

CONSENSUS OF HETEROGENEOUS MULTI-AGENT SYSTEMS WITH LINEAR AND NONLINEAR DYNAMICS*

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Abstract In this paper, we perform an in-depth study about the consensus problem of heterogeneous multi-agent systems with linear and nonlinear dynamics. Specifically, this system is composed of two classes of agents respectively described by linear and nonlinear dynamics. By the aid of the adaptive method and Lyapunov stability theory, the mean consensus problem is realized in the framework of first-order case and second-order case under undirected and connected networks. Still, an meaningful example is provided to verify the effectiveness of the gained theoretical results. Our study is expected to establish a more realistic model and provide a better understanding of consensus problem in the multi-agent system.

Keywords Consensus, multi-agent systems, heterogeneous, linear and nonlinear dynamics, adaptive control.

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1. Introduction

Recent years have witnessed an inordinate amount of attention and increasing interest from various scientific communities is attracted by the consensus problems of multi-agent systems. It has been an emerging research direction of control theory and came into widespread use, such as sensor networks, formation, satellite clusters, robotic systems, etc. One aim of the consensus research is to resolve the issue that a team of agents reaches an agreement on a common value by negotiating with their neighbors [11].

A long line of research in this field has contributed a bunch of works focusing on the solution of the fascinating consensus problem. For example, Olfati-Saber and Murray [12] proposed a classical research framework of consensus problem for first-order integrator multi-agent systems under fixed and switching networks. The proposed analysis method and consensus protocols provide inspiration for the upcoming researchers. Inspired by the prominent work [12], many consensus algorithms were introduced for first-order integrator multi-agent systems in the face of various conditions, such as consensus with time delay [5], consensus in uncertain

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communication environments [1], with leader-following networks [4], etc. Moreover, researchers [8, 10, 17] also paid their attention to the consensus of second-order multi-agent systems and nonlinear multi-agent systems.

Thus far, as in the usual model in this field, the agents in the multi-agent systems are described by the homogeneous dynamics. However, such assumptions need to be relaxed when considering the real systems, among which the dynamics of agents may be heterogeneous. It is understandable that consensus problem of heterogeneous multi-agent systems are more challenging, though more suitable to the practical situation, since more complicated dynamics will be involved in situations like these. Also, there is a growing literature on this meaningful issue. For instance, Zheng & Wang [18] investigated the consensus problems in the framework of heterogeneous multi-agent systems, which are composed of first-order linear agents and second-order linear agents. When the communication topology is undirected, fixed and connected, the proposed linear consensus protocol and saturated consensus protocol are effective in guaranteeing a realizable agreement by all agents. Afterwards, a special case is considered in [2], where the control input of partial first-order linear agents is bounded due to the limitation of actuators. A new consensus protocol with input saturation is proposed to solve the consensus problem of heterogeneous multi-agent systems. Along this line, previous research has also focused on the finite-time consensus [20] and consensus without velocity measurement information [19] of heterogeneous multi-agent systems, respectively.

Besides the linear one, past works have also made great contribution in studying the consensus problem of multi-agent systems with different nonlinear dynamics. By introducing novel decentralized adaptive strategies, Liu etc. [7] investigated second-order consensus of multi-agent systems with heterogeneous nonlinear dynamics and time-varying delays. Combination with the pinning control method and Lyapunov stability theory helps to realize the consensus. In [15], Wang etc. studied the consensus problem for cooperative heterogeneous agents with nonlinear dynamics in a directed network. Here, the LMI and Lyapunov theory were employed to effectively obtain the global bounded consensus. Led by the proposed consensus protocol, the agents with different nonlinear dynamics would converge to a bounded region. Furthermore, the more challenging cases with linear and nonlinear dynamics were also probed into by some researchers. For example, Liu etc. [9] investigated the heterogeneous multi-agent systems composed of first-order linear agents, second-order linear agents and Euler-Lagrange nonlinear agents. Under undirected networks, the proposed consensus protocols can guarantee all agent to reach a common value on position information.

In this context, motivated by the referred works [9, 10, 15] and in an attempt to explore a more realistic scenario, we consider the heterogeneous multi-agent systems with linear and nonlinear dynamics. And more remarkably, by employing the adaptive method and Lyapunov stability theory, the mean consensus problem can be solved for first-order case and second-order case under undirected and connected networks. Our work affords novelty to the present literatures by that the proposed heterogeneous multi-agent system composed of linear and nonlinear dynamics agents in our settings, is more challenging than the current general case. At this point, the adaptive method is adopted to solve the problem that the designed feedback gain is required sufficiently large.

The rest of this paper is organized as follows. In section 2, some preliminaries are briefly outlined and the problem description is given. The analysis for first-order

and second-order case are shown in section 3 and section 4, respectively. A example is given in section 5, whereas in the last section we summarize our findings and provide concluding comments.

Notation 1. $\mathbb{R}^{n \times n}$ and $\mathbb{C}^{n \times n}$ denote the set of $n \times n$ real and complex matrices. $\mathbf{1}_N$ is an N dimensional column vector with all components 1. I_N denotes the identity matrix. For a square matrix X , $X > 0$ means it is positive definite. $\|\bullet\|$ represents Euclidean norm.

2. Preliminaries

In this section, some important knowledge about algebraic graph and problem description are provided here.

2.1. Graph theory

Algebraic graph theory is a paradigm tool for investigating the consensus, thus we first briefly introduce some fundamental knowledge on graph theory [3]. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted directed graph of order n with the set of nodes $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and an adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ with nonnegative elements a_{ij} . $e_{ij} = (v_i, v_j)$ denotes an edge from node v_j to v_i . The set of neighbors of node v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. The graph \mathcal{G} is undirected if $a_{ij} = a_{ji}$ for any i, j . We define an undirected graph is connected if there exists a path between any two distinct nodes of the graph. The degree matrix $\mathcal{D} = \{d_1, d_2, \dots, d_n\} \in \mathbb{R}^{n \times n}$ of graph \mathcal{G} is a diagonal matrix, where diagonal $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$ for $i = 1, 2, \dots, N$. Then the Laplacian matrix of \mathcal{G} is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Next, some key lemmas and assumptions are given as follows.

Definition 2.1 (Invariant set, [14]). A set S is an invariant set for a dynamic system $\dot{x} = f(x)$ if every trajectory $x(t)$ which starts from a point in S remains in S for all time.

Lemma 2.1 (LaSalle's invariance principle, [14]). *Consider an autonomous system of the form $\dot{x} = f(x)$, with f continuous and let $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ be a scalar function with continuous first partial derivatives. Assume that*

- (i) when $\|x\| \rightarrow \infty, V(x) \rightarrow \infty$;
- (ii) for any $x \in \mathbb{R}^n, \dot{V}(x) \leq 0$.

Let S be the set of all points within \mathbb{R}^n where $\dot{V}(x) = 0$ and M be the largest invariant set in S . Then, every solution $x(t)$ originating in \mathbb{R}^n tends to M as $t \rightarrow \infty$.

Lemma 2.2 (Yu, [16]). *For any vector $x, y \in \mathbb{R}^n$ and positive definite matrix $G \in \mathbb{R}^{n \times n}$, the following matrix inequality hold:*

$$2x^T y \leq x^T G x + y^T G^{-1} y.$$

Assumption 2.1. All the agents move in a bounded region consistently in sense that there exists a compact set $\mathbb{S} = \mathbb{S}(x_i) \in \mathbb{R}^n$ (for first-order case), $\mathbb{S} = \mathbb{S}(x_i, v_i) \in \mathbb{R}^{n \times n}$ (for second-order case), and all the agents starting with $x_i(0)$ and $v_i(0)$ are always in \mathbb{S} .

Proposition 2.1 (Zorich, [21]). *A function that is continuous on a closed interval is bounded on that interval. Moreover, there is a point in the interval where the function assumes its maximum value and a point where it assumes its minimal value.*

2.2. Problem description

As previously mentioned, we aim to respectively study the first-order case and second-order case, and the description of first-order one is provided as follows.

Consider a multi-agent system of size N (agents) and the corresponding dynamics of each agent is described as follows.

$$\begin{cases} \dot{x}_i = u_i, & i = 1, 2, \dots, M, \\ \dot{x}_i = f(x_i, t) + u_i, & i = M + 1, M + 2, \dots, N, \end{cases} \quad (2.1)$$

where $x_i, u_i \in \mathbb{R}^p$ represent the position and control input vectors of agent i , respectively. $f(x_i, t)$ is a continuous nonlinear function, and satisfies the Lipschitz condition.

Under the assumption of the undirected and connected communication topology, the consensus protocol when applying adaptive method and adaptive strategies is determined by

$$u_i = \sum_{j=1}^N c_{ij}(t) a_{ij} (x_j - x_i), \quad i = 1, 2, \dots, N, \quad (2.2)$$

$$\dot{c}_{ij}(t) = K a_{ij} (x_j - x_i)^T (x_j - x_i), \quad i, j = 1, 2, \dots, N, \quad (2.3)$$

where $K > 0$ is the scalar gain, and $c_{ij}(t) \geq 0$ is the coupling weight.

$c_{ij}(0) = c_{ji}(0)$ makes it clear that the coupling weighted matrix $C = [c_{ij}]$ is symmetrical. A new Laplacian matrix can be defined as follows.

$$\hat{L} = \begin{bmatrix} c_{11}a_{11} & c_{12}a_{12} & \dots & c_{1N}a_{1N} \\ c_{21}a_{21} & c_{22}a_{22} & \dots & c_{2N}a_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ c_{N1}a_{N1} & c_{N2}a_{N2} & \dots & c_{NN}a_{NN} \end{bmatrix}. \quad (2.4)$$

In addition, different with the first-order case, each agent in the scenarios of the second-order one moves according to the following dynamics.

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i, & i = 1, 2, \dots, M, \\ \dot{x}_i = Ax_i + Bu_i + f(x_i, t), & i = M + 1, M + 2, \dots, N, \end{cases} \quad (2.5)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = [0, 1]^T$$

and $f(x_i, t)$ is a continuous nonlinear function, and meets with the Lipschitz condition. As for $x_i = [x_{xi}, x_{vi}]^T$, x_{xi} and x_{vi} specify the position and velocity information of agent i , respectively.

The consensus protocol with an adaptive law for second-order case is expressed by

$$u_i = F \sum_{j=1}^N c_{ij}(t) a_{ij}(x_i - x_j), \quad i = 1, 2, \dots, N, \quad (2.6)$$

$$\dot{c}_{ij} = K_1 a_{ij}(x_i - x_j)^T \Gamma (x_i - x_j), \quad i = 1, 2, \dots, N, \quad (2.7)$$

where K_1 is positive constant, $c_{ij}(t)$ denotes the time-varying coupling weight between agent i and j with $c_{ij}(0) = c_{ji}(0)$, and $F \in \mathbb{R}^{p \times n}$ and $\Gamma \in \mathbb{R}^{n \times n}$ are the feedback gain matrices.

For notational simplicity in the following analysis, we only pay attention to the case of $p = 1$. However, the analysis is valid for any dimension, and the expression should be rewritten in terms of the Kronecker product.

3. First-order system

Here we first consider the first-order multi-agent systems. Inspired by the fact that nonlinear dynamics or heterogeneous dynamics of the involved agents has received fewer attention thus far, our interest is primarily in the consensus issue of heterogeneous multi-agent systems with linear and nonlinear dynamics. In this work, the proposed protocol and adaptive law are beneficial for realizing the mean consensus.

With the protocol 2.2 and adaptive law 2.3, the system 2.1 is rewritten as

$$\begin{cases} \dot{x}_i = \sum_{j=1}^N c_{ij}(t) a_{ij}(x_j - x_i), & i = 1, 2, \dots, M, \\ \dot{x}_i = f(x_i, t) + \sum_{j=1}^N c_{ij}(t) a_{ij}(x_j - x_i), & i = M + 1, M + 2, \dots, N. \end{cases} \quad (3.1)$$

By the Lyapunov direct method and LaSalle's invariance principle, the mean consensus is solved and described by theorem 3.1 as follows.

Theorem 3.1. *We suppose that the graph \mathcal{G} is fixed, undirected and connected, and Assumption 2.1 holds. Thus, in the presence of the consensus protocol 2.2 and adaptive law 2.3, the mean consensus of system 2.1 can be solved, and each coupling weight c_{ij} converges to some finite value.*

Proof. First, we define the error variable by $e_i = x_i - \frac{1}{N} \sum_{j=1}^N x_j$. From 3.1, we gain

$$\begin{cases} \dot{e}_i = \sum_{j=1}^N c_{ij}(t) a_{ij}(e_j - e_i) - \frac{1}{N} \sum_{j=M+1}^N f(x_j, t), & i = 1, 2, \dots, M, \\ \dot{e}_i = f(x_i, t) + \sum_{j=1}^N c_{ij}(t) a_{ij}(e_j - e_i) - \frac{1}{N} \sum_{j=M+1}^N f(x_j, t), & i = M + 1, M + 2, \dots, N. \end{cases} \quad (3.2)$$

Consider the Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=1}^N e_i^T e_i + \sum_{i=1}^N \sum_{j=1}^N \frac{(c_{ij} - m)^2}{4K}, \quad (3.3)$$

where m is a positive constant and determined later.

Differentiating respect to along 3.3, we gain

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^N e_i^T \left[\sum_{j=1}^N c_{ij}(t) a_{ij} (e_j - e_i) \right] - \sum_{i=1}^N e_i^T \left[\frac{1}{N} \sum_{j=M+1}^N f(x_j) \right] \\
&+ \sum_{i=M+1}^N e_i^T f(x_i) + \sum_{i=1}^N \sum_{j=1}^N \frac{(c_{ij} - m)}{2} a_{ij} (e_j - e_i)^T (e_j - e_i) \\
&= -m e^T \mathcal{L} e + \sum_{i=M+1}^N e_i^T f(x_i), \tag{3.4}
\end{aligned}$$

where $e = [e_1, e_2, \dots, e_N]^T$, and \mathcal{L} denote the Laplacian matrix of graph \mathcal{G} .

Since \mathcal{G} is connected, zero is the simple eigenvalue of \mathcal{L} and all other eigenvalue are positive [13]. There exists a unitary matrix $U \in \mathbb{R}^{N \times N}$, such that $U^T \mathcal{L} U = \Lambda = \text{diag}(0, \lambda_2, \dots, \lambda_N)$. $\mathbf{1}$ and $\mathbf{1}^T$ are the left and right eigenvectors of \mathcal{L} corresponding to the zero eigenvalue, respectively. Let $U = [\mathbf{1}, Y]$ with $Y \in \mathbb{R}^{N \times (N-1)}$, and after a variable transformation, i.e. $\xi = [\xi_1, \xi_2, \dots, \xi_N]^T = U^T e$, it is easy to get

$$\xi_1 = \mathbf{1}^T e = \sum_{i=1}^N e_i = 0. \tag{3.5}$$

Assumption 2.1 and Proposition 2.1 can safely lead to the results that both $\|x_i\|$ and $\|f(x_i)\|$ are bounded. Suppose that the upper are ω_1 and ω_2 , respectively. After performing the variable transformation, equation 3.4 is rewritten as

$$\begin{aligned}
\dot{V} &\leq -m \xi^T \Lambda \xi + \sum_{i=M+1}^N e_i^T \omega_2 \\
&= -m \lambda_2 \|\xi_2\|^2 - m \lambda_3 \|\xi_3\|^2 - \dots - m \lambda_N \|\xi_N\|^2 \\
&+ \omega_2 \sum_{i=M+1}^N \|U(i, 2)\| \|\xi_2\| + \omega_2 \sum_{i=M+1}^N \|U(i, 3)\| \|\xi_3\| \\
&+ \dots + \omega_2 \sum_{i=M+1}^N \|U(i, N)\| \|\xi_N\|. \tag{3.6}
\end{aligned}$$

The conditions of $\lambda_i > 0, i = 2, 3, \dots, N$ and sufficiently large $m > 0$, will result in $\dot{V} \leq 0$. Then the mean consensus problem can be solved if $e_i \rightarrow 0$ as $t \rightarrow \infty$ according to the LaSalle's Invariance principle. \square

4. Second-order system

Next, we shift our attention to the second-order multi-agent systems. By LMI tools and adaptive method, the consensus problem of heterogeneous multi-agent systems with linear and nonlinear dynamics is solved.

Herein it is necessary to introduce the significant lemma used in our analysis.

Lemma 4.1 (Li, [6]). *Suppose that \mathcal{G} is connected. The N agents described by $\dot{x}_i = Ax_i + Bu_i$ reach consensus under the protocol as $u_i = cK \sum_{j=1}^N a_{ij}(x_i - x_j)$ with $K = -B^T P^{-1}$ and the coupling weight $c \geq (\frac{1}{\lambda_2})$, where λ_2 is the smallest nonzero eigenvalue of \mathcal{L} and $P > 0$ is a solution to the following linear matrix inequality:*

$$AP + PA^T - 2BB^T < 0. \quad (4.1)$$

By means of the protocol 2.6 and adaptive law 2.7, the system 2.5 is rewritten as

$$\begin{cases} \dot{x}_i = Ax_i + BF \sum_{j=1}^N c_{ij}(t) a_{ij}(x_i - x_j), & i = 1, 2, \dots, M, \\ \dot{x}_i = f(x_i, t) + Ax_i + BF \sum_{j=1}^N c_{ij}(t) a_{ij}(x_i - x_j), & i = M + 1, M + 2, \dots, N. \end{cases} \quad (4.2)$$

Motivated by 4.1, we obtain the consensus results described as theorem 4.1 in the following.

Theorem 4.1. *Suppose the graph \mathcal{G} is fixed, undirected and connected, and Assumption 2.1 holds. The adopted consensus protocol 2.6 with $F = -B^T P^{-1}$ and adaptive law 2.7 with $\Gamma = P^{-1} B B^T P^{-1}$, help to realize the mean consensus of system 2.5. Moreover, P is a solution of the LMI 4.1 and 4.3, and each coupling weight c_{ij} converges to some finite value.*

$$AP + PA^T - 2BB^T < -lI_2, \quad (4.3)$$

where $l > 0$ is sufficiently large.

Proof. The analysis is similar to theorem 3.1. Define the error variable $e_i = x_i - \frac{1}{N} \sum_{j=1}^N x_j$. From 4.2, we have

$$\begin{cases} \dot{e}_i = Ae_i + BF \sum_{j=1}^N c_{ij}(t) a_{ij}(e_i - e_j) - \frac{1}{N} \sum_{j=M+1}^N f(x_j, t) \\ i = 1, 2, \dots, M, \\ \dot{e}_i = f(x_i, t) + Ae_i + BF \sum_{j=1}^N c_{ij}(t) a_{ij}(e_i - e_j) - \frac{1}{N} \sum_{j=M+1}^N f(x_j, t) \\ i = M + 1, M + 2, \dots, N. \end{cases} \quad (4.4)$$

Consider the Lyapunov function candidate

$$V = \sum_{i=1}^N e_i^T P^{-1} e_i + \sum_{i=1}^N \sum_{j=1}^N \frac{(c_{ij} - m)^2}{2K_1}, \quad (4.5)$$

where m is a positive constant and determined later.

Differentiating respect to along 4.4, we have

$$\begin{aligned}
\dot{V} &= 2 \sum_{i=1}^N e_i^T P^{-1} \dot{e}_i + \sum_{i=1}^N \sum_{j=1}^N \frac{(c_{ij} - m)}{K_1} \dot{c}_{ij} \\
&= 2 \sum_{i=1}^N e_i^T P^{-1} \left[A e_i + B F \sum_{j=1}^N c_{ij}(t) a_{ij} (e_i - e_j) \right] \\
&\quad + 2 \sum_{i=1}^N e_i^T P^{-1} f(x_i, t) + \sum_{i=1}^N \sum_{j=1}^N (c_{ij} - m) a_{ij} (e_i - e_j)^T (e_i - e_j). \quad (4.6)
\end{aligned}$$

Since $F = -B^T P^{-1}$, $\Gamma = P^{-1} B B^T P^{-1}$, let $\bar{e}_i = P^{-1} e_i$, it is easy to see that

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^N \bar{e}_i^T (A P + P A^T) \bar{e}_i + 2 \sum_{i=M+1}^N \bar{e}_i^T f(x_i, t) - 2m \sum_{i=1}^N \sum_{j=1}^N \mathcal{L}_{ij} \bar{e}_i^T B B^T \bar{e}_j \\
&= \bar{e}^T [I_N \otimes (A P + P A^T) - 2m \mathcal{L} \otimes B B^T] \bar{e} + 2 \sum_{i=M+1}^N \bar{e}_i^T f(x_i, t), \quad (4.7)
\end{aligned}$$

where $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \dots, \bar{e}_N^T]^T$, and \mathcal{L} denotes the Laplacian matrix of graph \mathcal{G} .

There exists a unitary matrix $U \in \mathbb{R}^{N \times N}$, such that $U^T \mathcal{L} U = \Lambda = \text{diag}(0, \lambda_2, \dots, \lambda_N)$. Let $U = [1, Y]$ with $Y \in \mathbb{R}^{N \times (N-1)}$. After the variable transformation of $\xi = [\xi_1^T, \xi_2^T, \dots, \xi_N^T]^T = (U^T \otimes I_2) \bar{e}$, we get

$$\xi_1 = (\mathbf{1}^T \otimes I_2) \bar{e} = 0, \quad (4.8)$$

$$\begin{aligned}
\dot{V} &= \xi^T [I_N \otimes (A P + P A^T) - 2m \Lambda \otimes B B^T] \xi + 2 \sum_{i=M+1}^N \bar{e}_i^T f(x_i, t) \\
&\leq \sum_{i=2}^N \xi_i^T (A P + P A^T - 2m \lambda_i B B^T) \xi_i + 2\omega_2 \sum_{i=M+1}^N \sum_{j=2}^N \|(U_{ij} \otimes I_2) \xi_j\|. \quad (4.9)
\end{aligned}$$

After choosing an appropriate m satisfying the condition of $m \lambda_i \geq 1, i = 2, \dots, N$, according to the lemma 4.1, we have

$$A P + P A^T - 2m \lambda_i B B^T \leq A P + P A^T - 2 B B^T < 0. \quad (4.10)$$

There exists a unitary matrix $U_1 \in \mathbb{R}^{2 \times 2}$, such that $U_1^T (A P + P A^T - 2m \lambda_i B B^T) U_1 = \Lambda_1 = \text{diag}(\lambda_{11}, \lambda_{22})$ with $\lambda_{ii} < 0, i = 1, 2$. Similar with that pointed out above, the variable transformation of $\delta_i = [\delta_x, \delta_v]^T = U_1^T \xi_i$ leads to

$$\begin{aligned}
\dot{V} &\leq \sum_{i=2}^N \delta_i^T \text{diag}(\lambda_{11}, \lambda_{22}) \delta_i + 2\omega_2 \sum_{i=M+1}^N \sum_{j=2}^N \|(U_1 \otimes U_{ij}) \delta_j\| \\
&= \sum_{i=2}^N \lambda_{11} \|\delta_{ix}\|^2 + \sum_{i=2}^N \lambda_{22} \|\delta_{iv}\|^2 + \sum_{i=M+1}^N \sum_{j=2}^N 2\omega_2 \|(U_1(1, 1) + U_1(2, 1)) U_{ij}\| \|\delta_{jx}\| \\
&\quad + \sum_{i=M+1}^N \sum_{j=2}^N 2\omega_2 \|(U_1(1, 2) + U_1(2, 2)) U_{ij}\| \|\delta_{jv}\|. \quad (4.11)
\end{aligned}$$

Sufficiently large $l > 0$ will support that $-\lambda_{ii} > 0, i = 1, 2$ is sufficiently large and thus $\dot{V} \leq 0$. Then by LaSalle's Invariance principle, it follow that $e_i \rightarrow 0$, as $t \rightarrow \infty$. The mean consensus problem is solved. \square

5. Examples

Further, it is worthy to verify the effectiveness of the mentioned theoretical results and we present a plausible example as follows.

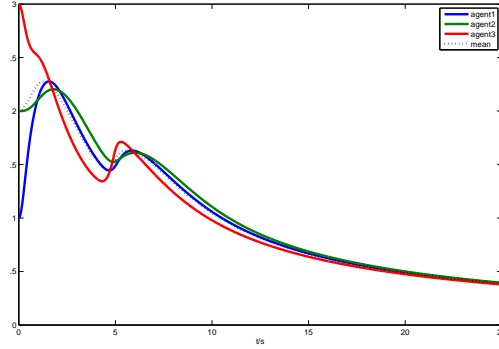


Figure 1. State trajectories of first-order system

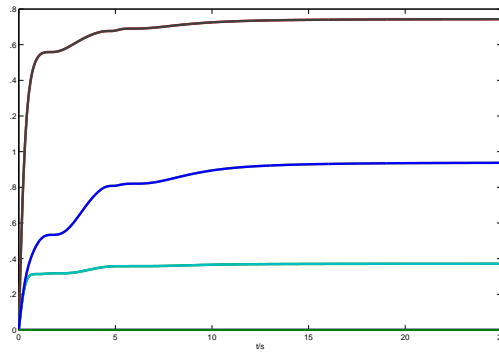


Figure 2. Trajectories of coupling weight c_{ij} of first-order system

Example 5.1. To illustrate, we consider a heterogeneous multi-agent system with 3 agents. Agent 1 and 2 are described by the first-order linear dynamics, while agent 3 is formulated as the first-order nonlinear dynamics. The nonlinear function is denoted by $f(x_i, t) = \sin(x_i t)$. Here, the coupling weight is set as $c_{ij}(0) = c_{ji}(0) = 0$. The communication topology among the agent is described by

$$A_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

The state trajectories of first-order system 2.1 and coupling weight c_{ij} are summarized by Figure 1 and Figure 2, respectively. Notably, our results afford clear evidences that each agents will converge to the mean state and the coupling weight will converge to some finite value.

6. Conclusion

The consensus problem of multi-agents has become a blooming and meaningful topic for researchers. Previous research has identified a series of solutions to the consensus behaviors. And yet, the established model can be extended and made more realistic in a variety of ways. This paper proposes and studies the consensus problem of a heterogeneous multi-agent system composed of linear agents and non-linear agents concurrently. In this setting, the mean consensus problem is solved for first-order case and second-order case under undirected and connected networks, with recourse to the adaptive method and Lyapunov stability theory. Moreover, a plausible example is provided to illustrate validness of the proposed consensus protocols. Our work offers a new perspective on the heterogeneity of multi-agent and its roles in influencing the collective behaviors of the involved system.

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