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STABILITY ANALYSIS OF AN ENTERPRISE COMPETITIVE MODEL WITH TIME DELAY*

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Abstract A three-dimensional enterprise competitive model with time delay is considered. Where the delay is regarded as bifurcation parameters. By analyzing the corresponding characteristic equation of positive equilibrium,the local stability of positive equilibrium is regarded. By using the normal form method and center manifold theorem, we give the formula for determining the direction of the Hopf bifurcation and the stability of bifurcating periodic solutions. Numerical simulations are shown to illustrate the obtained results.

Keywords Hopf bifurcation, stability, delay, enterprise competitive model, periodic solutions.

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1. Introduction

In recent years, in the fields of economic, the changes of enterprises' number caused by their competition are becoming more and more highlighted. Using the Ecological Theory to study the principle of enterprises' competition has become an important method. An increasing number of scholars are studying this issues, and have made some achievements. Hannan [3] synthesized the basic of Organizational Ecology and developed it, at the same time, he put forward the mathematical model for measuring enterprise's development, transition and succession. Moore's [6] Business Ecosystem Coevolution argued that, in the context of world economy mutually melting and environment increasingly deteriorating, enterprises should formulate their development strategy in an enterprise ecosystem perspective.

As an essential part of national economic system, investment enterprise has an important contribution to macro-Economics' development, decompressing of employment and stability of the society. Development of investing enterprise depends on competing for projects's competition. Their relationship is very similar to the relationship between predator and prey in the predator-prey models. Meanwhile, competitions also exist among investment enterprises and this competitions are similar to the relationship between two kinds of predators in predator-prey models. This similarities provide a new way for us to study the investment behaviors. In this paper, we consider the delay Enterprise Competive Model beside on the predator-prey model in ecology and analyzed the stability of this model.

Hypotheses of this model are as follows:

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(1) Enterprise cluster in one area can be regarded as a a population of ecosystem, and investment enterprise number per unit area can be accurately described by a variable.

(2) In the competition of investment market, acquirement of investing items is the main aspect of enterprise's competition for their source.

(3) According to investing items' geographical scope, we classify these enterprises into two parts, i.e., big investment enterprises and small and medium-size investment enterprise.

(4) Entrepreneurs are rational to considerate that there will be few competitions with other investment enterprises nearby their company.

(5) In one area, if investment items are more enough, entrepreneurs set up to establish investment enterprise. And suppose this process will cost $\tau(\tau > 0)$ times.

(6) Based on Hasting's predator-prey mode ([1, 4]), Liu Kai [5] built the model to describe the relationship of investment enterprises and investment items.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = (a - \mu x)x - d(z + y)x, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = cx(t - \tau)(z + y) - (m + e)y, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = my - bz, \end{cases} \tag{1.1}$$

where x, y, z present invest enterprises number per unit area of investment item, small and medium-size invest enterprise and big investment enterprise, respectively. Besides the natural tension ax, one area's investing item number suffers from the feedback from x's increase which can be represented by $-bx^2$. It should also satisfy the S curve in the long run. Logistic equation $x(a - \mu x)$ here represents investment item's variation with time t. Where a, μ and d respectively represent investment item's growth rate, inhibiting coefficient and coefficient that investment item fall prey to investment enterprise. Besides, c is the growth rate of small and medium-size investment enterprise along with the increase of the investment project, $x(t - \tau)$ is the investment item's number of accumulated over time τ, e is the coefficient that small and medium-size investment enterprise out of this industry, m is the coefficient of proportional small and medium-size investment enterprise transformed into big investment enterprise, $m + e$ is the reducing ratio of small and medium-size investment enterprise, b is the coefficient that big investment enterprise out of industry. All the coefficients above are positive.

Reference [5] considered the local stability, global stability and permanence of the equilibrium of system (1.1). But it ignored the limitation of market self-regulation. Therefore, it can not describe economic activities practically. In view of this, this paper consider the macroeconomic control effects in system (1.1), and obtain a new delayed enterprise competive model.

2. Stability analysis and Hopf bifurcation

In this paper, we focus our attention on the new competitive model with time delay which is described by

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = (a - \mu x)x - d(z + y)x, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = cx(z + y) - my - ey, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = my - bz + K[z - z(t - \tau)], \end{cases} \tag{2.1}$$

where $z(t-\tau)$ is the number of small and medium-size investment enterprise transformed into big investment enterprise accumulated over time τ. $K[z - z(t - \tau)]$ is the state's adjustment for business economic. $K > 0$ is the growth factor of support for the development of the big investment enterprise. On the contrary, $K < 0$ is the restraint coefficient of restraining the development of the big investment enterprise.

Clearly, system (2.1) always has a unique positive equilibrium $E^*(x^*, y^*, z^*)$, where

$$\begin{aligned} E^* &= (x^*, y^*, z^*) \\ &= \left(\frac{b(m+e)}{c(m+b)}, \frac{b(acm + abc - bm\mu - be\mu)}{cd(b+m)^2}, \frac{m(acm + abc - bm\mu - be\mu)}{cd(b+m)^2}\right). \end{aligned}$$

By analyzing the characteristic equation of the linearized system of system (2.1) at the positive equilibrium, we investigate the stability of the positive equilibrium and the existence of the local Hopf bifurcations occurring at the positive equilibrium.

The linearized system of (2.1) is

$$\dot{u} = Au(t) + Bu(t - \tau), \tag{2.2}$$

where

$$\begin{aligned} &u(t) = (x, y, z)^T, \quad A = (a_{ij})_{3\times 3}, \quad B = (b_{ij})_{3\times 3}, a_{11} = a - 2ux^* - dz^* - dy^*, \\ &a_{12} = a_{13} = -dx^*, a_{21} = c(y^* + z^*), \quad a_{22} = cx^* - m - e, \quad a_{23} = cx^*, \\ &a_{32} = m, \quad a_{33} = -b + K, \quad b_{33} = -K, \end{aligned}$$

all the others of a_{ij} and b_{ij} are 0.

The characteristic equation of system (2.2) is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 + \left(K\lambda^2 + a_4\lambda + a_5\right)e^{-\lambda\tau} = 0, \tag{2.3}$$

where

$$\begin{aligned} &a_1 = -a_{11} - a_{22} - a_{33}, \quad a_2 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{23}a_{32} - a_{12}a_{21}, \\ &a_3 = a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} - a_{11}a_{22}a_{33} - a_{13}a_{21}a_{32}, \\ &a_4 = -K(a_{11} + a_{22}), \quad a_5 = K(a_{11}a_{22} - a_{12}a_{21}). \end{aligned}$$

The equilibrium $E^*(x^*, y^*, z^*)$ is stable if all roots of (2.3) have negative real parts. Thus, we need to investigate the distribution of roots of Eq. (2.3). Obviously, $i\omega(\omega > 0)$ is a root of Eq. (2.3) if and only if ω satisfies

$$-i\omega^3 - a_1\omega^2 + a_2 i\omega + a_3 + \left(-K\omega^2 + a_4 i\omega + a_5\right)(\cos\omega\tau - i\sin\omega\tau) = 0. \tag{2.4}$$

Separating the real and imaginary parts, we have

$$\begin{cases} -\omega^3 + a_2\omega = -K\omega^2 \sin\omega\tau - a_4 w \cos\omega\tau + a_5 \sin\omega\tau, \\ -a_1\omega^2 + a_3 = K\omega^2\cos\omega\tau - a_4\omega\cos\omega\tau - a_5\sin\omega\tau, \end{cases} \tag{2.5}$$

which implies

$$\omega^6 + \left(a_1^2 - 2a_2 - K^2\right)\omega^4 + \left(a_2^2 - 2a_1a_3 + 2a_5K - a_4^2\right)\omega^2 + a_3^2 - a_5^2 = 0. \tag{2.6}$$

Let $z = \omega^2$ and denote

$$p = a_1^2 - 2a_2 - K^2, \quad q = a_2 - 2a_1a_3 + 2a_5K - a_4^2, \quad r = a_3^2 - a_5^2. \tag{2.7}$$

Then, Eq. (2.6) becomes

$$z^3 + pz^2 + qz + r = 0. \tag{2.8}$$

Denote

$$h(z) = z^3 + pz^2 + qz + r. \tag{2.9}$$

Hence, we have the following lemma.

Lemma 2.1. *For the polynomial Eq.* (2.9), *we have the following results.*

(i) If $r < 0$, Eq. (2.9) *has at least one positive root.*

(ii) If $r < 0$, Eq. (2.9) *has at least one positive root if and only if there exists a $z^* > 0$,such that $h'(z^*) = 0$ and $h(z^*) \geq 0$.*

Suppose that the Eq. (2.9) has positive roots. Without loss of generality, we assume that it has three positive rootsdenoted by z_1, z_2, z_3, respectively. Then

$$\omega_1 = \sqrt{z_1}, \quad \omega_2 = \sqrt{z_2}, \quad \omega_3 = \sqrt{z_3}.$$

According to (2.5), we have

$$\cos\omega\tau = \frac{a_4\omega^2(\omega^2 - a_2) + (a_1\omega^2 - a_3)(a_5 - K\omega^2)}{a_4^2\omega^2 + (K\omega^2 - a_5)^2}.$$

Thus, if we denote

$$\tau_k^{(j)} = \frac{1}{\omega_k}\left\{\cos^{-1}\left(\frac{a_4\omega_k^2(\omega_k^2 - a_2) + (a_1\omega_k^2 - a_3)(a_5 - K\omega_k^2)}{a_4^2\omega_k^2 + (K\omega_k^2 - a_5)^2}\right) + 2\pi j\right\}, \tag{2.10}$$

where $k = 1, 2, 3$; $j = 0, 1, 2, ...$, then $\pm i\omega_k$ is a pair of purely imaginary roots of Eq. (2.3) with τ_k^j. Define

$$\tau_0 = \tau_{k_0}^{(0)} = \min_{k\in\{1,2,3\}}\left\{\tau_\kappa^{(0)}\right\}, \quad \omega_0 = \omega_{\kappa_0}. \tag{2.11}$$

Note that when $\tau = 0$, Eq. (2.3) becomes

$$\lambda^3 + (K + a_1)\lambda^2 + (a_2 + a_4)\lambda + a_3 + a_5 = 0. \tag{2.12}$$

Till now, we can employ a result from Ruan and Wei [8] to analyze Eq. (2.3), which is stated as follows.

Lemma 2.2. *Consider the exponential polynomial*

$$\begin{aligned} &P\left(\lambda, e^{-\lambda\tau^1}, e^{-\lambda\tau^2}, ..., e^{-\lambda\tau^m}\right) \\ =&\lambda^n + P_1^{(0)}\lambda^{n-1} + \cdots + P_{n-1}^{(0)}\lambda + P_n^{(0)} + \left[P_1^{(1)}\lambda^{n-1} + \cdots + P_{n-1}^{(1)}\lambda + P_n^{(1)}\right] e^{-\lambda\tau_1} \\ &+ \cdots + \left[P_1^{(m)}\lambda^{n-1} + \cdots + P_{n-1}^{(m)}\lambda + P_n^{(m)}\right] e^{-\lambda\tau_m}, \end{aligned}$$

where $\tau_i \geq 0 \quad (i = 1, 2, 3, ..., m)$ *and* $P_j^{(i)} (i = 0, 1, 2, ...m; j = 1, 2, ..., n)$ *are constants.*

As $(\tau_1, \tau_2, ..., \tau_m)$ *vary, the sum of the order of the zeros of* $P(\lambda, e^{-\lambda\tau_1}, ..., e^{-\lambda\tau_m})$ *on the open right half plane can change only if a zero appears on or crosses the imaginary axis.*

By Lemma 2.1 and 2.2, we can obtain the following results on the distribution of roots of the transcendental Eq. (2.3).

Lemma 2.3. *Suppose that* $z_k = \omega_k^2$ *and* $h'(z_k) \neq 0$, *where* $h(z)$ *is defined by* (2.9). *Then,* $\dfrac{d(\mathrm{Re}\lambda(\tau_k^{(i)}))}{d\tau} \neq 0$ *and* $\dfrac{d(\mathrm{Re}\lambda(\tau_k^{(j)}))}{d\tau}$ *has the same sign with* $h'(z_k)$.

Proof. Substituting $\lambda(\tau)$ into Eq. (2.3) and differentiating the resulting equation in τ, we obtain

$$\begin{aligned} &\left\{3\lambda^2 + 2a_1\lambda + a_2 + \left[2K\lambda + a_4 - \tau\left(K\lambda^2 + a_4\lambda + a_5\right)\right] e^{-\lambda\tau}\right\} \frac{d\lambda}{d\tau} \\ =&\lambda\left(K\lambda^2 + a_4\lambda + a_5\right) e^{-\lambda\tau}, \end{aligned}$$

and then

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{\left(3\lambda^2 + 2a_1\lambda + a_2\right) e^{\lambda\tau}}{\lambda\left(K\lambda^2 + a_4\lambda + a_5\right)} + \frac{2K\lambda + a_4}{\lambda\left(K\lambda^2 + a_4\lambda + a_5\right)} - \frac{\tau}{\lambda}. \tag{2.13}$$

It follows from (2.5) that

$$\begin{aligned} &\left[\lambda\left(K\lambda^2 + a_4\lambda + a_5\right)\right]_{\tau=\tau_k^{(j)}} = -iK\omega_k^3 - a_4\omega_k^2 + ia_5\omega_k, \\ &\left[\left(3\lambda^2 + 2a_1\lambda + a_2\right) e^{\lambda\tau}\right]_{\tau=\tau_k^{(j)}} = \left(a_2 - 3\omega_k^2 + 2a_1 i\omega_k\right)\left(\cos\omega_k\tau_k^{(j)} + i\sin\omega\omega_k\tau_k^{(j)}\right), \\ &\left[2K\lambda + a_4\right]_{\tau=\tau_k^{(j)}} = a_4 + 2Ki\omega_k. \end{aligned} \tag{2.14}$$

From (2.13), (2.14) and (2.7), we can obtain

$$\begin{aligned} &\left[\frac{\mathrm{Re}\ d(\lambda(\tau))}{d\tau}\right]^{-1}_{\tau=\tau_k^{(j)}} \\ =&\mathrm{Re}\left[\frac{\left(3\lambda^2 + 2a_1\lambda + a_2\right) e^{\lambda\tau}}{\lambda\left(K\lambda^2 + a_4\lambda + a_5\right)}\right] + \mathrm{Re}\left[\frac{2K\lambda + a_4}{\lambda\left(K\lambda^2 + a_4\lambda + a_5\right)}\right] \\ =&\frac{1}{\Lambda}\left\{a_4\omega_k^2(3\omega_k - a_2)\cos\omega_k\tau_k^{(j)} - (3\omega_k^2 - a_2)(a_5\omega_k - K\omega_k^3)\sin\omega_k\tau_k^{(j)}\right. \\ &\left. -2\omega_k(a_5\omega_k - K\omega_k^3)(a_1\cos\omega_k\tau_k^{(j)} + 2K) + 2a_1a_4\omega_k^3\sin\omega_k\tau_k^{(j)} - a_4^2\omega_k^2\right\} \end{aligned}$$

$$=\frac{1}{\Lambda}\left\{3\omega_k^6+2(a_1^2-2a_2-K^2)\omega_k^4+(a_2^2-2a_1a_3+2a_5K-a_4^2)\omega_k^2\right\}$$
$$=\frac{1}{\Lambda}\left\{3\omega_k^6+2p\omega_k^4+q\omega_k^2\right\}=\frac{\omega_k^2}{\Lambda}\left\{3\omega_k^4+2p\omega_k^2+q\right\}=\frac{z_k}{\Lambda}h'(z_k),$$

where $\Lambda=a_4^2\omega_k^4+(K\omega_k^3-a_5\omega)^2$. Thus, we have

$$\text{sign}\left[\frac{\text{Re }d(\lambda(\tau))}{d\tau}\right]_{\tau=\tau_k^{(j)}}=\text{sign}\left[\frac{\text{Re }d(\lambda(\tau))}{d\tau}\right]^{-1}_{\tau=\tau_k^{(j)}}=\frac{z_k}{\Lambda}h'(z_k)\neq 0,$$

where $\Lambda, z_k>0$. We conclude that $\dfrac{\text{Re }d(\lambda(\tau))}{d\tau}$ has the same sign with $h'(z_k)$. This completes the proof. □

Note that when $\tau=0$, Eq. (2.3) becomes Eq. (2.12), Routh-Hurwitz criterion implies that

(H_1) if $K+a_1>0$, $a_3+a_5>0$ and $(K+a_1)(a_2+a_4)-(a_3+a_5)>0$, all roots of Eq. (2.3) with $\tau=0$ have negative real parts.

(H_2) if $K+a_1>0$, $a_3+a_5>0$ and $(K+a_1)(a_2+a_4)-(a_3+a_5)<0$, Eq. (2.12) have one negative real root and one pair of conjugate complex roots with positive real parts.

3. Direction and stability of the Hopf bifurcation

In this section, we obtain the conditions under which a family of periodic solutions bifurcate from the steady state at the critical value of τ. Following the ideals of Hassard et al. ([1,2,7,9,10]) by the normal form and the center manifold theory we derive the explicit formula for determining the properties of the Hopf bifurcation at the critical value of τ.

For the sake of simplicity of notation, we denote the critical values $\tau=\tau_k=k_j^{(k)}$, and denote the pair of purely imaginary roots of Eq. (2.3) as $\pm i\omega_k$.

Let $X=x-x^*$, $Y=y-y^*$, $Z=z-z^*$, then the system (2.1) can be written as

$$\begin{cases}\dfrac{\mathrm{d}X}{\mathrm{d}t}=(a-2\mu x^*-dz^*-dy^*)X-dx^*(Z+Y),\\ \dfrac{\mathrm{d}Y}{\mathrm{d}t}=c(z^*+y^*)X-(m+e)Y+cx^*(Z+Y),\\ \dfrac{\mathrm{d}Z}{\mathrm{d}t}=mY-bZ+K\left[Z-Z(t-\tau)\right].\end{cases}\tag{3.1}$$

Let $\mu=\tau-\tau_k$, then $\mu=0$ is the Hopf bifurcation value of system (3.1). Let $t=\tau t$, then the system (3.1) can be rewritten as a functional differential equation in $\mathbb{C}\left([-1,0],\mathbb{R}^2\right)$,

$$\dot{x}(t)=L_\mu(x_t)+f(\mu,x_t),\tag{3.2}$$

where $x(t)=(x_1(t),x_2(t),x_3(t))^T\notin\mathbb{R}^3$ and $L_\mu:\mathbb{C}\to\mathbb{R}^3$, $f:\mathbb{R}\times\mathbb{C}\to\mathbb{R}^3$ are given respectively by

$$
\begin{aligned}
&L_\mu(\Phi)\\
&= (\tau_0+\mu)\begin{pmatrix} a-2\mu x^*-dz^*-dy^* & -dx^* & -dx^* \\ c(z^*+y^*) & cx^*-(m+e) & cx^* \\ 0 & m & -b+K \end{pmatrix}\begin{pmatrix}\phi_1(0)\\ \phi_2(0)\\ \phi_3(0)\end{pmatrix}\\
&\quad+(\tau_0+\mu)\begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -K\end{pmatrix}\begin{pmatrix}\phi_1(-1)\\ \phi_2(-1)\\ \phi_3(-1)\end{pmatrix}
\end{aligned}
\tag{3.3}
$$

and

$$f(\tau,\phi)=(\tau_0+\mu)\begin{pmatrix} -\mu\phi_1^2(0)-d\phi_1(0)(\phi_2(0)+\phi_3(0))\\ c\phi_1(0)(\phi_2(0)+\phi_3(0))\\ 0\end{pmatrix}.$$

From the discussions in Section 2, we know that if $\mu=0$, then system (3.2) undergoes a Hopf bifurcation at the positive equilibrium E^* and the associated characteristic equation of system (3.2) has a pair of simple imaginary roots $\pm i\tau^{(j)}\omega_0$.

By the Riesz representation theorem, there exists a function $\rho(\theta,\mu)$ of bounded variation for $\theta\in[-1,0]$, such that

$$L_\mu\phi=\int_{-1}^{0}\mathrm{d}\rho(\theta,0)\phi(0),\quad \phi\in\mathbb{C}.$$

That is to say

$$
\begin{aligned}
\rho(\theta,\mu)&=(\tau_0+\mu)\begin{pmatrix} a-2\mu x^*-dz^*-dy^* & -dx^* & -dx^* \\ c(z^*+y^*) & cx^*-(m+e) & cx^* \\ 0 & m & -b+K \end{pmatrix}\sigma(\theta)\\
&\quad+(\tau_0+\mu)\begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -K\end{pmatrix}\sigma(\theta+1),
\end{aligned}
$$

where σ is Dirac-delta function.

For $\phi\in\mathbb{C}^1([-1,0],\mathbb{R}^3)$

$$A(\mu)\phi=\begin{cases}\dfrac{\mathrm{d}\phi(\theta)}{\mathrm{d}\theta}, & \theta\in[-1,0),\\ \int_{-1}^{0}\mathrm{d}\rho(\mu,s)\phi(s), & \theta=0,\end{cases}$$

and

$$R(\mu)\phi=\begin{cases}0, & \theta\in[-1,0),\\ f(\mu,\phi), & \theta=0.\end{cases}$$

Then, when $\theta \in [-1, 0)$ and $x_t(\theta) = x(t+\theta)$, system (3.1) is is equivalent to

$$\dot{x}(t) = A(\mu)x_t + R(\mu)x_t. \tag{3.4}$$

For $\psi \in \mathbb{C}^1\left([0,1], (\mathbb{R}^3)^*\right)$, define

$$A^*\psi(s) = \begin{cases} -\dfrac{\mathrm{d}\psi(s)}{\mathrm{d}s}, & s \in [-1, 0), \\ \int_{-1}^{0} \mathrm{d}\rho^T(t,0)\psi(-t), & s = 0, \end{cases}$$

and a bilinear inner product

$$\langle \psi(s), \phi(\theta) \rangle = \dot{\psi}(0)\phi(0) - \int_{-1}^{0}\int_{\varepsilon=0}^{\theta} (\varepsilon - \theta)\mathrm{d}\rho(\theta)\phi(\varepsilon)\mathrm{d}\varepsilon, \tag{3.5}$$

where $\rho(\theta) = \rho(\theta, 0)$. Denote $A = A(0)$, then, A and A^* are adjoint operators.

From the previous section, we know that $\pm\omega\tau_0$ are eigenvalues of $A(0)$. Thus, they are also eigenvalues of A^*.

Suppose that $q(\theta) = D(1, \beta^*, \gamma^*)^T e^{is\omega\tau_0}$ is the eigenvector of $A(0)$ corresponding to $i\omega\tau_0$, then $A(0)q(\theta) = i\omega\tau_0 q(\theta)$. It follows from the definition of $A(0)$ and $\rho(\theta,\mu)$ that

$$\tau_0 \begin{pmatrix} -a_{11} + i\omega & -\, a_{12} & -\, a_{13} \\ -a_{21} & i\omega - a_{22} & -\, a_{23} \\ 0 & -\, a_{32} & i\omega - a_{33} + Ke^{i\omega\tau_0} \end{pmatrix} q(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

which yields

$$\begin{aligned} &q(\theta) = (1, \beta, \gamma)^T \\ =&\left(1, -\frac{a_{11}a_{23} - a_{12}a_{21} - a_{23}i\omega}{a_{12}(i\omega + a_{23} - a_{22})}, \frac{i\omega - a_{11}}{a_{12}} + \frac{a_{11}a_{23} - a_{12}a_{21} - a_{23}i\omega}{a_{12}(i\omega + a_{23} - a_{22})}\right)^T. \end{aligned}$$

Similarly, it can be verified that $q^*(0) = D(1, \beta^*, \gamma^*)^T e^{is\omega\tau_0}$ is the eigenvector of A^* corresponding to $-i\omega\tau_0$, then

$$q^*(0) = D(1, \beta^*, \gamma^*)^T = D\left(1, -\frac{a_{11} + i\omega}{a_{21}}, \frac{(a_{11} + i\omega)(a_{22} + i\omega) - a_{12}a_{21}}{a_{21}a_{32}}\right)^T.$$

By (3.5), we have

$$\begin{aligned} &\langle q(s), q(0) \rangle \\ =&\bar{D}(1, \bar{\beta^*}, \bar{\gamma^*})(1, \beta, \gamma)^T \\ &- \int_{-1}^{0}\int_{\xi=0}^{\theta} \bar{D}(1, \bar{\beta^*}, \bar{\gamma^*})e^{-i(\xi-\theta)\omega\tau_0}\mathrm{d}\rho(\theta)(1, \beta, \gamma)^T e^{-i(\xi-\theta)\omega\tau_0}\mathrm{d}\xi \\ =&\bar{D}\left(1 + \beta\bar{\beta^*} + \gamma\bar{\gamma^*} - \int_{-1}^{0}(1, \bar{\beta^*}, \bar{\gamma^*})\theta e^{i\theta\omega\tau_0}\mathrm{d}\rho(\theta)(1, \beta, \gamma)^\gamma\right) \\ =&\bar{D}\left(1 + \beta\bar{\beta^*} + \gamma\bar{\gamma^*} - K\gamma\bar{\gamma^*}\tau_0 e^{-i\omega\tau_0}\right). \end{aligned}$$

Thus, we choose $\bar{D} = \dfrac{1}{1 + \beta\bar{\beta^*} + \gamma\bar{\gamma^*} - K\gamma\bar{\gamma^*}\tau_0 e^{-i\omega\tau_0}}$, such that $\langle q(s), q(\theta) \rangle = 1$.

In the following, we follow the ideas in Hassard et al. [2] and and by using the same notations as there to compute the coordinates describing the center manifold C_0 at $\mu = 0$.

On the center manifold C_0, define

$$Z(t) = \langle q^*, x_t \rangle, \quad W(t, \theta) = x_t - 2\mathrm{Re}\{z(t)q(\theta)\}. \tag{3.6}$$

We have

$$W(t, \theta) = (z(t), \bar{z}(t), \theta)$$

and

$$W(z, \bar{z}, \theta) = W_{20}(\theta)\frac{z^2}{2} + W_{11}(\theta)z\bar{z} + W_{02}(\theta)\frac{\bar{z}^2}{2} + \cdots,$$

where z and $\bar{z}$ are local coordinates for center manifold C_0 in the direction of q^* and $\bar{q^*}$. Note that W is real if x_t is real. We consider only real solutions. For the solution $x_t \in C_0$ of (3.5), since $\mu = 0$, we have

$$\begin{aligned}\dot{z}(t) =& i\tau_0\omega z + \langle \bar{q^*}(0), f(0, W(z, \bar{z}, \theta)) + 2\mathrm{Re}\{zq(\theta)\}\rangle \\ =& i\tau_0\omega z + \bar{q^*}(0)f(0, W(z, \bar{z}, \theta)) + 2\mathrm{Re}\{zq(\theta)\} \\ =& i\tau_0\omega z + \bar{q^*}(0)f_0(z, \bar{z}),\end{aligned}$$

where

$$g(z, \bar{z}) = \bar{q^*}(0)f_0(z, \bar{z}) = g_{20}\frac{z^2}{2} + g_{11}z\bar{z} + g_{02}\frac{\bar{z}^2}{2} + g_{21}\frac{\bar{z}^2\bar{z}}{2} + \cdots. \tag{3.7}$$

From (3.6), we have

$$x_t(\theta) = (x_{1t}(\theta), x_{2t}(\theta), x_{3t}(\theta))^T = W(t, \theta) + zq(\theta) + z\bar{q}(\theta)$$

and

$$q(\theta) = (1, \beta, \gamma)^T e^{i\theta\omega\tau_0},$$

then, we have

$$\begin{aligned}x_{1t}(0) &= z + \bar{z} + W_{20}^{(1)}(0)\frac{z^2}{2} + W_{11}^{(1)}(0)z\bar{z} + W_{02}^{(1)}(0)\frac{\bar{z}^2}{2} + o\left(|z, \bar{z}|^3\right),\\ x_{2t}(0) &= \beta z + \bar{\beta}\bar{z} + W_{20}^{(2)}(0)\frac{z^2}{2} + W_{11}^{(2)}(0)z\bar{z} + W_{02}^{(2)}(0)\frac{\bar{z}^2}{2} + o\left(|z, \bar{z}|^3\right),\\ x_{3t}(0) &= \gamma z + \bar{\gamma}\bar{z} + W_{20}^{(3)}(0)\frac{z^2}{2} + W_{11}^{(3)}(0)z\bar{z} + W_{02}^{(3)}(0)\frac{\bar{z}^2}{2} + o\left(|z, \bar{z}|^3\right),\end{aligned}$$

together with (3.7) it follows that

$$
\begin{aligned}
& g(z,\bar{z}) \\
=& \bar{q^*}(0) f_0(z,\bar{z}) = \tau_0 \bar{D}(1, \bar{\beta^*}, \bar{\gamma^*}) \begin{pmatrix} -\mu x_{1t}^2(0) - d x_{1t}(0)(x_{2t}(0) + x_{3t}(0)) \\ c x_{1t}(0)(x_{2t}(0) + x_{3t}(0)) \\ 0 \end{pmatrix} \\
=& \tau_0 \bar{D} \left[-\mu + (\beta+\gamma)(c\bar{\beta}^* - d) \right] z^2 + \tau_0 \bar{D} \left[-2\mu + (\bar{\beta} + \beta + \bar{\gamma} + \gamma)(c\bar{\beta}^* - d) \right] z\bar{z} \\
& + \tau_0 \bar{D} \left[-\mu + (\bar{\beta} + \bar{\gamma})(c\bar{\beta}^* - d) \right] \bar{z}^2 \\
& + \tau_0 \bar{D} \Big\{ \left[(\beta+\gamma)(c\bar{\beta}^* - d)(c\bar{\beta}^* - d) - 2\mu \right] W_{11}^{(1)}(0) \\
& + \frac{1}{2} \left[(\bar{\beta} + \bar{\gamma})(c\bar{\beta}^* - d) - 2u \right] W_{20}^{(1)}(0) + (c\bar{\beta}^* - d) \left[W_{11}^{(2)}(0) + W_{11}^{(3)}(0) \right] \\
& + \frac{1}{2}(c\bar{\beta}^* - d) \left[W_{20}^{(2)}(0) + W_{20}^{(3)}(0) \right] \Big\} z^2 \bar{z} \\
=& \tau_0 \bar{D} K_1 z^2 + \tau_0 \bar{D} K_2 z\bar{z} + \tau_0 \bar{D} K_3 \bar{z}^2 + \tau_0 \bar{D} K_4 z^2 \bar{z},
\end{aligned}
$$

where

$$
\begin{aligned}
& K_1 = -\mu + (\beta+\gamma)(c\bar{\beta}^* - d), \\
& K_2 = -2\mu + (\bar{\beta} + \beta + \bar{\gamma} + \gamma)(c\bar{\beta}^* - d), \\
& K_3 = -\mu + (\bar{\beta} + \bar{\gamma})(c\bar{\beta}^* - d), \\
& K_4 = \left[(\bar{\beta} + \bar{\gamma})(c\bar{\beta}^* - d) - 2u \right] W_{11}^{(1)}(0) + \frac{1}{2} \left[(\bar{\beta} + \bar{\gamma})(c\bar{\beta}^* - d) - 2u \right] W_{20}^{(1)}(0) \\
& \qquad + (c\bar{\beta}^* - d) \left[W_{11}^{(2)}(0) + W_{11}^{(3)}(0) \right] + \frac{1}{2}(c\bar{\beta}^* - d) \left[W_{20}^{(2)}(0) + W_{20}^{(3)}(0) \right].
\end{aligned}
$$

Comparing the coefficients with (3.7), we have

$$
g_{20} = 2\tau_0 \bar{D} K_1, \quad g_{11} = \tau_0 \bar{D} K_2, \quad g_{02} = 2\tau_0 \bar{D} K_3, \quad g_{21} = 2\tau_0 \bar{D} K_4.
$$

In order to determine g_{21}, in the sequel, we need to compute $W_{20}(\theta)$ and $W_{11}(\theta)$. From (3.4) and (3.7), we have

$$
\begin{aligned}
\dot{W} &= x_0 - \bar{z} q - \bar{z} \bar{q} \\
&= \begin{cases} AW - 2\mathrm{Re}\left\{ \bar{q^*}(0) f_0 q(\theta) \right\}, \theta \in [-1, 0), \\ AW - 2\mathrm{Re}\left\{ \bar{q^*}(0) f_0 q(\theta) \right\} + f_0, \theta = 0, \end{cases} \\
&\triangleq AW + H(z, \bar{z}, \theta),
\end{aligned} \tag{3.8}
$$

where

$$
H(z, \bar{z}, \theta) = H_{20} \frac{z^2}{2} + H_{11}(\theta) z\bar{z} + H_{02}(\theta) \frac{\bar{z}^2}{2} + \cdots, \tag{3.9}
$$

Since the coefficients are equal, we can obtain

$$
\begin{aligned}
& (A - 2i\omega\tau_0) W_{20}(\theta) = -H_{20}(\theta), \quad AW_{11}(\theta) = -H_{11}(\theta) \cdots, \\
& H(z, \bar{z}, \theta) = -\bar{q^*}(0) f_0 q(\theta) - q^*(0) \bar{f}_0 \bar{q}(\theta) = -g q(\theta) - \bar{g} \bar{q}(\theta).
\end{aligned} \tag{3.10}
$$

Comparing the coefficients with (3.9) gives

$$H_{20}(\theta) = -g_{20}q(\theta) - \bar{g}_{02}\bar{q}(\theta), \quad H_{11}(\theta) = -g_{11}q(\theta) - \bar{g}_{11}\bar{q}(\theta). \tag{3.11}$$

From (3.11) and the definition of A, we can get

$$\dot{W}_{20}(\theta) = 2i\omega\tau_0 W_{20}(\theta) + g_{20}q(\theta) + \bar{g}_{02}\bar{q}(\theta).$$

Notice that $q(\theta) = (1, \beta, \gamma)^T e^{i\theta\omega\tau_0}$, we have

$$W_{20}(\theta) = \frac{ig_{20}}{\omega\tau_0} q(0)e^{i\theta\omega\tau_0} + \frac{i\bar{g}_{20}}{2\omega\tau_0}\bar{q}(0)e^{i\theta\omega\tau_0} + E_1 e^{2i\theta\omega\tau_0}, \tag{3.12}$$

where $E_1 = \left(E_1^{(1)}, E_1^{(2)}, E_1^{(3)}\right) \in \mathbb{R}^3$ is a constant vector.
Similarly, we can also obtain

$$W_{11}(\theta) = -\frac{ig_{11}}{\omega\tau_0} q(0)e^{i\theta\omega\tau_0} + \frac{i\bar{g}_{11}}{\omega\tau_0}\bar{q}(0)e^{i\theta\omega\tau_0} + E_2, \tag{3.13}$$

where $E_2 = \left(E_2^{(1)}, E_2^{(2)}, E_2^{(3)}\right) \in \mathbb{R}^3$ is also a constant vector.

In what follows, we will seek appropriate E_1 and E_2. From the definition of A and (3.10), we obtain

$$\begin{aligned} &\int_{-1}^{0} \mathrm{d}\rho(\theta) W_{20}(\theta) = 2i\omega\tau_0 W_{20}(0) - H_{20}(0) \\ \text{and } &\int_{-1}^{0} \mathrm{d}\rho(\theta) W_{11}(\theta) = -H_{11}(\theta), \end{aligned} \tag{3.14}$$

where $\rho(\theta) = \rho(0, \theta)$.

From (3.8),we have

$$H_{20}(0) = -g_{20}q(0) - \bar{g}_{02}\bar{q}(0) + 2\tau_0 \begin{pmatrix} -\mu - d(\beta+\gamma) \\ c(\beta+\gamma) \\ 0 \end{pmatrix} \tag{3.15}$$

and

$$H_{11}(0) = -g_{11}q(0) - \bar{g}_{11}\bar{q}(0) + 2\tau_0 \begin{pmatrix} -\mu - \mathrm{Re}(\beta+\gamma) \\ c\mathrm{Re}(\beta+\gamma) \\ 0 \end{pmatrix}. \tag{3.16}$$

Substituting (3.12), (3.13), (3.15) and (3.16) into (3.14) and noticing that

$$\left(i\tau_0\omega I - \int_{-1}^{0} e^{i\theta\omega\tau_0} d\eta(\theta)\right) q(0) = 0 \text{ and } \left(-i\tau_0\omega I - \int_{-1}^{0} e^{-i\theta\omega\tau_0} d\eta(\theta)\right) \bar{q}(0) = 0,$$

we obtain

$$\left(2i\tau_0\omega I - \int_{-1}^{0} e^{2i\theta\omega\tau_0} d\eta(\theta)\right) E_1 = 2\tau_0 \begin{pmatrix} -\mu - d(\beta+\gamma) \\ c(\beta+\gamma) \\ 0 \end{pmatrix}.$$

It follows that

$$E_1^{(1)} = \frac{2}{M}\begin{vmatrix} -\mu - d(\beta+\gamma) & dx^* & dx^* \\ c(\beta+\gamma) & 2i\omega - cx^* + (m+e) & -cx^* \\ 0 & -m & 2i\omega + b - K + Ke^{-2i\omega\tau_0} \end{vmatrix},$$

$$E_1^{(2)} = \frac{2}{M}\begin{vmatrix} 2i\omega - a + 2\mu x^* + dz^* + dy^* & -\mu - d(\beta+\gamma) & dx^* \\ -c(z^*+y^*) & c(\beta+\gamma) & -cx^* \\ 0 & 0 & 2i\omega + b - K + Ke^{-2i\omega\tau_0} \end{vmatrix},$$

$$E_1^{(3)} = \frac{2}{M}\begin{vmatrix} 2i\omega - a + 2\mu x^* + dz^* + dy^* & -\mu - dx^* & -\mu - d(\beta+\gamma) \\ -c(z^*+y^*) & 2i\omega - cx^* + (m+e) & c(\beta+\gamma) \\ 0 & -m & 0 \end{vmatrix},$$

where

$$M = \begin{vmatrix} 2i\omega - a + 2\mu x^* + dz^* + dy^* & dx^* & dx^* \\ -c(z^*+y^*) & 2i\omega - cx^* + (m+e) & -cx^* \\ 0 & -m & 2i\omega + b - K + Ke^{-2i\omega\tau_0} \end{vmatrix}.$$

In the same way, we can obtain E_2. Thus, we have

$$E_2^{(1)} = \frac{2}{G}\begin{vmatrix} -\mu - \mathrm{Re}(\beta+\gamma) & dx^* & dx^* \\ c\mathrm{Re}(\beta+\gamma) & -cx^* + (m+e) & -cx^* \\ 0 & -m & b \end{vmatrix},$$

$$E_2^{(2)} = \frac{2}{G}\begin{vmatrix} -a + 2\mu x^* + dz^* + dy^* & -\mu - \mathrm{Re}(\beta+\gamma) & dx^* \\ c(z^*+y^*) & c\mathrm{Re}(\beta+\gamma) & -cx^* \\ 0 & 0 & b \end{vmatrix},$$

$$E_2^{(3)} = \frac{2}{G}\begin{vmatrix} -a + 2\mu x^* + dz^* + dy^* & dx^* & -\mu - \mathrm{Re}(\beta+\gamma) \\ -c(z^*+y^*) & -cx^* + (m+e) & c\mathrm{Re}(\beta+\gamma) \\ 0 & -m & 0 \end{vmatrix},$$

where

$$G = \begin{vmatrix} -a + 2\mu x^* + dz^* + dy^* & dx^* & dx^* \\ -c(z^*+y^*) & -cx^* + (m+e) & -cx^* \\ 0 & -m & b \end{vmatrix}.$$

Hence we can determine $W_{20}(\theta)$ and $W_{11}(\theta)$. Therefore, each g_{ij} is determined by the parameters and delay in (2.1). Thus, we can compute the following values:

$$c_1(0) = \frac{i}{2\omega\tau_0}\left(g_{11}g_{20} - 2|g_{11}|^2 - \frac{|g_{02}|^2}{3}\right) + \frac{g_{21}}{2}, \quad \mu_2 = -\frac{\mathrm{Re}\{c_1(0)\}}{\mathrm{Re}\{\lambda'(\tau_0)\}},$$

$$T_2 = -\frac{\mathrm{Im}\{c_1(0)\} + \mathrm{Im}\{\lambda'(\tau_0)\}}{\omega\tau_0}, \quad \beta_2 = 2\mathrm{Re}\{c_1(0)\},$$

which determine the quantities of bifurcating periodic solutions in the center manifold at the critical value τ_k, i.e., μ_2 determines the directions of the Hopf bifurcation: if $\mu_2 > 0$ ($\mu_2 < 0$), the Hopf bifurcation is supercritical (subcritical) and the bifurcation exist for $\tau > \tau_0 (< \tau_0)$; β_2 determines the stability of the bifurcation periodic solutions: the bifurcating periodic solutions are stable (unstable) if $\beta_2 < 0 (> 0)$; and T_2 determines the period of the bifurcating periodic solutions: the period increase (decrease) if $T_2 > 0 (< 0)$.

4. Numerical Simulation

In order to better understand the nature of the system, we perform the numerical simulation and analyze the system. We choose $a = 0.2$; $b = 0.15$; $c = 0.4$; $d = 0.2$; $e = 0.4$; $m = 0.01$; $\mu = 0.1$; $K = 0.2$. By a simple calculation, we can easily get

$$\omega_0 = 0.191856479, \quad \tau_0 = 6.05451339.$$

When $\tau = 5.46 < \tau_0$, the system tends to be stable and after a period of macroeconomic regulation and control, the numbers of the small and medium-sized investment companies and large investment company will eventually tend to a equilibrium $E^*(0.9609, 0.4871, 0.0325)$, see Fig.4.1; when $\tau = 6.056 > \tau_0$, we can get the periodic solution of the system, see Fig.4.2.

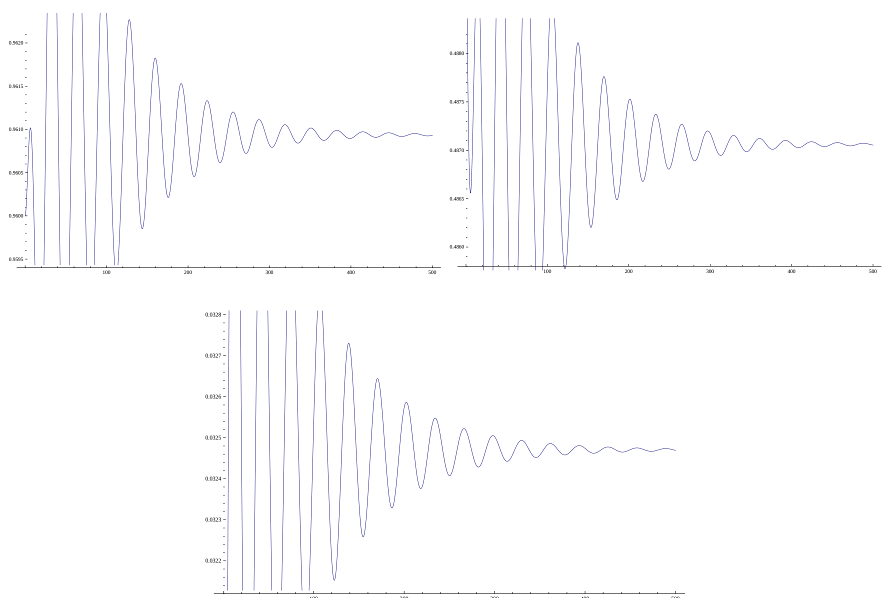

Figure 1. The trajetctories and phase graphs of system (2.1) with $\tau = 5.46 < \tau_0$, $K = 0.2$, E^* become local stable.

5. Conclusion

In this paper, we have proposed an enterprise competitive model with time delay in the number of large investment company. We have discussed the local asymptotic

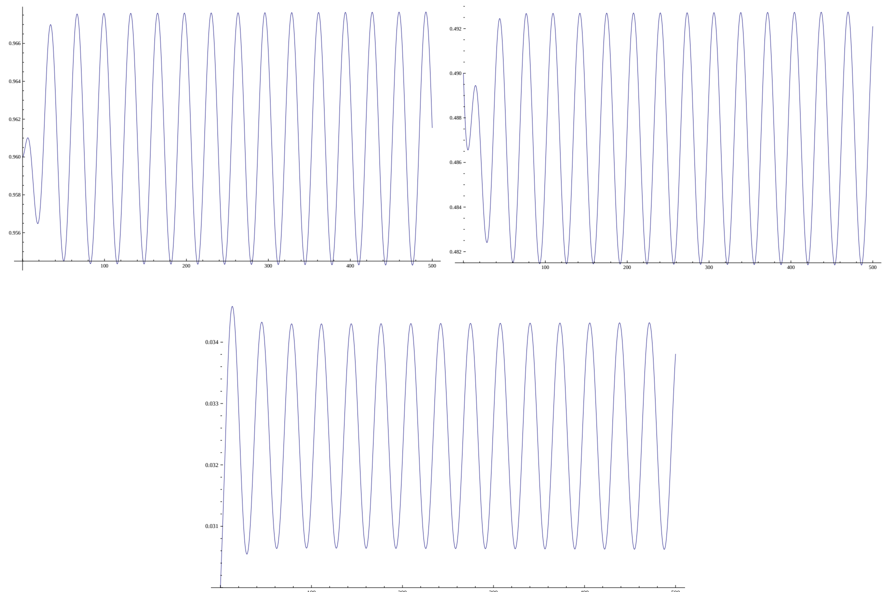

Figure 2. The trajetctories and phase graphs of system (2.1) with $\tau = 6.056 > \tau_0, K = 0.2$, a stable periodic solution bifurcate from the equilibrium E^*.

stability of this model, and further shown the macroeconomic control to the market at different times will have different impacts on the entire market.

When the macroeconomic control to the market within the critical value of time, it will promote the steady development of the market; otherwise, it will lead to confusion in the market. In this paper, we provide the right time to carry on the macroeconomic control to the market. And a guidance for the maintenance of the stale development of the investment market is shown.

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