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STABILITY FOR A NOVEL TIME-DELAY FINANCIAL HYPERCHAOTIC SYSTEM BY ADAPTIVE PERIODICALLY INTERMITTENT LINEAR CONTROL

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Abstract In this paper, we get a time-delay new financial hyperchaotic system by modifying an old financial hyperchaotic system. we study the stability of a time-delay financial hyperchaotic system via adaptive periodically intermittent linear control method. Stability is obtained by using Lyapunov stability theorem, adaptive update laws and differential inequalities. Moreover, some numerical simulations are performed to show the advantage of the applications of this method.

Keywords Time-delay financial hyperchaotic systems, adaptive periodically intermittent linear control, adaptive update laws.

MSC(2010) 34K20, 34C28, 34H10.

1. Introduction

During the past decades, the control and synchronization of chaos has been an important research topic and developed extensively. The synchronization and control of chaos has been studied widely due to its importance in theory and potential applications in various areas, such as secure communication, biological systems, mechanics, neural network, information science and so on [1, 6]. So far, there have been a lot of investigations on this subject. Many important fundamental results for the synchronization and control of nonlinear systems have been found by some researchers in the field of physics, engineering, biology and mathematics. Pecora and Carrol had found chaos synchronization and proposed a successful method to synchronize two identical chaotic systems with different initial conditions [2, 13].

To synchronize and stabilize chaotic systems, some important control methods have been put forward such as state feedback control [17], impulsive control [4, 19], sliding control [5–16], predictive feedback control [14], intermittent control [7, 20], et al. Recently, intermittent control of nonlinear system has drawn increasing interest in process control, ecosystem management and communication.

Comparing with continuous control method, intermittent control is more efficient when the system output is measured intermittently rather than continuously, and its cost is lower. It has been widely used in engineering fields for its practicability and ease of implementation in engineering control. In view of those merits, a lot of researches have been studied in recent years. As a discontinuous feedback control, the control time of intermittent is periodic. In every period, the time when the controller works is denoted as work time and the rest is regarded as rest time.

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In [8], authors got the synchronization conditions of chaotic systems with time delay with the method of intermittent linear state feedback control, Lyapunov function and linear matrix inequality. Huang et al. discussed parameter mismatches of a delayed chaotic system by using intermittent linear state feedback in [9]. In [10], authors investigated projective synchronization of a hyperchaotic system via periodically intermittent control, but in this paper, authors did not considerate time delay of hyperchaotic system. In [11], authors displayed an exponential stabilization and synchronization of time-varying delays neural networks via periodically intermittent control. Yu et al. studied synchronization of delay nonlinear systems via periodically nonlinear intermittent control in [18]. In [3], some simple criteria were derived for the exponential synchronization of complex dynamical networks under pinning periodically intermittent control. Last year, Sun et al. investigated synchronization of delayed complex dynamical networks via adaptive periodically intermittent control [15].

To the best of our knowledge, the above works have not researched on adaptive intermittent linear. Therefore, we construct an adaptive intermittent linear controller to analyze stabilization of hyperchaotic system.

Chinese and western economy occurred chaotic phenomena in 1985. Chaotic phenomena was shown in 2007 global economic crisis. Based on the existence of in financial market, we firstly constructed a financial hyperchaotic system in [17].

Control and synchronization of the financial chaotic or hyperchaotic system has significance [4]. With the development of economy, the financial system has attracted more and more attention. There exists a chaos phenomenon in economic and financial systems, which means that the system itself has intrinsic instability, and generally it is harmful to systems. However, the financial hyperchaotic system shows more complex dynamical behaviors.

In real life, financial system shows more sophisticated phenomenon and hence we modify the old system model in [17]. Accordingly, in this paper, we will give a novel time-delay financial hyperchaotic system and analyze its stabilization. Although many methods have been proposed to stabilize the financial hyperchaotic system like state feedback control. In the process of control, sample controllers are more effective in practical and ease of implement. In fact, the fewer the controllers, the lower the costs. Therefore, discontinuous control is much fitter for the realistic situations. Adaptive update laws stabilize the financial hyperchaotic system faster.

All in above, we will investigate stability of a new financial hyperchaotic system via adaptive periodically intermittent linear control. Then, we will give its stabilization criterion and simulation results with the method by using MATLAB.

The rest of this paper is organized as follows. In section 2, the novel time-delay financial hyperchaotic system is given. In section 3, adaptive periodically intermittent control scheme is introduced. In section 4, adaptive periodically intermittent control scheme of a time-delay financial hyperchaotic system for stability is shown. In section 5, numerical simulations are presented to verify the effectiveness of the theoretical results. Finally, the conclusions are drawn in section 6.

2. A time-delay financial hyperchaotic System

Yu et al. proposed a financial hyperchaotic system without time-delay in [17]. However, time-delay phenomena often happens in real practice. To show the complexity in real life, we add time-delay to the system in [17] and get a novel time-delay financial hyperchaotic system:

$$\begin{cases} \dot{x} = z + (y-a)x + w(t-\tau), \\ \dot{y} = 1 - by - x^2, \\ \dot{z} = -x - cz, \\ \dot{w} = -dxy - kw(t-\tau), \end{cases} \tag{2.1}$$

where the interest rate x, the investment demand y, the price exponent z, and the average profit margin w are the state variables and a, b, c, d, k are positive parameters,τ is the time-delay. If $\tau = 0$, the system (2.1) changes into the system in [17]. There are three unstable equilibrium points: $P_0(0, 1/b, 0, 0)$, $P_{1,2}(\pm\theta, \frac{k+ack}{c(k-d)}, \mp\frac{\theta}{c}, \frac{d\theta(1+ac)}{cd-ck})$,where $\theta = \sqrt{\frac{kb+abck}{c(d-k)} + 1}$. Letting parameters $a = 0.9, b = 0.2, c = 1.5, d = 0.2$ and $k = 0.17$ and $\tau = 0$, we calculated the four Lyapunov exponents with Wolf algorithm to be $L_1 = 0.034432, L_2 = 0.018041, L_3 = 0$ and $L_4 = -1.1499$. The Lyapunov exponents of system (2.1) is presented in Figure 1. When $\tau = 0$, the 3-dimensional phase portraits of financial hyperchaotic system (2.1) are shown in Figure 2 (a)-(d). More dynamics behaviors about system (2.1) would be found in [17].

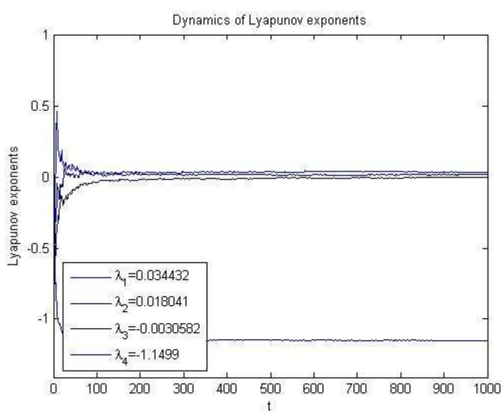

Figure 1. Lyapunov exponents spectrum of system (2.1) when $\tau = 0$.

Remark 2.1 (The background of financial hyperchaotic system in [17])**.** The system in [5] was constructed in the background of the global economic crisis occurred in 2007 and still exists nowadays. As this global economic crisis did not caused the great depression like in 1920s-1930s, so the system is a weak hyperchaotic finance system. Although the largest Lyapunov exponent of the system is relatively small, the hyperchaotic financial system exactly reflects the global economic crisis.

Remark 2.2 (The reason for adding time-delay to the old system in [17])**.** Time-delay phenomenon is often encountered in many practical control systems such as communication systems, electrical networks, financial systems, engineering systems and process control systems. Whenever time delay is presented in the systems, some erratic behavior could occur, bringing about oscillations, instability and poor

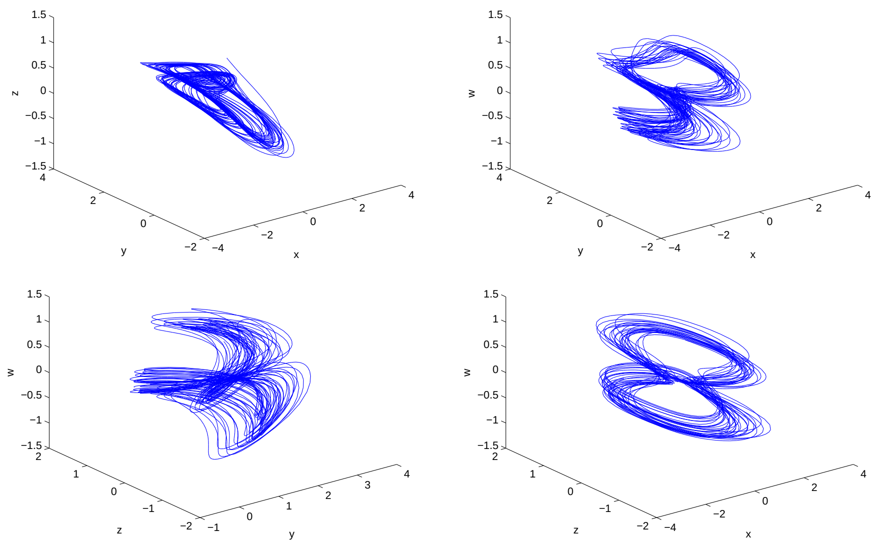

Figure 2. Phase portraits of hyperchaotic finance system(2.1) when $\tau = 0$.

performances. On the other hand, one can design the delayed feedback controller to improve the controlling effect. Therefore, the stability analysis and control design of time-delay systems are of great importance for both theoretical and practical reasons.

The financial system (2.1) is an extremely complex nonlinear system which is composed of many elements. In nonlinear financial systems, time delay is also a very important factor. Due to a lot of uncertainty, the general differential equations can not fully describe some economic phenomena. Therefore, time delay is added in Eq.(2.1).

3. Adaptive periodically intermittent control scheme

Consider a class of certain hyperchaotic system with time delay:

$$\dot{x} = Ax(t) + f(x(t)) + g(x(t-\tau)). \tag{3.1}$$

The controlled system is designed as:

$$\dot{x} = Ax(t) + f(x(t)) + g(x(t-\tau)) + u(t), \tag{3.2}$$

where $x \in R^n$ is the state vector of the systems (3.1) and (3.2), $A \in R^{n\times n}$, f, g: $R^{n\times n} \to R^{n\times n}$ are continuous nonlinear functions and $f(0) = 0, g(0) = 0$, τ is the time delay and $u(t)$ is the adaptive controller of the system (3.2). The $u(t)$ is a adaptive periodically intermittent controller and it is defined as:

$$u\left(t\right) = \begin{cases} -Kx(t), \quad nT \leq t \leq nT + \delta, \\ 0, \quad nT + \delta < t \leq \left(n+1\right)T, \end{cases} (i = 1,\ 2,\ 3,\ \cdots,\ n) \tag{3.3}$$

$$\dot{k}_i = \begin{cases} \lambda_i x_i^T x_i, & nT \le t \le nT + \delta, \\ 0, & nT + \delta < t \le (n+1)T, \end{cases} \quad (i = 1,\ 2,\ 3,\ \cdots,\ n)$$

where $u(t) = (u_1(t), u_2(t), ..., u_n(t))$, $K = (k_1(t), k_2(t), ..., k_n(t))$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$, $T > 0$ is the control period, and $\delta > 0$ is called the control width. Our goal is to design suitable δ, T, K such that the system (3.2) is stable.

Assumption 3.1. Let $f(x), g(x(t-\tau))$ are bounded functions, that is there exist constants matrices L and P, for any x, such that $\|f(x(t))\|^2 \le L\|x(t)\|^2$, with $L = diag\{l_1,\ l_2,\ \cdots,\ l_n\} \ge 0, \|g(x(t-\tau))\|^2 \le P\|x(t-\tau)\|^2$ with $P = diag\{p_1,\ p_2,\ \cdots,\ p_n\} \ge 0$.

Lemma 3.1 (Sanchez and Perez)**.** *For any vectors $x, y \in R^m$ and a positive-defined matrix $Q \in R^{m\times m}$, the matrix inequality $2x^T y \le x^T Q x + y^T Q^{-1} y$ holds.*

Lemma 3.2 (Huang, Li and Liu [8])**.** $V(t) \le M \exp(-\frac{\rho}{\omega}t),\ M = \|V(0)\|_\tau\, e^{(v_1+v_2)\omega} e^{\rho}$ *can be obtained, if there exist positive constants u_1, u_2, v_1, v_2 such that the following condition holds:*

(i) $\rho = r(\delta - \tau) - (v_1 + v_2)(\omega - \delta) > 0$,

(ii) $\begin{cases} \dot{V}(t) \le -u_1 V(t) + u_2 V(t-\tau), & n\omega \le t < n\omega + \delta, \\ \dot{V}(t) \le v_1 V(t) + v_2 V(t-\tau), & n\omega + \delta \le t < (n+1)\omega, \end{cases}$

where r is the unique positive solution to $-r = -u_1 + u_2 e^{r\tau}$, τ is the time delay, δ is the control width, ω is the control period.

Theorem 3.1 (Stabilization criterion)**.** *System (3.2) is stable if there exist positive constants $\alpha_1,\ \alpha_2,\ d_1, d_2, d_3$ and $d_1 > d_2$, such that the following conditions hold:*

(i) $A^T + A + \alpha_1^{-1} I + \alpha_2^{-1} I + \alpha_1 L + d_1 I \le 0$,

(ii) $A^T + A + \alpha_1^{-1} I + \alpha_2^{-1} I + \alpha_1 L - d_3 I \le 0$,

(iii) $\eta = r(\delta - \tau) - (d_2 + d_3)(T - \delta) > 0$,

where r is the unique positive solution to $-r = -d_1 + d_2 e^{r\tau}$, A is the matrix of system (3.1) and (3.2), A^T is the translation of matrix A, L is a positive constant in Assumption 3.1, I is the identity matrix.

Proof. Construct the following positive-defined Lyapunov function:

$$V(t) = x^T(t)x(t) + \sum_{i=1}^{n} \frac{k_i^2}{\lambda_i}. \tag{3.4}$$

Calculate the derivation $\dot{V}(t)$ with respect to time along the trajectories of the controlled system (3.2). For $nT \le t < nT + \delta$, using Assumption 3.1, Lemma 3.1 and condition (i) in Theorem 3.1, we get the following estimate:

$$\begin{aligned}
\dot{V}(t) &= \dot{x}^T(t)x(t) + x^T(t)\dot{x}(t) + 2\sum_{i=1}^{n} \frac{k_i}{\lambda_i}\dot{k}_i \\
&= (Ax(t) + f(x(t)) + g(x(t-\tau)) - Kx(t))^T x(t) + x^T(t)(Ax(t) + f(x(t)) \\
&\quad + g(x(t-\tau)) - Kx(t)) + 2\sum_{i=1}^{n} k_i x_i^2
\end{aligned}$$

$$\begin{aligned}
&= x^T(t)(A^T + A - K^T - K + 2K)x(t) + 2f^T(x(t))x(t) + 2g^T(x(t-\tau))x(t)\\
&\leq x^T(t)(A^T + A - K^T - K + 2K)x(t) + \alpha_2 g^T(x(t-\tau))g(x(t-\tau))\\
&\quad + \alpha_2^{-1}x^T(t)x(t) + \alpha_1 f^T(x(t))f(x(t)) + \alpha_1^{-1}x^T(t)x(t)\\
&\leq x^T(t)(A^T + A + \alpha_1^{-1} + \alpha_2^{-1})x(t) + \alpha_1 x^T(t)Lx(t)\\
&\quad + \alpha_2 x^T(t-\tau)Px(t-\tau)\\
&\leq x^T(t)(A^T + A + \alpha_1^{-1} + \alpha_2^{-1} + \alpha_1 L + d_1 I)x(t) + \alpha_2 PV(t-\tau) - d_1 V(t)\\
&\leq -d_1 V(t) + d_2 V(t-\tau).
\end{aligned}$$

Note that $d_1 > 0$, $d_2 = \alpha_2 P$ and P is a positive matrix in Assumption 3.1. Then we have

$$\dot{V}(t) \leq -d_1 V(t) + d_2 V(t-\tau), nT \leq t < nT + \delta. \tag{3.5}$$

For $nT + \delta \leq t < (n+1)T$, using Assumption 3.1, Lemma 3.1 and condition (ii) in Theorem 3.1, we get the following estimate:

$$\begin{aligned}
\dot{V}(t) &= \dot{x}^T(t)x(t) + x^T(t)\dot{x}(t)\\
&= (Ax(t) + f(x(t)) + g(x(t-\tau)))^T x(t) + x^T(t)(Ax(t) + f(x(t)) + g(x(t-\tau)))\\
&= x^T(t)(A^T + A)x(t) + 2f^T(x(t))x(t) + 2g^T(x(t-\tau))x(t)\\
&\leq x^T(t)(A^T + A)x(t) + \alpha_2 g^T(x(t-\tau))g(x(t-\tau)) + \alpha_2^{-1}x^T(t)x(t)\\
&\quad + \alpha_1 f^T(x(t))f(x(t)) + \alpha_1^{-1}x^T(t)x(t)\\
&\leq x^T(t)(A^T + A + \alpha_1^{-1} + \alpha_2^{-1} + \alpha_1 L - d_3 I)x(t) + \alpha_2 PV(t-\tau) + d_3 V(t)\\
&\leq d_3 V(t) + d_2 V(t-\tau).
\end{aligned}$$

Note that $d_3 > 0$, $d_2 = \alpha_2 P$ and P is a positive matrix in Assumption 3.1. Then we have

$$\dot{V}(t) \leq d_3 V(t) + d_2 V(t-\tau), nT + \delta \leq t < (n+1)T. \tag{3.6}$$

By Lemma 3.2, we have

$$V(t) \leq \bar{M} \exp \frac{-\eta t}{T}, t > 0, \tag{3.7}$$

where $\bar{M} = \parallel V(0) \parallel_\tau \exp[(d_2 + d_3)T]\exp(\eta), \eta = r(\delta - \tau) - (d_2 + d_3)(T - \delta) > 0$. By Eqs. (3.4) and (3.7), we have

$$\parallel x(t) \parallel^2 \leq \bar{M} \exp(\frac{-\eta t}{T}), t > 0.$$

Therefore, we obtained:

$$\parallel x(t) \parallel \leq \sqrt{\bar{M} \exp(\frac{-\eta t}{T})}, t > 0.$$

The above inequality implies that the controlled system is exponential stabilization. The proof is complete. □

Remark 3.1. Let λ be the largest eigenvalue of $A^T + A$. If we replace the first two conditions in Theorem 1 by the two scalar equalities $d_1^* = -\lambda - \alpha_1^{-1} - \alpha_2^{-1} - \alpha_1 L, d_3^* = \lambda + \alpha_1^{-1} + \alpha_2^{-1} + \alpha_1 L$, where $d_1^* > d_1, d_3^* > d_3$, it is easy to see that Theorem 3.1 also holds. The results are rewritten as the following corollary.

Corollary 3.1. *The controlled system is globally exponentially stable, if there exist a positive constant* P *and* $0 < \delta < 1$*, such that* $\eta = r(\delta - \tau) - (d_2 + d_3^*)(T - \delta) > 0$*, where* $d_1^* = -\lambda - \alpha_1^{-1} - \alpha_2^{-1} - \alpha_1 L, d_3^* = \lambda + \alpha_1^{-1} + \alpha_2^{-1} + \alpha_1 L$.

Based on Lyapunov stability theory, we studied stable in the controlled system. By using the adaptive intermittent linear control, the controlled system can be stabilized easily. Some sufficient conditions for stabilization via adaptive intermittent linear control are derived rigorously. So we can apply this method to control financial hyperchaotic system.

4. Stability of system (2.1) via adaptive periodically intermittent linear control

In this section, we stabilize the unstable equilibrium point $P_0(0, 1/b, 0, 0)$ of the time- delay financial hyperchaotic system by periodically intermittent linear control method. At first, we make a transformation of the financial hyperchaotic system(2.1) at the equilibrium point P_0. Thus we get a new hyperchaotic system, which has an unstable equilibrium point P_0'. Therefore, we can use Lyapunov stability theory and Theorem 3.1 to stabilize it. Let

$$\begin{cases} x_1 = x, \\ x_2 = y - \frac{1}{b}, \\ x_3 = z, \\ x_4 = w, \end{cases}$$

the system can be written as:

$$\begin{cases} \dot{x}_1 = x_3 + \left(\frac{1}{b} - a\right) x_1 + x_4(t - \tau) + x_1 x_2, \\ \dot{x}_2 = -b x_2 - x_1^2, \\ \dot{x}_3 = -x_1 - c x_3, \\ \dot{x}_4 = -\frac{d}{b} x_1 - k x_4(t - \tau) - d x_1 x_2. \end{cases} \tag{4.1}$$

The controlled hyperchaotic system is described as follows:

$$\begin{cases} \dot{x}_1 = x_3 + \left(\frac{1}{b} - a\right) x_1 + x_4(t - \tau) + x_1 x_2 + u_1, \\ \dot{x}_2 = -b x_2 - x_1^2 + u_2, \\ \dot{x}_3 = -x_1 - c x_3 + u_3, \\ \dot{x}_4 = -\frac{d}{b} x_1 - k x_4(t - \tau) - d x_1 x_2 + u_4, \end{cases} \tag{4.2}$$

where the controllers are designed as follows:

$$u_i(t) = \begin{cases} -k_i x_i(t), & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T, \end{cases} \qquad (i = 1,\ 2,\ 3,\ 4)$$

and the adaptive laws are given as:

$$\dot{k}_i = \begin{cases} \lambda_i x_i^2, & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T. \end{cases} \quad (i = 1,\ 2,\ 3,\ 4)$$

In system (4.2), we select parameters $a = 0.9, b = 0.2, c = 1.5, d = 0.2$ and $k = 0.17$. Therefore, the linear matrix $A = \begin{bmatrix} 4.1 & 0 & -1 & -1 \\ 0 & -0.2 & 0 & 0 \\ 1 & 0 & -1.5 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$, the nonlinear function vector $f(x) = (x_1x_2, -x_1^2, 0, -0.2x_1x_2)^T$, and the time-delay function vector $g(x(t-\tau)) = (x_4(t-\tau), 0, 0, -0.17x_4(t-\tau))^T$. We choose positive matrices $L = P = 0.5I, \alpha_1 = \alpha_2 = 1, d_1 = 1, d_2 = 0.5, d_3 = 1$, then system (4.2) satisfies all conditions in Theorem 3.1. From Theorem 3.1, system (4.2) is prove to be stabled at the equilibrium point $P_0'(0,0,0,0)$. Consequently, the system (2.1) is stable at the equilibrium point $P_0(0, \frac{1}{b}, 0, 0)$.

Unstable equilibrium point $P_{1,2}$ also stable by using adaptive intermittent control method. Those proofs omitted here because they are similar with that of stabilizing the unstable equilibrium point P_0.

5. Numerical simulations

To verify the results, some numerical simulations are performed. The financial hyperchaotic system (2.1) is rewritten as system (4.1) at unstable equilibrium point P_0. It shows chaotic behavior when the parameters are chosen as $a = 0.9, b = 0.2, c = 1.5, d = 0.2, k = 0.17$ and the initial values of the control system (4.1) are designed as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (-1, -2, -3, -4), \tau = 0.5$. Using MATLAB, time evolutions of (x_1, x_2, x_3, x_4) are system (4.1) is demonstrated in Figure 3.

To verify the effectiveness of adaptive intermittent linear control method, we choose system (4.2) as the controlled system. The parameters are chosen as $a = 0.9, b = 0.2, c = 1.5, d = 0.2, k = 0.17$. The initial values of the controlled system (4.2) are designed as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (4, 3, 2, 1)$ and the time delay is $\tau = 0.5$. The initial values of the unknown parameters are designed $\dot{k}_i = 1 (i = 1, 2, 3, 4)$ and $\lambda_1 = 12, \lambda_2 = 12, \lambda_3 = 12, \lambda_4 = 12$ respectively. Assuming the controllers are switched on the $T = 1s$ and $= 0.8s$. we get the time evaluation of states in the controlled system (4.2) with adaptive intermittent linear control, which is shown in Figure 4. Figure 5 presents the evolution of $\|x(t)\|$ of the system (4.2) with adaptive intermittent linear control. Figure 6 shows the estimations of adaptive gains $k_i (i = 1, 2, 3, 4)$ of financial hyperchaotic systems (4.2).

Comparing Figure 3 with Figure 4, we find that the time evaluations of $x_i(t) (i = 1, 2, 3, 4)$ of system (4.1) without controller present irregular phenomenon in Figure 3, in other words, system (4.1) shows chaotic behavior. However, it is stabilized quickly by adding adaptive intermittent linear control (see Figure 4). It verifies the feasibility and effectiveness of adaptive intermittent linear control method.

In order to illustrate the advantage of the adaptive intermittent control method, we give a numerical simulation of system (4.2) with intermittent linear control

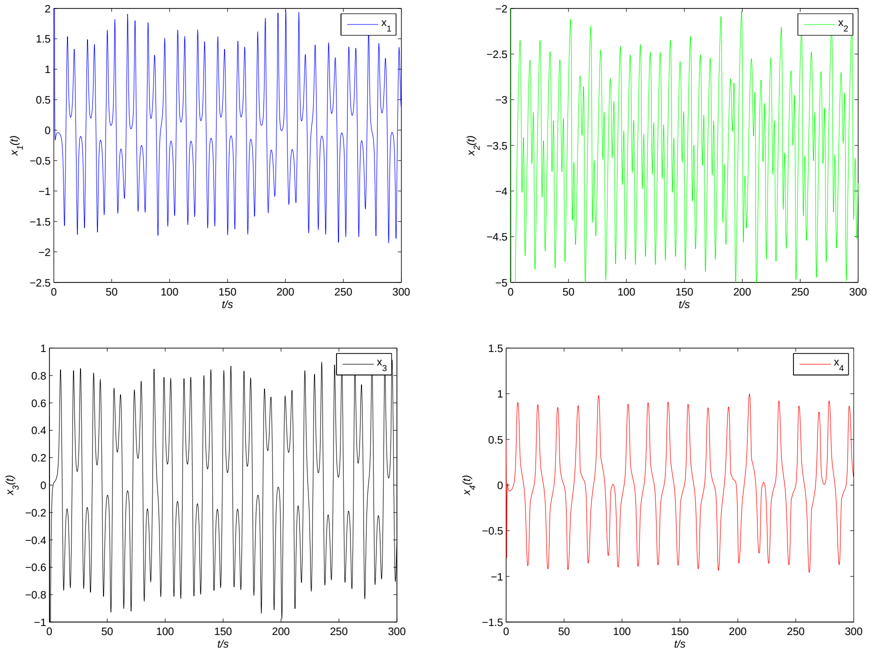

Figure 3. Time evolutions of x_1, x_2, x_3, x_4 of system (4.1).

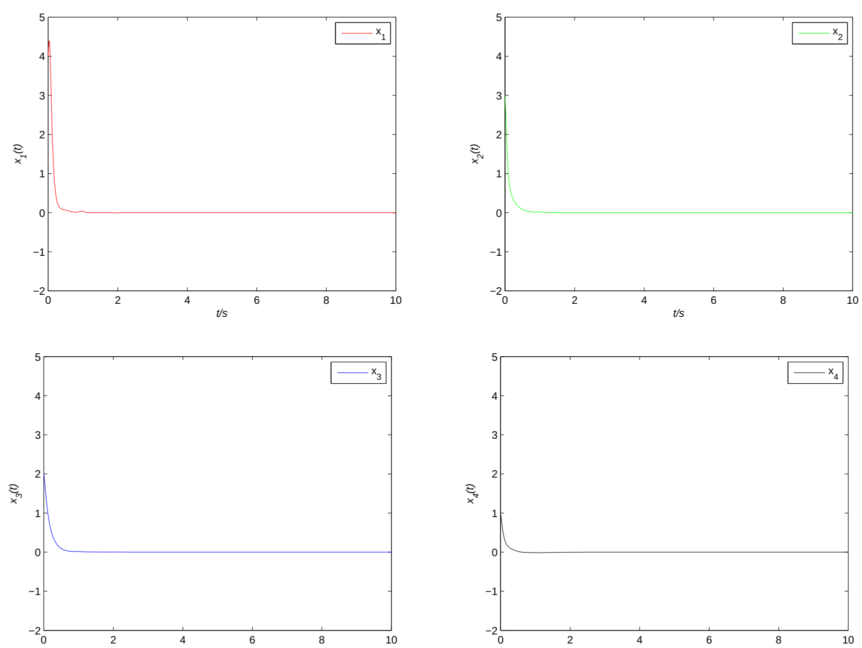

Figure 4. Time evolutions of x_1, x_2, x_3, x_4 of system (4.2)with adaptive intermittent linear control.

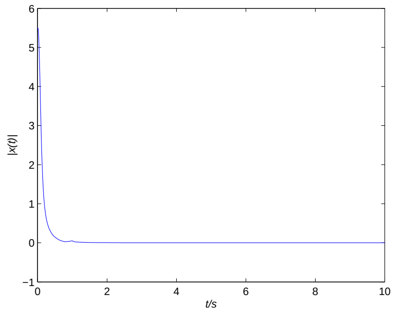

Figure 5. Time evolutions of $|x(t)|$ of system (4.2)with adaptive intermittent linear control.

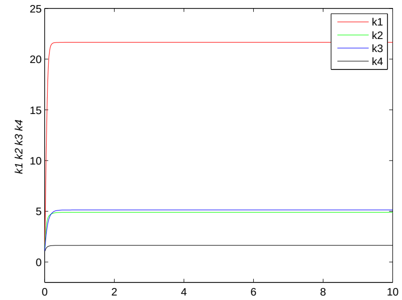

Figure 6. Time evolutions of adaptive gain $k_i, (i = 1, 2, 3, 4)$ of the system (4.2) with adaptive intermittent linear control.

and make a comparison between intermittent linear control method and adaptive intermittent linear control. We also choose system (4.2) as the controlled system. The parameters are chosen as $a = 0.9, b = 0.2, c = 1.5, d = 0.2, k = 0.17$. The initial values of the controlled system are also chosen as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (4, 3, 2, 1)$ and the time delay is $\tau = 0.5$. The control gains are designed as $k_i = 4(i = 1, 2, 3, 4)$. Assuming the controllers are switched on $T = 1s$ and $= 0.8s$, we get the time evaluation of states in the controlled system (4.2) with intermittent linear control (see Figure 7).

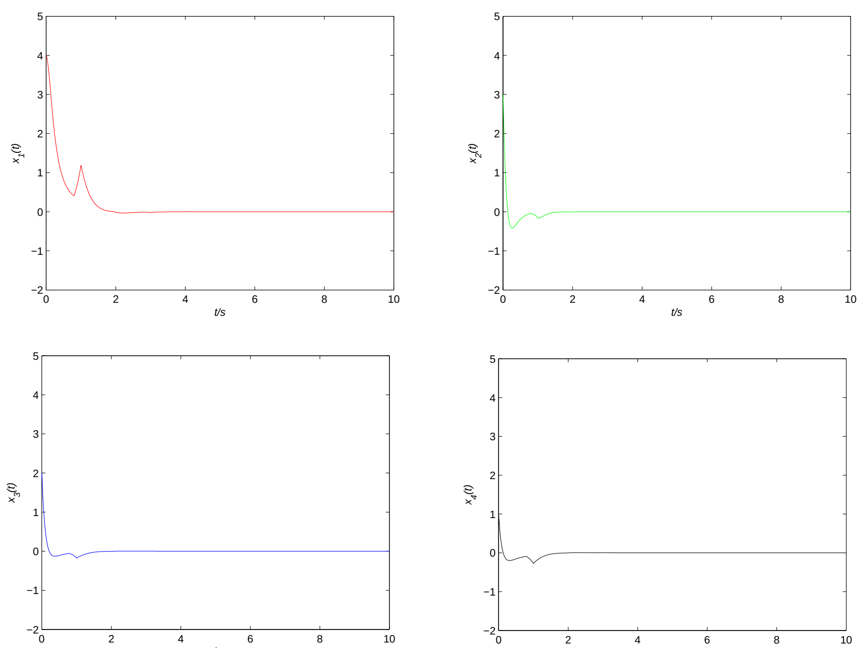

Figure 7. Time evolutions of x_1, x_2, x_3, x_4 of system (4.2)with intermittent linear control.

From Figures 4 and 7, it is easy to find that the states with adaptive intermittent linear control method have smaller fluctuation than those with intermittent linear control method. It is obvious that the states of controlled system are stabilized faster under the adaptive intermittent linear method than those only under intermittent linear control.

All the above results show that the advantage, correctness and effectiveness of adaptive intermittent linear control method. As we all know, intermittent control is a discontinuous control. Because controllers does not work in the full time period, it needs fewer controllers in real process than continuous control like adaptive control, and it saves much cost in real process. Therefore, the adaptive intermittent control method is superior to others not only on convergence velocity but also on the cost.

Remark 5.1. Adaptive controllers have the merit of simple design, but continuous control needs more energy. Intermittent controllers can save much energy with simple design and less economy consumption. The new adaptive intermittent controller constituted in this study integrates the advantages of adaptive controller and intermittent controller. Therefore, the proposed method in this study can be applied to many fields, such as secure communication and commercial systems.

6. Conclusion

In this paper, the adaptive intermittent control method of a time-delay financial hyperchaotic system has been analyzed. With less conservative conditions, the controlled system could be stable at its equilibrium points by Lyapunov stability theory and differential inequalities. Adaptive intermittent control speeds up stabilization velocity of the system. Besides, adaptive update laws are proposed to m stabilize the control gain. Finally, numerical simulations have been presented to verify effectiveness and correctness of adaptive intermittent control method.

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