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AN ARITHMETIC-GEOMETRIC MEAN INEQUALITY APPROACH FOR DETERMINING THE OPTIMAL PRODUCTION LOT SIZE WITH BACKLOGGING AND IMPERFECT REWORK PROCESS*

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Abstract Some classical studies on economic production quantity (EPQ) models with imperfect production processes have focused on determining the optimal production lot size. However, these models neglect the fact that the total production-inventory costs can be reduced by reworking imperfect items for a relatively small repair and holding cost. To account for the above phenomenon, we take the out of stock and rework into account and establish an EPQ model with imperfect production processes, failure in repair and complete backlogging. Furthermore, we assume that the holding cost of imperfect items is distinguished from that of perfect ones, as well as, the costs of repair, disposal, and shortage are all included in the proposed model. In addition, without taking complex differential calculus to determine the optimal production lot size and backorder level, we employ an arithmetic-geometric mean inequality method to determine the optimal solutions. Finally, numerical examples and sensitivity analysis are analyzed to illustrate the validity of the proposed model. Some managerial insights are obtained from the numerical examples.

Keywords Production, random defective rate, failure in repair, backlogging, arithmetic-geometric mean inequality.

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1. Introduction

It is implicitly assumed that the production process functions perfectly at all times (i.e., all items are assumed to be of perfect quality) in the traditional economic production quantity (EPQ) model. However, this is not always true. In real production environments, it is often observed that defective items are produced due to imperfect production processes or other factors. These defective items must be reworked, rejected, or, if they have reached the customers, refunded. In all such cases, substantial costs are incurred, and quality-related costs cannot be ignored. To incorporate this more realistic situation and to study the effects of imperfect quality on lot sizes, scholars have developed various analytical models involving

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quality-related issues. Rosenblatt and Lee [22] have studied the effects of an imperfect production process on the optimal production cycle time for the classical economic manufacturing quantity (EMQ) model. Porteus [21] found a relationship between process quality control and lot sizing. Lee and Rosenblatt [16] considered the problem of a joint optimal production quantity and maintenance schedule in the classical EMQ model. Zhang and Gerchak [33] studied joint lot sizing and inspection policy in an economic order quantity (EOQ) model with random yield. Cheng [8] developed an EOQ model with demand-dependent unit production cost and imperfect production processes. Paknejad et al. [19] considered the number of defective items in a lot to be a random variable and examined the effect of defective items on the operational characteristics of a continuous-review (s, Q) system. Salameh and Jaber [23] considered the issue that arises when defective items are sold as a single batch at a discounted price and a 100% screening process of the lot is conducted. Ben-Daya [1] formulated an integrated model with joint determination of the EPQ and preventive maintenance levels under an imperfect process. Lin et al. [17] examined an integrated production-inventory model for imperfect production processes under inspection schedules. Sarker and Moon [26] considered an EPQ model with inflation in an imperfect production system. Hsu and Hsu [14] proposed two economic production quantity models with imperfect production processes, inspection errors, planned backorders, and sales returns. Tai [28] developed two economic production quantity models for deteriorating/imperfect quality items with a rework process. Hou et al. [13] extended Porteus [21] and took the maintenance cost into account to develop an optimal lot size model for defective items with a constant probability when the system is out-of-control. Sana [24], Sarker et al. [27], Sarker [25], Yoo et al. [32], Pal et al. [20] and the scholars they cite have discussed other studies on the issues of imperfect production systems.

All of the above studies about EPQ/EMQ models with imperfect production processes focus on determining the optimal lot size. However, these models neglect the total production-inventory costs, which can be reduced by reworking and repairing defective items with a relatively small repair and holding cost. Treviño-Garza et al. [30] developed two algorithms to determine jointly both the optimal replenishment lot size and the optimal number of shipments for a family of economic production quantity inventory models for an integrated vendor-buyer system considering that production system generates defective products, in which the rework cost is involved. Numerous studies on the problems of imperfect quality EPQ/EMQ models with reworking have been undertaken by Liu and Yang [18], Hayek and Salameh [12], Chiu [9], Jamal et al. [15], Chiu [10], Chiu et al. [11], Taleizadeh et al. [29], Cárdenas-Barrón et al. [6], Cárdenas-Barrón et al. [4, 5], Cárdenas-Barrón et al. [3], Wee et al. [31], Cárdenas-Barrón et al. [7] and the scholars they cite.

In this paper, we first establish an EPQ model with imperfect production process and failure in repair, where the holding cost of perfect items is distinguished from that of imperfect ones. In addition, to reflect the real market, we assume that shortages are allowed and completely backlogged. Hence, the cost of repair, disposal, and shortage are all included in the proposed model. Then, instead of complex differential calculus, we use an arithmetic-geometric mean inequality approach to determine the optimal production lot size and backorder level. It is already well established that the arithmetic mean is always greater than or equal to the geometric mean; that is, for any two positive real numbers, say u and v, we have

$$\frac{u+v}{2} \geq \sqrt{uv}.$$

The equality holds only if $u = v$. Finally, the numerical examples and sensitivity analysis are presented to illustrate the proposed model.

2. Model Description and Formulations

2.1. Notation and Assumptions

The following notation and assumptions are used in this article.

Notation:

P	Production rate per unit time
x	The proportion of imperfect quality items produced, a random variable with a known probability density function in the interval [0, a], where $0 < a < 1$
λ	Demand rate per unit time
C	Production cost per item including inspection cost
d	The production rate of imperfect items during the regular production process per unit time, where $d = Px$
P_1	The rate of reworking of imperfect items per unit time
θ_1	The proportion of reworked items that are irreparable, a random variable with a known probability density function in the interval [0, δ], where $0 < \delta \leq 1$
d_1	The production rate of scrap items during the rework process, where $d_1 = P_1\theta_1$
Q	Production lot size for each cycle
B	Allowable backorder level
K	Setup cost for each production run
C_R	Repair cost for each imperfect item reworked including inspection cost
C_S	Disposal cost per scrap item produced during the rework process
h	Holding cost per perfect item per unit time
h_1	Holding cost for each imperfect item being reworked per unit time
b	Shortage cost per item per unit time
H_1	Maximum level of on-hand inventory when regular production process stops
H	Maximum level of on-hand inventory in units, when the reworking ends
$TC(Q, B)$	Inventory total cost per cycle
$TCU(Q, B)$	Inventory total cost per unit time.

Assumptions:

(1) Production rate for perfect items is higher than the demand rate, *i.e.*, $(1-x)P > \lambda$.

(2) All imperfect items must be reworked and the reworked items may be irreparable during the rework process. For simplicity, we assume that the inspection time is very short, and therefore can be neglected.

(3) The proportion of reworked items that are irreparable, θ_1, is independent of the proportion of imperfect items produced x. That is, θ_1 and x are independent random variables.

(4) To guarantee the optimal production lot size exists and is larger than zero, we assume the condition $UV - W^2 > 0$ holds in this study, where the values of U, V and W are defined hereafter.

2.2. Mathematical formulation

First, a short problem description is provided. A constant product rate P was considered during the regular production uptime. The process may randomly generate x percent of imperfect items at a production rate $d = Px$ when all produced items are inspected. Thus, produced items fall into two groups: perfect and imperfect. The production rate for perfect items $(1-x)P$ is higher than the demand rate λ. All imperfect items were assumed to be reworkable at a rate of P_1, and the rework process starts when the regular production process ends. Since the rework process is not perfect, a random portion θ_1 of the reworked items were irreparable and became scrap items. The production rate of scrap items was $d_1 = P_1\theta_1$. Due to the randomness of x and θ_1, d and d_1 were also random variables. However, when observations of x and θ_1 were obtained, x and θ_1 became real constants. In this situation, the production-inventory system followed the pattern depicted in Figure 1. From Figure 1, the expressions of production uptime t_1 and t_5, reworking time t_2, production downtime t_3 and t_4, the maximum levels of on-hand inventory H_1 and H, the production rate of scrap items during the rework process d_1 and the cycle length T could be obtained, and are given as follows:

$$t_1 = \frac{H_1}{P-d-\lambda}, \tag{2.1}$$

$$t_5 = \frac{B}{P-d-\lambda}, \tag{2.2}$$

$$t_2 = \frac{xQ}{P_1} = \frac{dQ}{P_1 P}, \tag{2.3}$$

$$t_3 = \frac{H}{\lambda}, \tag{2.4}$$

$$t_4 = \frac{B}{\lambda}, \tag{2.5}$$

$$H_1 = (P-d-\lambda)\frac{Q}{P} - B, \tag{2.6}$$

$$H = H_1 + (P_1 - d_1 - \lambda)t_2 = Q\left(1 - \frac{\lambda}{P} - \frac{d_1 d}{P_1 P} - \frac{d\lambda}{P_1 P}\right) - B, \tag{2.7}$$

$$d_1 = \frac{\theta_1 x Q}{t_2}, \tag{2.8}$$

and

$$T = t_1 + t_2 + t_3 + t_4 + t_5 = \frac{Q(1-\theta_1 x)}{\lambda}. \tag{2.9}$$

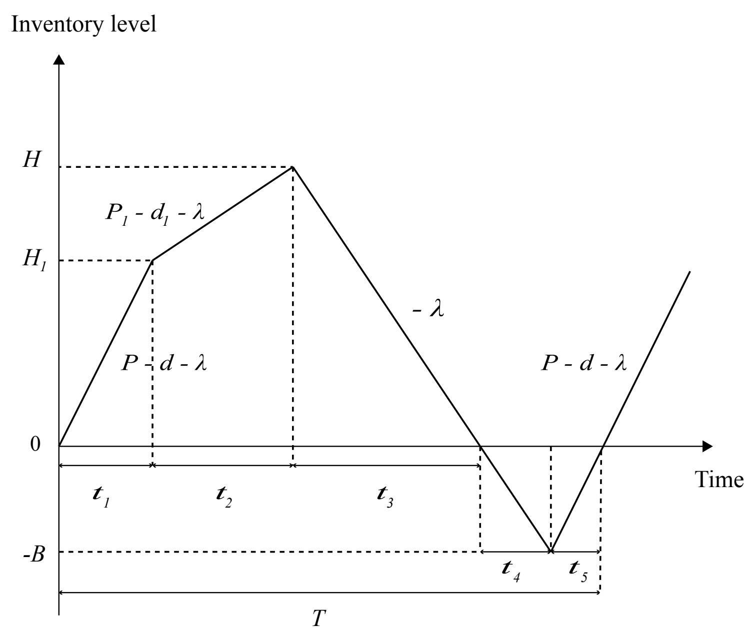

Figure 1. Graphical representation of production-inventory system.

Each produced item was inspected to judge its quality. Because all imperfect items must be reworked, perfect and imperfect items should be held in distinct areas, respectively. Thus, we must distinguish between the holding cost of perfect items and that of imperfect ones. Based on the above mentioned scenario and Figure 1, the holding cost for perfect items is $h\left(\frac{H_1}{2}t_1 + \frac{H_1+H}{2}t_2 + \frac{H}{2}t_3\right)$. The holding cost for imperfect items being reworked is $h_1\left[\frac{d(t_1+t_5)}{2}(t_1+t_5) + \frac{d(t_1+t_5)t_2}{2}\right]$. In addition, shortages were allowed and completely backlogged. Therefore, when the observations of random variables x and θ_1 were obtained, the inventory total cost per cycle could be expressed as

$$\begin{aligned}
TC(Q,B) &= \text{production cost} + \text{repair cost} + \text{disposal cost} + \text{setup cost} + \\
&\quad \text{holding cost for perfect items} + \text{holding cost for imperfect items} \\
&\quad \text{reworked} + \text{shortage cost} \\
&= CQ + C_R xQ + C_S xQ\theta_1 + K + h\left(\frac{H_1}{2}t_1 + \frac{H_1+H}{2}t_2 + \frac{H}{2}t_3\right) \\
&\quad + h_1\left[\frac{d(t_1+t_5)}{2}(t_1+t_5) + \frac{d(t_1+t_5)t_2}{2}\right] + \frac{b}{2}B(t_4+t_5) \\
&= CQ + C_R xQ + C_S xQ\theta_1 + K \\
&\quad + \frac{1}{2\lambda}\left(\frac{1-x}{1-x-\lambda/P}\right)(b+h)B^2 - \frac{h}{\lambda}(1-\theta_1 x)QB
\end{aligned}$$

$$+\left\{\frac{(h_1-h)}{2}\left(\frac{x}{P}+\frac{x^2}{P_1}\right)+\frac{h}{2}\left[\frac{1}{\lambda}\left(1-\frac{\lambda}{P}\right)-\frac{2\theta_1 x}{\lambda}\left(1-\frac{\lambda}{P}\right)\right.\right.$$
$$\left.\left.+\frac{\theta_1^2x^2}{\lambda}\left(1-\frac{\lambda}{P_1}\right)\right]\right\}Q^2. \quad (2.10)$$

In order to cope with the randomness of x and θ_1, we considered the expected total cost per cycle $\mathrm{E}[TC(Q,B)]$ and the expected cycle length $\mathrm{E}(T)$, where $\mathrm{E}(T)=Q[1-E(\theta_1x)]/\lambda$. Hence, by using the renewal reward theorem, the expected inventory total cost per unit time was given by:

$$\begin{aligned}\mathrm{E}[TCU(Q,B)] &= \mathrm{E}[TC(Q,B)]/\mathrm{E}(T)\\ &= \frac{1}{2Q[1-\mathrm{E}(\theta_1x)]}\left\{2\lambda Q[C+C_R\mathrm{E}(x)+C_S\mathrm{E}(\theta_1x)]\right.\\ &\quad +2K\lambda+\left[(b+h)\mathrm{E}\left(\frac{1-x}{1-x-\lambda/P}\right)\right]B^2-2h[1-\mathrm{E}(\theta_1x)]QB\\ &\quad +\left[\lambda(h_1-h)\mathrm{E}\left(\frac{x}{P}+\frac{x^2}{P_1}\right)+h\left(1-\frac{\lambda}{P}\right)[1-2\mathrm{E}(\theta_1x)]\right.\\ &\quad \left.\left.+h(1+\frac{\lambda}{P_1})\mathrm{E}(\theta_1^2x^2)\right]Q^2\right\}. \end{aligned} \quad (2.11)$$

By assumption, θ_1 and x are independent random variables, hence $E(\theta_1x)=E(\theta_1)E(x)$ and $E=(\theta_1^2x^2)=E(\theta_1^2)E(x^2)$. Furthermore, for convenience, we let

$$U=(b+h)\mathrm{E}\left(\frac{1-x}{1-x-\lambda/P}\right)>0,$$
$$W=h[1-\mathrm{E}(\theta_1x)]=h[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]>0,$$

and

$$\begin{aligned}V =&\lambda(h_1-h)\mathrm{E}\left(\frac{x}{P}+\frac{x^2}{P_1}\right)+h\left(1-\frac{\lambda}{P}\right)[1-2\mathrm{E}(\theta_1x)]+h\left(1+\frac{\lambda}{P_1}\right)\mathrm{E}(\theta_1^2x^2)\\ =&\lambda(h_1-h)\mathrm{E}\left(\frac{x}{P}+\frac{x^2}{P_1}\right)+h\left(1-\frac{\lambda}{P}\right)[1-2\mathrm{E}(\theta_1)\mathrm{E}(x)]\\ &+h\left(1+\frac{\lambda}{P_1}\right)\mathrm{E}(\theta_1^2)\mathrm{E}(x^2).\end{aligned}$$

Therefore, Equation (2.11) can be rewritten as

$$\begin{aligned}\mathrm{E}[TCU(Q,B)] = &\frac{\lambda\left\{C+[C_R+Cs\mathrm{E}(\theta_1)]\mathrm{E}(x)\right\}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}+\\ &\frac{1}{2Q[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}\left\{2K\lambda+UB^2-2WQB+VQ^2\right\}.\end{aligned} \quad (2.12)$$

3. Optimal replenishment policy using arithmetic-geometric mean inequality

In this section, we first use a simple algebraic operation to find the relation between production lot size and backorder level. Next, an arithmetic-geometric mean inequality approach is employed to find the optimal production lot size. Once the optimal production lot size, Q^*, is obtained, based on the relation between production lot size and backorder level, the optimal backorder level, B^*, follows.

By using algebraic operations in Equation (2.12), we get

$$\begin{aligned}&\mathrm{E}[TCU(Q,B)]\\&=\frac{\lambda\{C+[C_R+C_S\mathrm{E}(\theta_1)]\mathrm{E}(x)\}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}\\&\quad+\frac{1}{2Q[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}\left\{U\left(B-\frac{W}{U}Q\right)^2+2K\lambda+\left(V-\frac{W^2}{U}\right)Q^2\right\}.\end{aligned}\tag{3.1}$$

For minimizing $\mathrm{E}[TCU(Q,B)]$, we let the square term in Equation (3.1) equal to zero, then

$$B=\frac{W}{U}Q,\tag{3.2}$$

and Equation (3.1) can be reduced as follows:

$$\begin{aligned}\mathrm{E}[TCU(Q)]=&\frac{\lambda\{C+[C_R+C_S\mathrm{E}(\theta_1)]\mathrm{E}(x)\}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}\\&+\frac{K\lambda}{Q[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}+\frac{[V-(W^2/U)]Q}{2[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}.\end{aligned}\tag{3.3}$$

When $UV-W^2>0$, the three conditions proposed by Cárdenas-Barrón [2] could be verified. Therefore, the arithmetic-geometric mean inequality can be used as optimization method to minimize the expected inventory total cost per unit time. That is, we have

$$\mathrm{E}[TCU(Q)]\geq\frac{\lambda\{C+[C_R+C_S\mathrm{E}(\theta_1)]\mathrm{E}(x)\}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}+\frac{\sqrt{2K\lambda[V-(W^2/U)]}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)},$$

and the equality holds when

$$\frac{K\lambda}{Q[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}=\frac{[V-(W^2/U)]Q}{2[1-\mathrm{E}(\theta_1)\mathrm{E}(x)]}>0.$$

This implies the optimal production lot size (say Q^*) is given by

$$Q^*=\sqrt{\frac{2K\lambda}{V-(W^2/U)}}.\tag{3.4}$$

Therefore, the optimal backorder level can be obtained (say B^*) as

$$B^*=\frac{W}{U}Q^*=\frac{W}{U}\sqrt{\frac{2K\lambda}{V-(W^2/U)}}.\tag{3.5}$$

As a result, the minimum expected inventory total cost per unit time is

$$\mathrm{E}[TCU(Q^*,B^*)]=\frac{\lambda\{C+[C_R+C_S\mathrm{E}(\theta_1)]\mathrm{E}(x)\}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}+\frac{\sqrt{2K\lambda[V-(W^2/U)]}}{1-\mathrm{E}(\theta_1)\mathrm{E}(x)}.\tag{3.6}$$

4. Numerical Examples

Example 4.1. Given $P = 12,000$ units per year, $\lambda = 4,000$ units per year, $P = 600$ units per year, $K = \$200$ for each production run, $C =$\$2 production cost per item, $C_R = \$1$ repaired cost for each item reworked, $C_S = \$0.3$ disposal cost for each scrap item, $b = \$0.2$ per item backordered per year, $h = \$0.6$ per item per year and $h_1 = \$0.3$ per reworked item per year. In addition, the defective rate x is uniformly distributed over the interval [0, 0.1], and the percentage of defective items failing the reworking, θ_1, also follows the uniform distribution over the interval [0, 0.1], hence, $\mathrm{E}(x) = \mathrm{E}(\theta_1) = 0.05$. Since $UV - W^2 = 0.118358 > 0$, the arithmetic-geometric mean inequality can be used in this example. Substituting the given values of these parameters into Equations (3.4) and (3.5), we obtain the optimal production lot size $Q^* = 4083$ units, the optimal backorder level $B^*= 1981$ units, and the corresponding minimum annual expected inventory total cost, $\mathrm{E}[TCU(4083, 1981)] = \$8616$.

Example 4.2. We now study the effects of changes in the system parameters P, λ, P_1, K, b, h and h_1 on the optimal production lot size Q^*, the optimal backorder quantity B^* and the optimal annual expected inventory total cost $\mathrm{E}[TCU(Q^*, B^*)]$. Using the same data as in Example 4.1, the sensitivity analysis is performed by changing each of the parameters by -50%, -25%, +25% and +50%, taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 1.

Based on the computational results as shown in Table 1, we obtained the following managerial insights:

1. The optimal annual expected inventory total cost $\mathrm{E}[TCU(Q^*, B^*)]$, the optimal production lot size Q^* and the optimal backorder quantity B^* increase with an increase in the value of parameters λ and K. Moreover, Q^* and B^* are highly sensitive to the demand rate λ and the setup cost K. However, $\mathrm{E}[TCU(Q^*, B^*)]$ is highly sensitive to the demand rate λ, but only slightly sensitive to the setup cost K.
2. Q^* decreases while $\mathrm{E}[TCU(Q^*, B^*)]$ and B^* increase with an increase in the values of parameters P and h. Moreover,Q^* and B^* are highly sensitive to the production rate P, and moderately sensitive to the unit holding cost for perfect items h. However, $\mathrm{E}[TCU(Q^*, B^*)]$ is slightly sensitive to the production rate P and the unit holding cost for perfect items, h.
3. Q^* and B^* decrease while $\mathrm{E}[TCU(Q^*, B^*)]$ increases with an increase in the values of parameters P_1, b and h_1. Moreover, Q^* and B^* are highly sensitive to the unit shortage cost b, and moderately sensitive to the rate of reworking of imperfect items P_1 as well as the unit holding cost for imperfect item h_1. However, $\mathrm{E}[TCU(Q^*, B^*)]$is slightly sensitive to the unit shortage cost b, the rate of reworking of imperfect items P_1 and the unit holding cost for imperfect items h_1.

5. Conclusions

In this article, we establish an EPQ model with a failure in repair and complete backlogging to extend the model of Chiu et al. [11]. To reflect real production environments and economic realities, firstly, we considered that the rework process is

Table 1. Sensitivity analysis in various parameters changed for Example 4.2.

Parameter (initial value)	% Change in parameter	% Change in Q^*	B^*	$\mathrm{E}[TCU(Q^*, B^*)]$
	-50	38.80	-36.65	-0.57
P	-25	9.21	-10.50	-0.38
(12000)	+25	-4.53	5.86	0.22
	+50	-7.23	9.54	0.36
	-50	-37.42	-20.44	-48.63
λ	-25	-18.83	-7.82	-24.20
(4000)	+25	20.45	4.14	24.04
	+50	43.99	5.40	47.93
	-50	-29.29	-29.28	-1.34
K	-25	-13.40	-13.38	-0.60
(200)	+25	11.81	11.81	0.55
	+50	22.48	22.51	1.03
	-50	5.88	3.69	-0.15
P_1	-25	1.18	1.21	-0.05
(600)	+25	-0.69	-0.66	0.03
	+50	-1.17	-1.11	0.06
	-50	32.75	51.74	-1.11
b	-25	11.93	19.43	-0.45
(0.2)	+25	-7.86	-13.28	0.39
	+50	-13.50	-23.07	0.72
	-50	7.62	-13.88	-0.31
h	-25	2.33	-5.50	-0.10
(0.6)	+25	-1.00	4.24	0.05
	+50	-1.30	7.67	0.07
	-50	3.18	3.23	-0.14
h_1	-25	1.57	1.56	-0.07
(0.3)	+25	-1.47	-1.46	0.07
	+50	-2.89	-2.88	0.14

imperfect, leading to some scrap items. Thus, different holding costs for perfect and imperfect items were employed in the model. Secondly, shortages were allowed and were assumed to be completely backlogged in this study. Thirdly, we used an algebraic operation and a simple-to-use arithmetic-geometric mean inequality method to derive the optimal solution for the proposed model. Finally, numerical examples were given to verify the theoretical results and the sensitivity analysis of key model parameters was also performed. Some managerial insights were obtained as follows: (1) a higher value of production rate, P, causes higher values of backorder quantity and the annual expected inventory total cost, but a lower value of production lot size; (2) a higher value of setup cost, K, causes higher values of production lot size, backorder quantity, and annual expected inventory total cost; (3) a higher value of unit shortage cost, b, causes a higher value of annual expected inventory total cost, but lower values of production lot size and backorder quantity.

The model proposed in this article can be extended in several ways. For instance, we may extend the model to consider inflation rates and trade credits. In addition, we can relax the assumption that all imperfect items can be reworked and judge whether some imperfect items should be scraped before the rework process, in order to save the holding cost and repair cost for non-reworkable items. Furthermore, we may study the shipment policy for an EPQ model with multi-delivery and partial rework. Finally, we can consider that shortages are allowed and partially backlogged. The backorder rate is a decreasing function of the waiting time for the next replenishment.

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