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NETWORK SCIENCE FACES THE CHALLENGE AND OPPORTUNITY: EXPLORING "NETWORK OF NETWORKS" AND ITS UNIFIED THEORETICAL FRAMEWORK*

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Abstract In the era of big data, network science is facing new challenges and opportunities. This review article focuses on discussing one of the hottest subjects of network science - "network of networks" (NON). The main features, several typical examples and the main progress for NON are outlined, including the epidemic spreading in multilayer coupled networks. Finally the most challenging tasks for NON are proposed.

Keywords Network of networks, challenges and opportunities, multilayer coupled networks, spreading dynamics

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1. Introduction

Albert-László Barabási, one of network science international leader, pointed out in nature physics (2012) for the complexity in the special issue [4] that "as an example, reductionism is outdated. As a field, complexity research is exhausted; therefore, it is to take over the network. Based on the data of the mathematical model of complex systems is to provide a fresh perspective, rapid development has become a cross subject: network science". Network science has become an extensive cross emerging science since small-world and scale-free properties are found in $1998 - 1999$ [7,101], continuously raised a hot wave research all over the world, and has published many works [6, 8, 9, 11, 29, 44, 62, 75, 81]. Some typical network structural features are widely studied in the Internet, the World Wide Web, the transportation networks, brain networks, energy, economy, social network, and so on [5, 22, 26, 27, 70, 74]. These structural features have a significant impact on dynamical processes or functions, and thus provide a new understanding and perspective for real-world complex systems. The methodology of network science has been widely accepted and applied in various fields, as well as has been of far-reaching reality and long-term strategic significance.

Under the impact of a wave of big data, the number of scientific research on "network of Networks" (NON) increases rapidly, which can reflect complex systems in the real world more really, has attracted global attention [5, 10, 12, 13, 23, 28, 30,

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40, 60, 108] but the research difficulty is unprecedented due to complexity interplay between networks. The NON is also called as "super-network" or "hyper-network", which is made of different kinds of links between network nodes, depending on each other in distinct ways. As internal and interlayer structures of the NON is surprisingly complex, studying on it will be driven by big data analysis, which has become one of the most important research directions for network science and provides a rare development opportunity. In this article the main features, several typical examples and main progresses for NON are outlined. The spreading features and some progress of the epidemic spreading in multilayer coupled networks are discussed in a special section. Finally the most challenging tasks for NON are proposed.

2. Several typical examples for NON

The NON widely exists in nature and human society; examples abound, here just to name a few. One prominent example is that the brain network is a very typical "network of networks"; the author has a systemic review in the journal nature [5,10,12,13,23,30,108]. Neuroscientists construct the network [5,10,23,30,40,60,108] from the following three levels, micro scale (neurons), mesoscale (nerve clusters) and large scale (brain regions). These three levels are also associated with each other and nested. Brain scientists think functional networks must be rooted in the network structure in some way. As the relationship of the structure is complex, how it functions is still a completely unsolved mystery. The second prominent instance is the high-tech network [40], which is a typical super network with four levels: first level is the Z-Park (so called Beijing zhongguancun science park-round), it is an China's first state-level high-tech industrial development zone, and the largest "science" park in China, has been hailed as a Chinese "silicon valley" , which is the first level of Chinese high-tech super network, as the national high-tech network zone of the core, play a pioneer, demonstration and the huge impact in China. The second level is the national high-tech Park and the university science Park network. The third level is the national high technology industry network. The fourth level is the high and new technology enterprise network in all over the world. The four levels of high-tech super network have distinguishing feature each, depend on and influence each other research results imply the all levels have the similar small-world and scale-free properties, which are only with different power-law index and the characteristics of their respective, and network features can mutual transformation between layers.

The third prominent example is the national defense military network [60], which includes three levels: intelligence reconnaissance, command and control network and network fighting effect. The communication network, command and control network, early warning detection network and the space satellite network also form the four layers networks. It also contains a "network centric warfare (NCW)" which is such a form of network confrontation, and its nine core system of space network, its data is huge and unprecedented.

Cyberspace security is facing severe challenges, which are not only a pure science and technology competition, but closely related to political, social, military and other issues. It therefore is a comprehensive, multi-level and multi-field complex integrated security issues. It should be pointed out that the biggest threat to Chinese cyberspace security has been from the United State.

The fourth outstanding examples is “Internet of things”, it is connected to the Internet. The first, the core and foundation of the Internet of things is still the Internet, is in the Internet based on the extension and expansion of network; Second, the client extended and expanded into any goods and goods between, such as information exchange and communication. Through the Internet of things intelligence recognition technology and pervasive computing, ubiquitous network integration application, more specifically, it is through the sensor network to the Internet as a basic framework to network, logistics network, and other properties of different “network of networks”. Outstanding characteristic is the network interaction between multiple, multiple decision makers and multi-objective, there are both competition and cooperation, uncertainty, and so on.

The fifth outstanding example is Chinese Beidou satellite navigation system (BDS) which for global navigation satellite system (GNSS). BDS satellite is closely related to global and regional location services networks, which involves the positioning network, mobile social networks, location-based services and cloud computing network. The establishment of the GNSS location services provides a technical basis and space-time benchmark. Location services have already formed the broad user requirements according to different space environment. It can be divided into the ground service, ocean location services, aviation services and space location services according to different service. Based on service mode, it also can be divided into personal location services, mobile target location services, professional project location services, military services, and so on. The service can be classified as pure location, interests, location services, social location services, and so on. BDS will become an interdisciplinary, cross system of social networking services.

In additional, there exist many datasets that can be represented as NON, such as network of different transportation networks including flight networks, railway networks and road networks, network of ecological networks including species interacting networks and food webs, network of biological networks including gene regulation network, metabolic network and protein-protein interacting network, network of social networks and so on. Among them, many interdependent networks including critical infrastructures are embedded in space, and so on.

All in all, it can be seen from the above typical examples, we are faced with the unprecedented challenges of NON, but it can also offer a unique opportunity for greater science and technology developments, and post many difficult theoretical as well as practical challenges to both scientific research and technological developments.

3. Some Features of NON

NON is actually a typical complex open system, it mainly displays in: NON has at least one of the following characteristics of “many”: multilayer, multidimensionality, multiple attribute, multi-targets, multi-parameters, multi-standards and choice. The network is mutual nesting, interdependent, interrelated and influences each other. More specifically, there are following characteristics.

First, the NON system is two or more sub-networks, overlap, and any layers structure. It has connection between layer and layer. Such as the transportation network has the physical layer, business and management layer. The multilayer communication network protocol also has a connection between layer and layer. That is to say, NON nested with the network, or contained in the network, or the network which can be made of different community composed of complex network. Network node itself can be a complex network, the node with different qualitative, even the edge with different attributes. Basically, most of the NON are mutual nesting and many kinds of network integrating, the intersection of multiple networks. For example, the Internet is a typical NON system, it and telecom network, financial and social network such as cross each other, interdependence and mutual influence.

Second, NON general system is an open complex system, system of material, energy and information exchange between inside and outside, belongs to the non-equilibrium statistical ensemble, although the traditional network limited to equilibrium network system, but in fact it is an urgent need to develop effective non-equilibrium statistical theory and the method to deal with NON problem.

Third, for NON system, the interdependence between network nodes and the edges are likely to have complex nonlinear interaction, the diversity and complexity. Complex interactions between multiple networks and influence each other.

Fourth, NON's topology structure, function and dynamic space-time evolution characteristics are different from traditional single network; the rich of self-organization behavior characteristics may emerge.

Fifth, NON may have a pyramid structure forms [31, 39]: such as, the high-tech network has four levels of the pyramid, including: first level is the highest of Zhongguancun science Park so-called Z-Park; The second level is the national high-tech park enterprise network; The third level is the national high technology industry network (including all over the country relating to the high-tech industry); The fourth level is the world's high-tech enterprise network (can be represented by a top-500 enterprise network) around the world. As the complexity of life pyramid: the bottom of the pyramid is the first level, on behalf of the living system of traditional network cell function: genes, transcriptase, proteins, and metabolism of these networks, both regularity and structure level exist in various aspects of the special integration; Upward from the bottom of the second level in the metabolism of these components form control gene identification and network, then the third level is a function module in building community, the fourth level is the community be nested and produces a large scale of scale-free network structure, the study found: both the cell function network topology and the network of nature and society have a striking similarity, able to complete from information storage, processing to perform the whole project. Life, therefore, profoundly reveals the system complexity pyramid: from human body cell function network to the world wide web (WWW), there is a general principle of self organization, that is to say, from the molecular level micro network to the nature and the macro network world of human society, their organizational principles have similarities, such as the node degree distribution is power function $p = aK^b$ form, b for power, called scale-free feature.

Sixth, NON has multidimensional nature, for example, network traffic can be multidimensional, charges (generation price) is a function of flow. As for the logistics network, energy network, supply chain network, there are many traffic. Can have multiple parameters to depict?

In addition, there exist many datasets that can be represented as NON, such as network of different transportation networks including flight networks, railway networks and road networks, network of ecological networks including species interacting networks and food webs, network of biological networks including gene regulation network, metabolic network and protein-protein interacting network, network of social networks and so on.

All in all, visible from the above typical examples, we are faced with the problem of "network of networks" unprecedented challenges.

4. Some research progresses for NON

In 2014, S. Boccaleti, G. Bianconi and R. Criado et al. have published a review article on Physics Report which titled with "Structure and dynamics of multilayer networks" [14]. No matter from theory or from the empirical study, this review article written by 12 scholars, who share the common features, and they have summarized and commented on the main progress and application of "network of networks" quite systematic and comprehensive so far. Due to the fast growth of this field, there are many definitions of different types of NON, such as interdependent networks, interconnected networks, multilayered networks, multiplex networks and many others. Among them, many interdependent networks including critical infrastructures are embedded in space, introducing spatial constraints. We think, therefore, their definition about the "network of networks" is the first formal definition of mathematics, basically covers so far in the literature of the existing related definition of "network of networks", and makes the network of networks begin to enter a normative research stage. In this review we here just added some work related to the Chinese scholars.

At present the main explorations focus on multilayers networks depending on each other. In recent years, the research mainly focused on the super network based on real world network features of research. In 2008 Wang et al. published a book about "Super network theory and its application" [76], which main preliminary introduction concepts and theory of super graph and some applications in management field [100]. Zlatic et al. studied topological properties of random three hypergraph and its application in tagged social network [110]. Zhang et al. proposed a hypergraph growth model [109] based on the user of background and object, labels, double preferential attachment mechanism; Gao and his cooperators summarized the relevant interdependent network [48] that is mainly theoretical analysis framework and the main progress of NON, as well as reveal the NON cascading collapse process, based on the percolation phase transition theory to explain the cascade of mutations in the NON physical mechanism. Recently Gao et al. review the progress again from a single network to a network of networks [49], including spatially embedded networks. Wang et al. based on the growth and preferential attachment mechanism generates super network model of the dynamic evolution [102]. Hu et al. built a super network evolution model and gave over distribution follows scale-free feature [58]. They also constructed an evolving model for hypergraph-structure-based scientific collaboration networks [59]. They study analytically the dynamics of authors' hyper-degree using mean-field approach. Results of both theoretical and numerical studies indicate that such a hyper-degree distribution follows a power-law decay. The proposed model may shed some light on the in-depth understanding of the structure and scientific collaboration networks. Guo et al. proposed a unified model hyper networks and studies evolving mechanisms of scale-free hyper networks and topological properties of the hyper network by using a Poisson process theory and a continuous technique. Their theoretical formula shows emergence of the strictly scaling law in hyper networks [57]. Liu et al. recently put forward three levels of super network evolution model [68], define the interlayer crossing ratio as new characteristics, reveals the evolution of the super network in the characteristic and applied to the top 500 domestic and foreign high-tech super network, the theory reveals some new topological properties which agrees with the empirical simulation well. We have also proposed three kinds of super network theory models with multilayer's [69], respectively, based on our Unified Hybrid Network Theory Formwork (UHNTF).The simulation results reveal some interesting phenomena, such as, the V-type curve of topological properties versus total hybrid ratio dr for the shortest path length L and the cluster coefficient r_c and proportionality coefficient C. These results provide a new way for super network further research and lay the foundation for further extending UHNTF application in practice.

In 2014, National Science review (NSR) published a special topic: Network Science. This special topic of the NSR presents several timely technical review and perspective, along with a highlight and interview, including NON and pointing to some new research directions in the field [18, 49]. In the same year Journal of Complex Systems and Complexity Science organized special issue on network science, there are several papers about progress of NON [40, 57, 65, 68, 69, 111]. In view of the actual NON or super network with large data, complexity, multiplicity and universality and other characteristics, in the exploration stage at present, NON is a new significant research direction and new areas for the network science and engineering application in the future, to explore the NON can not only stay in the past on the standard framework, must break through the single complex network research, in the big wave of data driven, innovative, pioneering new world, climb the new scientific peak, to promote wider application, it is the vitality of network science, charm and power.

5. Several progress of spreading dynamics on coupled networks

In this special section, we discuss the spread features and some research progress of the epidemic in the NON, which mainly focus on multilayer coupled networks. Epidemic spreading [2, 52, 82–85, 96, 97, 99, 105, 106] is one of the most successful application areas of the new science of networks [1, 73, 86, 87, 107], such as sexually transmitted diseases [50] and the H1N1 virus [61]. Currently, we only have a solid understanding of epidemic dynamics on single networks. But in the real world, a disease can be transmitted through a variety of pathways. For example, sexually transmitted diseases can spread both in heterosexual and homosexual networks of sexual contacts [66]. And another example is the spreading of epidemics propagated by human beings traveling via multiple transportation networks (e.g., aviation network and railway network). To make up the lack of understanding epidemic dynamics in coupled networks, Saumell-Mendiola et al. are the first to model the susceptible-infected-susceptible spreading on coupled networks [80]. They found that a global endemic state may arise in the coupled system, even though the epidemic is not able to propagate in any sub-network and even the number of coupling connections between two sub-networks is small. Almost in the same time, Dickison et al. studied the susceptible-infected-recovered processes on interconnected network systems, and found that two distinct regimes emerge with the decrease of coupling strength [21]. In strongly coupled networks, epidemics occur simultaneously across the entire system above a critical infection probability. While in weakly coupled networks, there is a mixed phase in which an epidemic outbreaks in one sub-network but does not spread to the coupled sub-network. Xu et al. studied the susceptible-infected-susceptible processes on multi-relational networks which a fraction p of the links carry a higher weight w_1 and the remaining $1-p$ carry a weight w_0 [94]. They found the fraction of infected nodes $\rho(p)$ shows a non-monotonic behavior, with ρ drops with p for small p and increases for large p. They also found there exists a moderate w_1/w_0 ratios which makes$\rho(p)$ exhibit a minimum. For large w_1/w_0 ratios, there exists an absorbing phase consisting only of healthy nodes within a range $p_L \leq p \leq p_R$, and an active phase with mixed infected and healthy nodes for $p < p_L$ and $p > p_R$. Wang et al. showed that in a different composition of the coupled networks, such as coupled Erdös-Rényi networks or the Erdös-Rényi network coupled with the scale-free network, the infection rates in the network and between the networks all have an influence on the epidemic threshold [88]. Buono et al. studied the dependence of the epidemic threshold on fraction $q > 0$ of shared nodes in a system composed of two layers and found that in the limit of $q \to 0$ the threshold is dominated by the layer with the smaller isolated threshold [16]. Wang et al. investigated epidemic spreading on interconnected networks at the level of metapopulation and found that both the interaction network topology and the mobility probabilities between subnetworks jointly influence the epidemic spread [90]. Zhao et al. proposed two kinds of the immunization strategies on multiplex networks, one is multiplex node-based random (targeted) immunization and the other is layer node-based random (targeted) immunization. They found both types of random immunization strategies show more efficiency in controlling disease spreading on multiplex Erdös-Rényi (ER) random networks; but targeted immunization strategies provide better protection on multiplex scale-free (SF) networks [104], and so on.

Social contagions [19, 20, 89] are another widespread spreading processes in complex networks. Unlike biological contagions (i.e., epidemic spreading) in which successive contacts result in disease transmission with independent probabilities, infection probability in social contagions depends on previous contacts since the multiple confirmation of the credibility and legitimacy of social contagions are needed. As the SIS and SIR models in epidemic spreading, the linear threshold model is the classical model for social contagions [53, 98]. In earlier studies, scholars mainly focus on single networks and studied the social contagions such as the adoption of innovations [17, 98], healthy activities [17] and microfinance [3], etc. However, with the development of communication technology, the online social networks emerged, such as Facebook and Twitter, which provide users with a variety of online interaction channels. Besides, in the traditional offline interactions, when an individual choosing which product to buy, ideas to adopt, and movement to join, he/she may be influenced by his/her friends, colleagues, family members, and other types of contacts. Brummitt et al. generalized the threshold cascade model into coupled networks, in which a node will be activated if a sufficiently large fraction of its neighbors in any sub-network are active [15]. They found that both combining two ER subnetworks with mean degrees z_1 and z_2 and splitting an ER network into two subnetworks with mean degrees z_1 and z_2 enlarge the cascade region. It's mainly because of these parameters, each subnetwork is too sparse or too dense to achieve global cascades, but they cooperatively achieve them when multiplex-coupled. Lee et al. considered the effects of response heterogeneity on the threshold model which assumes some nodes become active when, in at least one sub-network, a large enough fraction of neighbors are active; while the other nodes become active when, in all sub-networks, a large enough fraction of neighbors are active [63]. They found that varying the fractions of nodes following either rule facilitates or inhibits cascades. Other scholars also studied how the importance of different relationships [95] and the price of crossing between sub-networks [72] affect the social contagions in coupled networks [51]. Traditionally, epidemic spreading [2,82–85,96,97,99,106] and social contagions [19,20,53,89,92] have been studied independently, but in real-world situations there is always coupling or interplay between them. For example, when a disease suddenly emerges, the spread of disease results in elevated crisis awareness and thus facilitates the diffusion of the information about the disease [46, 47, 93], vice versa, the diffusion of the information promotes more people to take preventive measures and consequently suppresses the epidemic spreading [77]. The two types of spreading dynamics constitutes the asymmetrically interacting spreading dynamics in complex coupled networks, which leads to a new direction of research in network science [45]. In this area, Funk et al. first presented an epidemiological model by incorporating the spread of awareness in a well-mixed population, and found that the awareness based response can markedly reduce the final infection size [45]. Some other scholars also have considered the effects of information-driven [77, 103] and behavioral response [78, 79] on epidemic spreading, etc.

However, due to the difference in the epidemic and information spreading processes, the pathways for these two processes are different. Along this line, Granell et al. established a two susceptible-infected-susceptible (SIS) processes coupled model to investigate the inhibitory effect of the spreading of awareness on the epidemic dynamics in coupled networks, the results showed that the epidemic threshold is determined by the structures of the two respective sub-networks as well as the effective transmission rate of the awareness [54]. They also investigated the effect of mass media when all the agents are aware of the infection [55] which is the best case for stopping the epidemic. Wang et al. studied the asymmetrically interacting spreading dynamics based on a two susceptible-infected-recovered (SIR) processes coupled model in coupled networks, and found that the outbreak of disease can lead to the spread of information, leading to the increase of the epidemic threshold [91]. Other scholars also studied how the risk perception [71], the two-stage effects of awareness cascade [56] affect epidemic spreading in coupled networks [31].

Epidemic spreading and social contagions are ubiquitous in nature and human society. The former endangers human's life all the time and the latter always influences the decision-making and the behavior adoption of human. Such the two processes respectively or both of them simultaneously spreading in coupled networks are still an active and not yet consolidated research field, and leaves many unsolved problems. Besides, the addition of new sub-networks could make the existing theory, algorithms and analysis not fit and need some completely new ways. Therefore, it is very important and challengeable for the scholars to explore and research the spreading processes on coupled networks, and more widely in the field of network of networks. One of the problems is worth thinking about: for coupled networks coupling point to dynamic relevance theory, could you put forward the analysis framework? It relates to the overall system in the future to establish a unified theory of NON and its applications.

6. The Challenge and opportunity

Nature and human society show a variety of complex systems. How could the scholars describe the actual widespread "network of networks" from the view of network science so far? Yet the lack of perfect theoretical system, but also the lack of deep understanding, the main free from design, manage or control the complete theory of the network. So the network scientific theory is facing unprecedented severe challenges.

In 2015 the 10*th* China national conference on complex networks [24] and the 10*th* forum on China network science [25], common to refine the top ten network science subject, they are as follows.

1. NON research is the biggest challenging subject.
2. The influence of human behavior characteristics on the transmission dynamics.
3. Based on real data network transmission problem.
4. A multi-level dynamic network theory study and the space-time evolution problem.
5. A complex network robustness generation mechanism and network space safety problem.
6. Accurate theoretical description of complex network topology characteristics.
7. The nonlinear time series and network diagram of each table and characterization.
8. NON controllable, observability and control theory of the new method research.
9. The network dynamic structure changes.
10. Multilayer interaction in the network communication and its control.

In this paper, we only want to discuss top-1 subject since limit pages. Exploring and establishing a unified theory framework and the applications of NON are the most challenging frontier problem of the network science [24, 25, 42]. Because it needs to extract the most basic and the most essential network elements and interaction each other, and looking for a dynamic evolution characteristics of NON, expected to create a unified theory of NON framework, is a long way to go.

According to our previous research results and progresses [8, 11, 13, 32–36, 38, 41, 43, 64], we suggest that two possible unified roads (direction) may be possible.

Unified possible road # 1: How to go from a single network theory framework outreach to the NON? For example: from based on single unified hybrid network theory framework to develop the possibility of hybrid network theory for NON.

As we shown in Refs. [11, 32–36, 38, 41], nature and human social life as well as determine the probability of the random world, it is the unity of randomness and certainty of harmonious world. Considering the facts of randomness and certainty, whether physical, or biological networks and social network, is no exception, but a mixture of both deterministic and random degree and the different ways are in accordance with the concrete object, they always naturally coexist harmoniously self-organizing complex network system. As a result, a variety of mixing and preferred way has universality in nature and human society, this is the actual foundation and the real background of theory research, fully conform to the natural, social, physical, technical and the actual network of life.

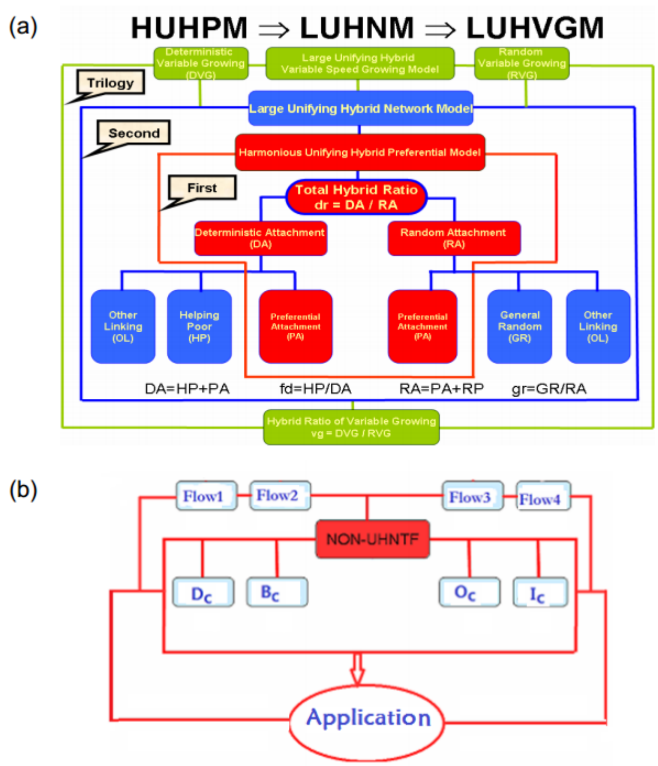

Figure 1. (a) A Unified hybrid network theory framework [11, 32–36, 38, 41, 43, 64], (b) Develop a new unified way (solution).

In order to describe the certainty and randomness of the harmony of the world and the complexity and diversity of growth process, we have put forward a unified hybrid network theory framework (UHNTF) form a trilogy of network theory models [11, 32–38, 41, 43, 44, 64, 67] as shown in Figure. 1, the theoretical analysis and simulation have revealed the evolution features of the UHNTF with multiple mixing ratio change, including the small-world effect and the scale-free properties as well as finding some new characteristics and phenomena, which are applied to some actual network, such as high-tech networks.

Figure.1 shows a UHNTF with basic ideas of a trilogy and schematic theory projects, in which the inner ring for the Harmonious Unifying Hybrid Preferential Model (HUHPM); the middle ring is a Large Unifying Hybrid Network Model (LUHNM); the outer ring is Large Unifying Hybrid Variable Growing Model (LUHVGM) or Unifying Hybrid Network Model with Variable Speed Growth (UHNMVSG). In the whole theoretical system introduces four hybrid ratios (dr,fd,gr,vg) and constitutes a unified trilogy of hybrid network model.

Using the above UHNTF, we can be flexible and appropriate regulation four hybrid ratios, study the characteristics of actual network from different angles, and applied to design the network structure and its characteristics. For example, the theory are used in high-tech networks [40, 64], such as the Z-Park companies

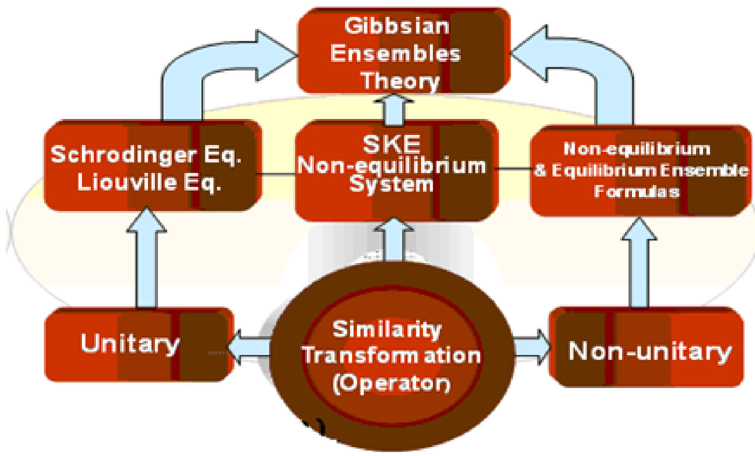

Figure 2. A Framework of Non-equilibrium & Equilibrium Ensemble Theory [11–13].

network is shifting growth each year, the network degree distribution with scale-free and small-world effect, features and accumulation degree distribution under different hybrid ratio have the power-law distribution and extensive mutual transformation between scale-free and stretched exponential distribution, which depend on the matching of four hybrid ratios. The theoretical result is in line with the actual network.

In order to promote above UHNTF of a single network to the network of networks. First, we will introduce some correlation parameters or ratios between multilayer networks to describe their interaction and influence. Such as, we maybe introduce interlayer cross ratio C_c, interlayer nodes ratio D_c, interlayer edge ratio B_c, interlayer coupling strength ratio O_c, the intersection layers ratio I_c, between any layers, and according to the need to phase in several parameter ratios, corresponding to different types of NON. Using these new parameter ratios between the layers as parametric meticulous depiction multi-layer hybrid network crossover and associated with different types, in order to further explore typical topology, function and dynamics characteristics of NON system, the relationships for new features and its variation law of NON should be searched.

At the same time, according to research needs for NON, the related flows between the layers will also be introduced, such as: information flow, energy flow, knowledge flow and so on, in order to further reflect and describe the interaction between all layers and influence.

Unified possible road # 2:

From the equilibrium state to non-equilibrium physics to set up a unified ensemble theory for NON, as shown in Figure2 [11–13]. As Albert-László Barabási pointed out: "we need to conquer the next frontier problem, is to understand the network in the dynamic process. But the problem is that we have almost as much dynamic phenomena and complex system....... although is varied, but if there's a possibility: the kinetic process has some common features? " Can predict: kinetics generality exists, but has not yet fully uncover their universal theoretical framework. Duo to there are rich and colorful non-equilibrium statistical physics process, so to find the theoretical basis of complex networks is dependent on the method of statistical mechanics, which requires innovation and development of statistical physics method.

How to establish a set of basic dynamic equations of the network science? Does Network science need Liouville equation and Schrodinger equation and Hamiltonian? Statistical physics contains two parts of the equilibrium and non-equilibrium state. How to extract network model Hamiltonian, is a bigger difficult point.

Brussels school headed by the Nobel laureate I. Prigogine has put forwarded the sub-dynamics equation and the related theory [11–13], which may be most suitable to a new equilibrium statistical physics basic equations, based on which we further derived a basic equations without the use of assumptions, only based on Rigged the Hilbert space (Liouville) and the correct operator algebra operation. Sub-dynamics theory, of course, also need to be further improved and developed. For example, we still need the perfect theory of function space expansion, projection density operator structure in the network model and so on. We propose a possible unified theory scheme as follows [11–13, 44]. First of all, the Liouville equation of equilibrium state closed statistical system must be established. Second, satisfying the equation is not simple original Liouville equation, but dynamic equation, the original Liouville operators has become intermediary (or collision) operator, by similarity transformation to connect between them, and corresponds to the irreversible system, the transformation is positive. In this way, by the original Liouville operators are non-unitary is similarity transformation and structure of "intermediary (or collision) operator" child brought by the dynamics equation is a new statistical physics basic equation. When the similarity transformation is about unitary, this equation is returned to the original Liouville equation of equivalent said, and "agents" (or collision) operator with the Liouville operators have the same spectrum structure, clearly, then their corresponding three ensemble calculation formula of the density operator is the same.

Simple Summary: This article reviewed the main features, several typical examples and the main research progress for "network of networks". The ten top challenging subjects faced by network science are pointed out, and we also put forward the exploration theory framework of the unify of the "network of networks" possible two roads. Because of the diversity, complexity and difficulty of the frontier, therefore, to explore the road of the unified process will be a long-term struggle, but as long as scientists all over the world widely cooperation and mutual efforts, we believe that network science's ultimate benefit mankind will be achieve it.

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