beta = [a_L(\beta), a_R(\beta)]$  which can be calculate as

$$\hat{\tau}_a^\beta = [a_L(\beta), a_R(\beta)],$$

where

$$a_L(\beta) = (a - l_{\nu_a}) + \frac{(1 - \beta)l_{\nu_a}}{1 - u_a},$$

and

$$a_R(\beta) = (a + r_{\nu a}) + \frac{(1 - \beta)r_{\nu a}}{1 - u_a}.$$

It can be easily proven that for  $\hat{\tau}_a = \langle (a, l_{\mu_a}, r_{\mu_a}; w_a), (a, l_{\nu a}, r_{\nu a}; u_a) \rangle \in \text{GTIFN}(\mathfrak{R})$  and for any  $\alpha \in [0, w_a], \beta \in [u_a, 1]$ , where  $0 \leq \alpha + \beta \leq 1$

$$\hat{\tau}_a^{\alpha, \beta} = \hat{\tau}_a^\alpha \wedge \hat{\tau}_a^\beta,$$

where the symbol “ $\wedge$ ” denotes the minimum between  $\hat{\tau}_a^\alpha$  and  $\hat{\tau}_a^\beta$ .

## 2.4. Order Relations of Intervals

Let  $A = [a_L, a_R]$  and  $B = [b_L, b_R]$  be a pair of arbitrary intervals. These can be classified as follows:

Type-I: Non-overlapping intervals;

Type-II: Partially overlapping intervals;

Type-III: Completely overlapping intervals;

These three types of intervals are shown in Figure-3(a), 3(b) and 3(c) for different situations.

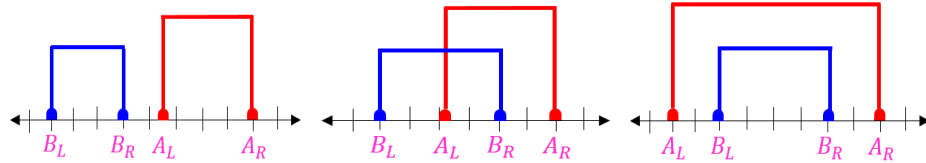


Figure. 3(a)

Figure. 3(b)

Figure. 3(c)

**Figure 3.** Different type of intervals

## 3. Model formulation

To formulate the models we make the following assumption:

- $c_{ijk}, \tilde{c}_{ijk}, \hat{c}_{ijk}$  = are the crisp, fuzzy, intuitionistic fuzzy unit transportation cost in STP respectively.
- $a_i, \tilde{a}_i, \hat{a}_i$  = are the available crisp, fuzzy, intuitionistic fuzzy source at supply point  $i$  respectively.
- $b_j, \tilde{b}_j, \hat{b}_j$  = are the crisp, fuzzy, intuitionistic fuzzy demand at demand point  $j$  respectively.
- $e_k, \tilde{e}_k, \hat{e}_k$  = are the crisp, fuzzy, intuitionistic fuzzy conveyance capacity for the conveyance  $k$  respectively.
- $x_{ijk}$  = the unknown quantity to be transported from  $i$ -th origin to  $j$ -th destination by means of  $k$ -th conveyance.

where  $i = 1, 2, \dots, m; j = 1, 2, \dots, n; k = 1, 2, \dots, l$  respectively.

### 3.1. Model 1: A Solid Transportation Problem with Mixed Constraint in Crisp Environment

Here all the parameters of STP are crisp in nature and with this assumption the mixed constraint STP model formulate as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l (c_{ijk} x_{ijk})$$

subject to,

$$\begin{aligned} \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\geq a_i, \quad i \in \alpha_1; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &= a_i, \quad i \in \alpha_2; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\leq a_i, \quad i \in \alpha_3, \\ \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\geq b_j, \quad j \in \beta_1; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &= b_j, \quad j \in \beta_2; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\leq b_j, \quad j \in \beta_3, \\ \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\geq e_k, \quad k \in \gamma_1; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &= e_k, \quad k \in \gamma_2; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\leq e_k, \quad k \in \gamma_3, \end{aligned}$$

where  $x_{ijk} \geq 0, i = 1, 2, \dots, m; j = 1, 2, \dots, n$  and  $k = 1, 2, \dots, l$  are all integers.  $\alpha_1, \alpha_2$  and  $\alpha_3$  are pairwise disjoint subset of  $\{1, 2, \dots, n\}$  such that  $\alpha_1 \cup \alpha_2 \cup \alpha_3 = \{1, 2, \dots, n\}$ ; Similarly  $\beta_1, \beta_2$  and  $\beta_3$  are pairwise disjoint subset of  $\{1, 2, \dots, m\}$  such that  $\beta_1 \cup \beta_2 \cup \beta_3 = \{1, 2, \dots, m\}$  and  $\gamma_1, \gamma_2$  and  $\gamma_3$  are pairwise disjoint subset of  $\{1, 2, \dots, l\}$  such that  $\gamma_1 \cup \gamma_2 \cup \gamma_3 = \{1, 2, \dots, l\}$ .

### 3.2. Model 2: A Solid Transportation Problem with Mixed Constraint in Fuzzy Environment

Here all the parameters of STP are fuzzy in nature and the model formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l (\tilde{c}_{ijk} x_{ijk})$$

subject to,

$$\begin{aligned} \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\geq \tilde{a}_i, \quad i \in \rho_1; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &= \tilde{a}_i, \quad i \in \rho_2; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\leq \tilde{a}_i, \quad i \in \rho_3, \\ \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\geq \tilde{b}_j, \quad j \in \theta_1; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &= \tilde{b}_j, \quad j \in \theta_2; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\leq \tilde{b}_j, \quad j \in \theta_3, \\ \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\geq \tilde{e}_k, \quad k \in \delta_1; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &= \tilde{e}_k, \quad k \in \delta_2; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\leq \tilde{e}_k, \quad k \in \delta_3, \end{aligned}$$

where  $x_{ijk} \geq 0, i = 1, 2, \dots, m; j = 1, 2, \dots, n$  and  $k = 1, 2, \dots, l$  are all integers.  $\rho_1, \rho_2$  and  $\rho_3$  are pairwise disjoint subset of  $\{1, 2, \dots, n\}$  such that  $\rho_1 \cup \rho_2 \cup \rho_3 = \{1, 2, \dots, n\}$ ; Similarly  $\theta_1, \theta_2$  and  $\theta_3$  are pairwise disjoint subset of  $\{1, 2, \dots, m\}$  such that  $\theta_1 \cup \theta_2 \cup \theta_3 = \{1, 2, \dots, m\}$  and  $\delta_1, \delta_2$  and  $\delta_3$  are pairwise disjoint subset of  $\{1, 2, \dots, l\}$  such that  $\delta_1 \cup \delta_2 \cup \delta_3 = \{1, 2, \dots, l\}$ .



### 3.3. Model 3: A Solid Transportation Problem with Mixed Constraint in Intuitionistic Fuzzy Environment

Here all the parameters of STP are intuitionistic fuzzy in nature and the respective model formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l (\hat{c}_{ijk} x_{ijk})$$

subject to,

$$\begin{aligned} \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\geq \hat{a}_i, \quad i \in \sigma_1; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &= \hat{a}_i, \quad i \in \sigma_2; & \sum_{j=1}^n \sum_{k=1}^l x_{ijk} &\leq \hat{a}_i, \quad i \in \sigma_3, \\ \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\geq \hat{b}_j, \quad j \in \varphi_1; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &= \hat{b}_j, \quad j \in \varphi_2; & \sum_{i=1}^m \sum_{k=1}^l x_{ijk} &\leq \hat{b}_j, \quad j \in \varphi_3, \\ \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\geq \hat{e}_k, \quad k \in \omega_1; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &= \hat{e}_k, \quad k \in \omega_2; & \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &\leq \hat{e}_k, \quad k \in \omega_3, \end{aligned}$$

where  $x_{ijk} \geq 0, i = 1, 2, \dots, m; j = 1, 2, \dots, n$  and  $k = 1, 2, \dots, l$  are all integers.  $\sigma_1, \sigma_2$  and  $\sigma_3$  are pairwise disjoint subset of  $\{1, 2, \dots, n\}$  such that  $\sigma_1 \cup \sigma_2 \cup \sigma_3 = \{1, 2, \dots, n\}$ ; Similarly  $\varphi_1, \varphi_2$  and  $\varphi_3$  are pairwise disjoint subset of  $\{1, 2, \dots, m\}$  such that  $\varphi_1 \cup \varphi_2 \cup \varphi_3 = \{1, 2, \dots, m\}$  and  $\omega_1, \omega_2$  and  $\omega_3$  are pairwise disjoint subset of  $\{1, 2, \dots, l\}$  such that  $\omega_1 \cup \omega_2 \cup \omega_3 = \{1, 2, \dots, l\}$ .

## 4. Solution Procedure

The proposed models are solved using two different techniques, genetic algorithm (GA) and LINGO, which based on the Generalized Reduced Gradient (GRG) Algorithm. The steps of GA are given as follows.

### 4.1. Genetic Algorithm (GA)

A genetic algorithm is a heuristic search process for optimization that resembles natural selection. GAs has been applied successfully in different areas. Genetic Algorithm for the Linear and Nonlinear Transportation Problem develop by G.A. Vignaux and Z. Michalewicz [20]. As the name suggests, GA is originated from the analogy of biological evolution. GAs consider a population of individuals. Using the terminology of genetics, a population is a set of feasible solutions of a problem. A member of the population is called a genotype, a chromosome, a string or a permutation. A genetic algorithm contains three operators - reproduction, crossover and mutation.

#### 4.1.1. Parameters

Firstly, we set the different parameters on which this GA depends. These are the number of generation (*MAXGEN*), population size (*POPSIZE*), probability of crossover (*PXOVER*), probability of mutation (*PMU*). There is no clear indication as to how large a population should be. If the population is too large, there

may be difficulty in storing the data, but if the population is too small, there may not be enough string for good crossovers. In our problem,  $POPSIZE = 500$ ,  $PXOVER = 0.6$ ,  $PMU = 0.9$  and  $MAXGEN = 5000$ .

#### 4.1.2. Chromosome representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosome are not effective in many non-linear physical problems. Since the proposed problem is non-linear, hence to overcome this difficulty, a real - number representation is used in this problem.

#### 4.1.3. Reproduction

Parents are selected at random with selection chances biased in relation to chromosome evaluations. Next to initialize the population, we first determine the independent and dependent variables from all (here 16) variables and then their boundaries.

#### 4.1.4. Crossover

Crossover is a key operator in the GA and is used to exchange the main characteristics of parent individuals and pass them on the children. It consist of two steps:

- (i) Selection for crossover: For each solution of  $P^1(T)$  generate a random number  $r$  from the range  $[0, 1]$ . If  $r < p_c$  then the solution is taken for crossover, where  $p_c$  is the probability of crossover.
- (ii) Crossover process: Crossover taken place on the selected solutions. For each pair of coupled solutions  $Y_1, Y_2$  a random number  $c$  is generated from the range  $[0, 1]$  and  $Y_1, Y_2$  are replaced by their offspring's  $Y_{11}$  and  $Y_{21}$  respectively where  $Y_{11} = cY_1 + (1 - c)Y_2, Y_{21} = cY_2 + (1 - c)Y_1$ , provided  $Y_{11}, Y_{21}$  satisfied the constraints of the problem.

#### 4.1.5. Mutation:

The mutation operation is needed after the crossover operation to maintain population diversity and recover possible loss of some good characteristics. It is also consist of two steps:

- (i) Selection for mutation: For each solution of  $P^1(T)$  generate a random number  $r$  from the range  $[0..1]$ . If  $r < p_m$  then the solution is taken for mutation, where  $p_m$  is the probability of mutation.
- (ii) Mutation process: To mutate a solution  $X=(x_1, x_2, \dots, x_K)$  select a random integer  $r$  in the range  $[1..K]$ . Then replace  $x_r$  by randomly generated value within the boundary of  $r^{th}$  component of  $X$ .

#### 4.1.6. Evaluation

Evaluation function plays the same role in GA as that which the environment plays in natural evolution. To this problem, the evaluation function is

$$eval(V_i) = \text{objective function value}$$

By Roulette wheel selection method the better chromosome are selected from the population to generate the next the improved chromosomes. Now new chromosomes are produced by arithmetic crossover and uniform mutation. The general outline of the algorithm is following:

```

begin
   $t \leftarrow 0$ 
  initialize Population( $t$ )
  evaluate Population( $t$ )
  while(not terminate-condition)
  {
     $t \leftarrow t + 1$ 
    select Population( $t$ ) from Population( $t-1$ )
    alter(crossover and mutate) Population( $t$ )
    evaluate Population( $t$ )
  }
  Print Optimum Result
end.

```

## 5. Numerical Example

In this section, a numerical example for the proposed STP with mixed constraint has been solved. For the unavailability of high capacitance computer, we consider  $i = 2, j = 2$  and  $k = 2$  with following input data. The problem can be extended for more number of sources, destinations and conveyances with high capacitance computer:

### 5.1. Input for three Models

Here we take the unit transportation cost, available source, the required demand and the capacity of the conveyances are as crisp number for model-1, triangular fuzzy number for model-2 and generalized intuitionistic triangular fuzzy number for model-3 respectively and given by Table 1.

### 5.2. Result of three proposed Models

With the numerical data given in Table 1, we solved the three proposed models of this paper using the above mentioned solution techniques and in each case we got global optimal solutions. These solutions are given by the Table 2. Observing the optimal transportation cost for different models, it found that in all cases GA gives the optimal value of objective function compare to LINGO for this particular problem. As we have been considered maximum generation is 5000 for GA, so it generate a the best optimal solution, whereas in LINGO we have noticed that after 4 iteration the solution obtained. So it is obvious that the solution so obtained using GA converge more to global optimal solution than LINGO. The graphical representation of change on total transportation cost obtained using two different algorithm i.e. GA and LINGO, for shipping of same amount of goods are shown in the Figure-4.

**Table 1.** Input for different models

Input for Model-1			
Crisp Unit Transportation Cost $c_{ijk}$			
	$j = 1$	$j = 2$	$k$
$i = 1$	3.25	2.75	1
$i = 1$	2.75	2.5	2
$i = 2$	3.5	3.0	1
$i = 2$	2.75	3.0	2
Available Source	Require Demand	Conveyance Capacity	
$a_1 = 35$	$b_1 = 10$	$e_1 = 11$	
$a_2 = 8$	$b_2 = 33$	$e_2 = 32$	
Input for Model-2			
Triangular Fuzzy Unit Transportation Cost $\tilde{c}_{ijk}$			
$i = 1$	(2,3,5)	(1,3,4)	1
$i = 1$	(1,3,4)	(2,2,4)	2
$i = 2$	(2,3,5)	(1,3,4)	1
$i = 2$	(1,3,4)	(2,3,4)	2
Available Source	Require Demand	Conveyance Capacity	
$\tilde{a}_1 = (20,24,28)$	$\tilde{b}_1 = (20,24,32)$	$\tilde{e}_1 = (0,2,8)$	
$\tilde{a}_2 = (4,6,12)$	$\tilde{b}_2 = (4,6,8)$	$\tilde{e}_2 = (24,28,32)$	
Input for Model-2			
Intuitionistic triangular Fuzzy Unit Transportation Cost $\hat{c}_{ijk}$			
$i = 1$	$\langle\langle (5,1,2;0.6), (5,1.5,2.6;0.3) \rangle\rangle$	$\langle\langle (6,1.5,2.5;0.5), (6,2.3;0.4) \rangle\rangle$	1
$i = 1$	$\langle (5,1.2,2.3;0.5), (5,1.7,2.8;0.3) \rangle$	$\langle (3,1,2.2;0.5), (3,1.5,2.7;0.3) \rangle$	2
$i = 2$	$\langle (6,1.4,2.8;0.5), (6,1.6,3.2;0.4) \rangle$	$\langle (3,1,2.2;0.5), (3,1.5,2.7;0.3) \rangle$	1
$i = 2$	$\langle (4,1.6,2;0.6), (4,1.7,2.6;0.3) \rangle$	$\langle (4,1.4,2.1;0.7), (4,1.5,2.6;0.2) \rangle$	2
Available Source	Require Demand	Conveyance Capacity	
$\hat{a}_1 = \langle\langle (8,2,1.5;0.6), (8,1.8,2;0.3) \rangle\rangle$	$\hat{b}_1 = \langle\langle (6,1.5,0.9;0.5), (6,1.2,1;0.4) \rangle\rangle$	$\hat{e}_1 = \langle\langle (31,4,2.5;0.5), (31,3,2.8;0.4) \rangle\rangle$	
$\hat{a}_2 = \langle\langle (32,4,2;0.5), (32,3,2;0.4) \rangle\rangle$	$\hat{b}_2 = \langle\langle (34,4.5,2.6;0.6), (34,3.6,3;0.3) \rangle\rangle$	$\hat{e}_2 = \langle\langle (9,2,1;0.7), (9,1.8,1.2;0.3) \rangle\rangle$	

### 5.3. Degree of efficiency evaluation

In order to validate the proposed methodology one numerical example for each environment of a STP with mixed type constraint solved using two different algorithm viz. genetic algorithm and GRG algorithm. The optimal solutions are presented in Table 2. From the result we observed that the genetic algorithm gives the optimal solution compare to GRG algorithm i.e. LINGO 13.0. Also we find out the relative error rate and standard deviation for the problems to test the order of convergency.

According to Kuo et al. [13] the relative error rate define as follows,

$$\text{Relative error rate} = \frac{|f^* - f_G^*|}{f_G^*} \times 100\%, \tag{5.1}$$

where  $f^*$  is the optimal solution obtained by LINGO and for our problem  $f_G^*$  is the optimal solution based on GA algorithm.

And the standard deviation(SD) is given by the formula,

$$\text{Standard deviation} = \sqrt{\frac{1}{N} \sum_{i=1}^N (f_{Mi}^* - f_{MA}^*)^2}, \tag{5.2}$$

where  $i = 1, 2, \dots, N$ .  $f_{Mi}^*$  represent the optimal solution for  $M$  method in the  $i^{\text{th}}$  execution and  $f_{MA}^*$  represent the average of  $N$  optimal solutions for  $M$  method. We execute our program on a personal computer with Intel Pentium duo processor (2.2 GHz, 800 MHz FSB)CPU and 3GB RAM using Dev-C++ and LINGO 13.0 version. We calculated the relative error rate and SD using the equations (5.1) and (5.2) respectively and listed in the Table 3. From the Table 3, it observed that the

**Table 2.** Optimal transported amount ( $x_{ijk}$ ) with optimal objective values

Solution using GA			Solution using LINGO		
Solution for model-1					
	$j = 1$	$j = 2$	$j = 1$	$j = 2$	
$i = 1$	4.6270	0.0759	0	0	$k = 1$
$i = 1$	0.7115	0.0085	0	2.1	$k = 2$
$i = 2$	3.1922	3.1273	0	1	$k = 1$
$i = 2$	1.4721	0.2114	0	0.68	$k = 2$
<i>Total cost = 42.4612</i>			<i>Total cost = 43.5</i>		
Solution for model-2					
$i = 1$	1.9413	0.6338	0	0	$k = 1$
$i = 1$	0.2787	1.1541	0	0	$k = 2$
$i = 2$	3.1499	2.2889	0	5.82	$k = 1$
$i = 2$	0.3659	2.2714	1	0	$k = 2$
<i>Total cost = 36.0559</i>			<i>Total cost = 38.5</i>		
Solution for model-3					
$i = 1$	0.8364	1.8047	0	8.21	$k = 1$
$i = 1$	0.6074	1.4747	0	0	$k = 2$
$i = 2$	1.0004	4.5390	0	19.2	$k = 1$
$i = 2$	2.4744	1.8286	0	0	$k = 2$
<i>Total cost = 32.6463</i>			<i>Total cost = 37.376</i>		

genetic algorithm (GA) has better solution compare to GRG algorithm (LINGO). Although it find here that the standard deviation of LINGO is smaller than those of genetic algorithm. which means that GRG algorithm (LINGO) is much more stable than GA.

#### 5.4. Replication based experimental evaluation for genetic algorithm(GA)

To test the convergency of genetic algorithm (GA), here we have made some experiment with GA by changing the different parameters by which GA has been developed. We have run the program several times by changing the parameters maximum generation number (MAXGEN), population size (POPSIZE), probability of mutation(PMU) etc. The findings are given by the Table 4. From the Table 4, it has been observed that, the parameters of GA viz. MAXGEN, POPSIZE, PMU and the corresponding transportation cost are inversely proportional to each other, i.e. when there is a decrease in these parameters the corresponding transportation cost have an increase. This phenomena happen due to the fundamental properties of GA. Actually GA is a iteration based algorithm, where initially number are generates randomly in the given range i.e. POPSIZE. So large POPSIZE creates a large number of generation, by which we can check optimality. In our problem we see that when the POPSIZE is 500 then the cost is minimum compare to other POPSIZE. Same conclusion also can be drawn for the other parameters i.e. MAXGEN and PMU. These changes of cost with respect to the different GA parameters are picturised by the Fig. 5 and Fig. 6.

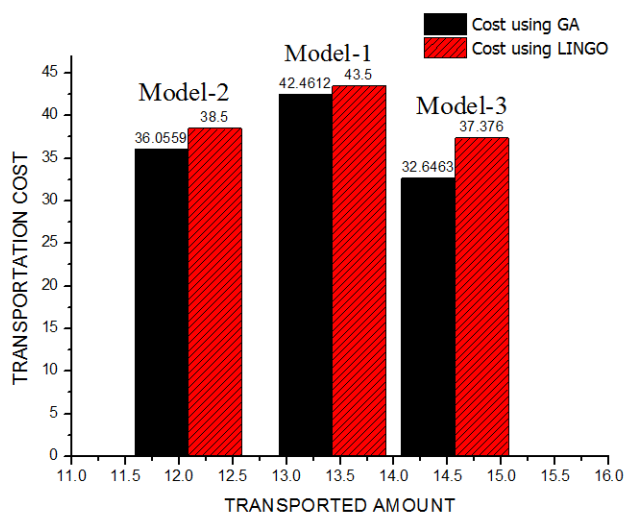


Figure 4. Cost Vs Total Transported amount for Different Models

Table 3. Comparison on results obtained using GA and LINGO

Methods →	→	Genetic Algorithm	LINGO
Model-1	Optimal cost	42.4612	43.5
	error rate on optimal cost	N/A	2.446%
	Standard deviation on optimal cost	0.0000000001	0
Model-2	Optimal cost	36.0559	38.5
	error rate on optimal cost	N/A	6.779%
	Standard deviation on optimal cost	0.000000002	0
Model-2	Optimal cost	35.6463	37.376
	error rate on optimal cost	N/A	2.941%
	Standard deviation on optimal cost	0.000000001	0

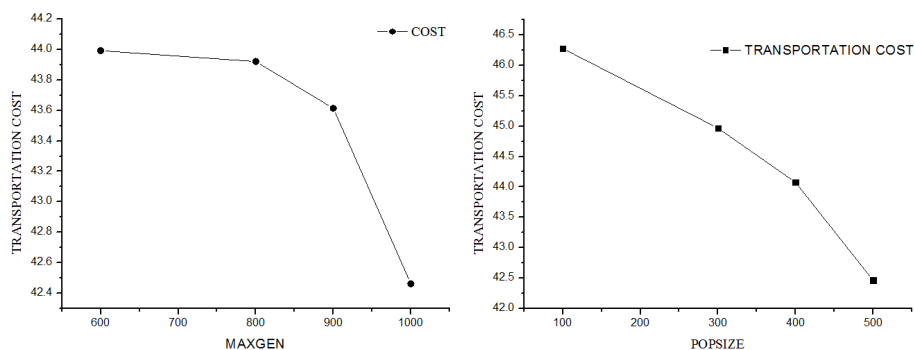
## 6. Conclusion

This paper deals with several ideas that can handle a STP with mixed type constraint in different environments. The major aspects of this study are given by the following.

1. We provide a meta-heuristic algorithm GA, to find the optimal solution of a solid transportation problem with mixed constraints i.e. both less than, more than and equal type constraints are exist in each side. The model is developed in crisp, fuzzy and Intuitionistic fuzzy environments.
2. From Table 2 it reveal that, for a mixed constraints, optimal solutions may not be balanced although problem is a balanced one, as it expected.
3. Also from Table 2, it fetches that, optimal amount of transportation need not proportionate to the unit transportation cost as like “matrix minimum methods”.
4. The criteria of mixed constraints STP also can be developed other environ-

**Table 4.** Effect on cost for different values of parameters(GA)

	MAXGEN	POPSIZE	PMU	Optimal cost
Model-1	1000	500	0.9	42.4612
	900	500	0.9	43.6149
	800	500	0.9	43.9223
	600	500	0.9	43.3336
	1000	500	0.9	42.4612
	1000	400	0.9	44.0757
	1000	300	0.9	44.9685
	1000	100	0.9	46.2795
	1000	500	0.9	42.4612
	1000	500	0.8	44.1630
	1000	500	0.7	45.6761
	1000	500	0.4	46.7326

**Figure 5.** Cost Vs Maximum generation(MAXGEN) and Population size(POPSIZE)

ments, like- stochastic, fuzzy stochastic, rough etc.

In this present investigation we have analyze a simple solid transportation problem using two different optimization algorithm viz. GA and LINGO, with numerical data. It has been revealed that the algorithm GA gives the best optimal solution in such case. The proposed models can be further extended with fixed charge solid transportation problem with a big size numerical illustration. All these extension keep in our mind.

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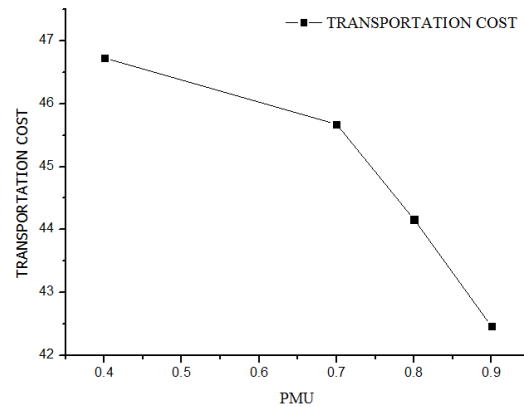


Figure 6. Transportation cost Vs Probability of mutation(PMU)

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