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STUDY ON DIVERGENCE MEASURES FOR INTUITIONISTIC FUZZY SETS AND ITS APPLICATION IN MEDICAL DIAGNOSIS

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Abstract As far as medical diagnosis problem is concerned, predicting the actual disease in complex situations has been a concerning matter for the doctors/experts. The divergence measure for intuitionistic fuzzy sets is an effective and potent tool in addressing the medical decision making problems. We define a new divergence measure for intuitionistic fuzzy sets (IFS) and its interesting properties are studied. The existing divergence measures under intuitionistic fuzzy environment are reviewed and their counter-intuitive cases has been explored. The parameter α is incorporated in the proposed divergence measure and it is defined as parametric intuitionistic fuzzy divergence measure (PIFDM). The different choices of the parameter α provide different decisions about the disease. As we increase the value of α, the information about the disease increases and move towards the optimal solution with the reduced in the uncertainty. Finally, we compare our results with the already existing results, which illustrate the role of the parameter α in obtaining the optimal solution in the medical decision making application. The results demonstrate that the parametric intuitionistic fuzzy divergence measure (PIFDM) is more comprehensive and effective than the proposed intuitionistic fuzzy divergence measure and the existing intuitionistic fuzzy divergence measures for decision making in medical investigations.

Keywords Intuitionistic fuzzy sets, intuitionistic fuzzy divergence measure, medical diagnosis, decision making.

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1. Introduction

Uncertainty removal in decision making has been one of the basic challenges in various types of applications. It has been significantly increasing in medical diagnosis problems due to which diagnose the disease becomes more complicated. Due to high level of uncertainty, it is very tough to predict the disease with which the patient is suffering from. Hence, dealing efficiently with uncertainty is absolutely essential for proper diagnosis of diseases. To cope with these problems, the theory of intuitionistic fuzzy sets (IFS), introduced by Atanassov [1] is an expedient tool over fuzzy sets, vague sets [4], interval-valued fuzzy sets [12] and many more. The theory of IFS has been utilized by various authors in miscellaneous disciplines. It makes possible to define the inexact medical information in terms of membership degree, non-membership degree and hesitation degree.

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In the last few years, divergence measure for intuitionistic fuzzy sets have been played an imperative role in reducing the uncertainty and help experts to take the decision about the patient's disease. This will in turn diminish uncertainty in cases where limited information is available to the experts, as it helps to diagnose the disease with maximum accuracy and less uncertainty. It has numerous applications in lot of fields such as pattern recognition [2, 6, 13, 15], group decision making [16], medical diagnosis [2, 3, 5, 8–11, 13, 14, 18], image processing [13] etc. Vlachos & Sergiadis [13] firstly proposed the divergence measure for IFS and shown its application in pattern recognition, image processing and medical diagnosis. Thereafter, many researchers Junjun *et al.* [7], Xia & Xu [16], Wei & Ye [15], Zhang & Jiang [18], Hung & Yang [6] have defined the different divergence measures for IFS and the findings have applied in variety of fields. However, Wei & Ye [15] pointed out the downside of Vlachos & Sergiadis [13] measure and modified the measure of Vlachos & Sergiadis [13].

In the present work, we will propose a new divergence measure under intuitionistic fuzzy phenomenon and extend it to the parametric form by incorporating the parameter α and defined it as parametric intuitionistic fuzzy divergence measure (PIFDM). The findings of the proposed intuitionistic fuzzy divergence measure and the parametric intuitionistic fuzzy divergence measure (PIFDM) are shown in the medical decision making problem.

The paper is arranged as follows: Section 2 provides some basic definitions, which are used in the analysis of the paper. Some already existing divergence measures for IFS are given in section 3. Section 4 introduces a new intuitionistic fuzzy divergence measure with its elegant properties and show by numerical examples that some of the already existing divergence measures for IFS do not satisfy the axioms of intuitionistic fuzzy divergence measures. Section 5 defines the parametric form of proposed divergence measure, Vlachos & Sergiadis [13] measure and Wei & Ye [15] measure. A numerical example is presented to demonstrate the applicability of the proposed divergence measures for the problem of medical diagnosis. Conclusion of the paper is given in Section 6.

2. Preliminaries

In this section, fundamental knowledge concerning about IFS and intuitionistic fuzzy divergence measure are introduced so as to smooth the analysis of the paper. For convenience, let $X = \{x_1,\ x_2,\ \ldots,\ x_n\}$ be used in throughout this article.

2.1. Intuitionistic Fuzzy Sets

Definition 2.1. An intuitionistic fuzzy set A defined on the universe of discourse X, introduced by Atanassov [1], given by the expression

$$A = \{\langle x,\ \mu_A(x),\ \nu_A(x)\rangle | x \in X\}, \tag{2.1}$$

where the functions $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the membership degree and the non-membership degree to A respectively. For every $x \in X$

$$0 \le \mu_A(x) + \nu_A(x) \le 1. \tag{2.2}$$

Further, we denote by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$ hesitation degree to A which basically expresses lack of information of whether x belongs to A or not. It is obvious that $0 \leq \pi_A(x) \leq 1$ for every $x \in X$. In fact, when $\mu_A(x) = 1 - \nu_A(x)$ for all $x \in X$, an intuitionistic fuzzy set is converted into a fuzzy set. Let $IFS(X)$ denote the family of all intuitionistic fuzzy sets in the finite universe $X = \{x_1, x_2, \ldots, x_n\}$and $A, B \in IFS(X)$ given by $A = \{\langle x, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i)\rangle | x_i \in X\}$ used throughout in this paper. Then some set operations can be defined as follows:

- **Complement of A**

$$A^C = \{\langle x, \nu_A(x_i), \mu_A(x_i)\rangle | x_i \in X\}.$$

- **Intersection of A and B**

$$A \cap B = \{\langle x, \min\{\mu_A(x_i), \mu_B(x_i)\}, \max\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}.$$

- **Union of A and B**

$$A \cup B = \{\langle x, \max\{\mu_A(x_i), \mu_B(x_i)\}, \min\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}.$$

- **Inclusion Relation**

$$A \subseteq B \text{ if and only if } \mu_A(x_i) \leq \mu_B(x_i) \text{ and } \nu_A(x_i) \geq \nu_B(x_i), \forall x_i \in X.$$

2.2. Intuistionistic Fuzzy Divergence Measures

Definition 2.2. Let $A, B \in IFS(X)$ be two intuitionistic fuzzy sets in X. A mapping $D : IFS(X) \times IFS(X) \to R$ is a divergence measure for IFS if it fulfils the following axioms [13].
M1. $D(A||B) \geq 0$.
M2. $D(A||B) = 0$ if and only if $A = B$.
M3. $D(A||B) = D\left(A^C||B^C\right)$.

The divergence measure for IFS should satisfy the above mentioned axioms M1 – M3. Next, we listed some of the existing intuitionistic fuzzy divergence measures, are as follows.

3. Review of intuitionistic fuzzy divergence measures

A number of divergence measures under intuitionistic fuzzy phenomenon have been proposed by the several researchers and practitioners in order to measure the degree of discrimination between the two IFS. The existing divergence measures between IFS are listed next.
Vlachos & Sergiadis [13]

$$D_{VS}(A||B) = \sum_{i=1}^{n}\left(\mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) + \nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right). \quad (3.1)$$

Vlachos & Sergiadis [13] also defined the symmetric version of measure (3.1), given by

$$D_{VS}^{sym}(A||B) = D_{VS}(A||B) + D_{VS}(B||A)$$
$$= \sum_{i=1}^{n} \begin{pmatrix} \mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) + \nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \\ +\mu_B(x_i)\ln\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) + \nu_B(x_i)\ln\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \end{pmatrix}. \tag{3.2}$$

Zhang & Jiang [18]

$$D_{ZY}(A||B)$$
$$= \sum_{i=1}^{n} \begin{pmatrix} \left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\ln\left(\frac{2(\mu_A(x_i)+1-\nu_A(x_i))}{(\mu_A(x_i)+1-\nu_A(x_i))+(\mu_B(x_i)+1-\nu_B(x_i))}\right) \\ +\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\ln\left(\frac{2(\nu_A(x_i)+1-\mu_A(x_i))}{(\nu_A(x_i)+1-\mu_A(x_i))+(\nu_B(x_i)+1-\mu_B(x_i))}\right) \end{pmatrix}. \tag{3.3}$$

Wei & Yei [15] & K. C. Hung [5]

$$D_{WY}(A||B) = \sum_{i=1}^{n} \begin{pmatrix} \mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) \\ +\nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \\ +\pi_A(x_i)\ln\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right) \end{pmatrix}. \tag{3.4}$$

The symmetric discrimination of measure (3.4) is given by

$$D_{WY}^{sym}(A||B) = D_{WY}(A||B) + D_{WY}(B||A)$$
$$= \sum_{i=1}^{n} \begin{pmatrix} \mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) + \nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \\ +\pi_A(x_i)\ln\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right) + \mu_B(x_i)\ln\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i}\right) \\ +\nu_B(x_i)\ln\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) + \pi_B(x_i)\ln\left(\frac{2\pi_B(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right) \end{pmatrix}. \tag{3.5}$$

Jujun *et al.* [7]

$$D_J(A||B) = \sum_{i=1}^{n}\left(\pi_A(x_i)\ln\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right) + \Delta_A(x_i)\ln\left(\frac{2\Delta_A(x_i)}{\Delta_A(x_i)+\Delta_B(x_i)}\right)\right), \tag{3.6}$$

where $\Delta_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)|$, denotes that how close the membership and non membership degrees are. The symmetric divergence measure of (3.6) are defined as follows

$$D_J^{sym}(A||B) = D_J(A||B) + D_J(B||A). \tag{3.7}$$

In the next section, we will propose a new divergence measure for intuitionistic fuzzy sets (IFS) and satisfies a number of additional properties apart from the basic axioms.

4. New Intuitionistic Fuzzy divergence Measure

Let $A, B, C \in IFS(X)$, then the intuitionistic fuzzy divergence measure of A against B is defined as

$$D\left(A||B\right) = -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right), \tag{4.1}$$

which represents the amount of discrimination information of one intuitionistic fuzzy set A from other intuitionistic fuzzy set B. Now, we demonstrate through some numerical examples that the measures $D_{VS}\left(A||B\right)$, $D_{ZY}\left(A||B\right)$ and $D_J\left(A||B\right)$ given by (3.1), (3.3) and (3.6), respectively do not satisfy the axioms, which an intuitionistic fuzzy divergence measure should comply.

4.1. Counter-intuitive Cases

Example 4.1. Let $A, B \in IFS(X)$, given by

$$A = \left\{\langle x_1, 0.44, 0.385\rangle, \langle x_2, 0.43, 0.39\rangle, \langle x_3, 0.42, 0.38\rangle\right\},$$
$$B = \left\{\langle x_1, 0.34, 0.48\rangle, \langle x_2, 0.37, 0.46\rangle, \langle x_3, 0.38, 0.45\rangle\right\}.$$

Then, we have the following results for the measures given by (3.1), (3.6) and (4.1), respectively for the above two IFS,

$$D_{VS}\left(A||B\right) = -0.0, D_J\left(A||B\right) = -0.0,$$

and

$$D\left(A||B\right) = 0.0027.$$

This is the violation of axiom M1 by the measures (3.1) and (3.6).

Example 4.2. Let $A, B \in IFS(X)$, given by

$$A = \left\{\langle x_1, 0.0, 0.5\rangle, \langle x_2, 0.5, 0.0\rangle, \langle x_3, 0.0, 0.0\rangle\right\},$$
$$B = \left\{\langle x_1, 0.5, 0.5\rangle, \langle x_2, 0.5, 0.5\rangle, \langle x_3, 0.5, 0.0\rangle\right\}.$$

Considering (3.1) and (4.1), we get

$$D_{VS}\left(A||B\right) = 0,$$

and

$$D\left(A||B\right) = 0.4033.$$

So, the measure given by (3.1) violates the axiom M2.

Example 4.3. Let $A, B \in IFS(X)$, given by

$$A = \left\{\langle x_1, 0.0, 0.0\rangle, \langle x_2, 0.5, 0.5\rangle\right\},$$
$$B = \left\{\langle x_1, 0.5, 0.5\rangle, \langle x_2, 0.0, 0.0\rangle\right\}.$$

Considering (3.3) and (4.1), we get

$$D_{ZY}\left(A||B\right) = 0,$$

and

$$D\left(A||B\right) = 1.$$

Example 4.4. Let $A, B \in IFS(X)$, given by

$$A = \{\langle x_1, 0.0, 0.5\rangle\},$$
$$B = \{\langle x_1, 0.5, 0.0\rangle\}.$$

Using (3.6) and (4.1), we get

$$D_J\left(A||B\right) = 0,$$

and

$$D\left(A||B\right) = 0.4150.$$

Example (4.3) and (4.4) are again the case of violation of axiom M2.

So, we can easily observe from the above examples that the measures introduced by Vlachos & Serigiadis [13], Zhang & Jiang [18] and Junjun *et al.* [7] failed to carry out the axioms M1 and M2, given in definition (2.2) , whereas the proposed measure given by (4.1) satisfies all the axioms of divergence measure. Therefore, the proposed divergence measure is a valid divergence measure for IFS and does not have any counter-intuitive cases.

4.2. Properties of the Proposed Intuitionistic Fuzzy Divergence Measure

Theorem 4.1. *Let $A, B, C \in IFS(X)$, then the proposed measure $D\left(A||B\right)$ given by (4.1) satisfies the following properties are given as follows:*

P1. $D\left(A||B\right) = D\left(B||A\right)$ *and* $0 \leq D\left(A||B\right) \leq 1$.

P2. $D\left(A||B\right) = 0$ *if and only if* $A = B$.

P3. $D\left(A\cap C||B\cap C\right) \leq D\left(A||B\right)$ *for every* $C \in IFS(X)$.

P4. $D\left(A\cup C||B\cup C\right) \leq D\left(A||B\right)$ *for every* $C \in IFS(X)$.

P5. $D\left(A||B\right) = D\left(A^C||B^C\right)$.

P6. $D\left(A||B^C\right) = D\left(A^C||B\right)$.

P7. $D\left(A||A^C\right) = 1$ *if and only if A is a crisp set.*

P8. $D\left(A||A^C\right) = 0$ *if and only if* $\mu_A(x_i) = \nu_A(x_i)$ *for all* $x_i \in X$.

P9. $D\left(A||A\cup B\right) = D\left(A\cap B||B\right) \leq D\left(A||B\right)$ *for* $A \subseteq B$ *and* $B \subseteq A$.

P10. $D\left(A\cap B||A\cup B\right) = D\left(A||B\right)$.

P11. $D\left(A||B\right) \leq D\left(A||C\right)$ *for* $A \subseteq B \subseteq C$.

P12. $D\left(B||C\right) \leq D\left(A||C\right)$ *for* $A \subseteq B \subseteq C$.

Proof. P1. The symmetry of measure (4.1) with respect to their argument is obvious. So, $D\left(A||B\right) = D\left(B||A\right)$. Further by virtue of arithmetic geometric mean inequality, we have

$$\sqrt{\mu_A(x_i)\mu_B(x_i)} \leq \frac{\mu_A(x_i) + \mu_B(x_i)}{2},$$

$$\sqrt{\nu_A(x_i)\nu_B(x_i)} \leq \frac{\nu_A(x_i)+\nu_B(x_i)}{2},$$

$$\sqrt{\pi_A(x_i)\pi_B(x_i)} \leq \frac{\pi_A(x_i)+\pi_B(x_i)}{2}.$$

On adding the above three equations and take summations on both sides, we get

$$\begin{aligned}
&\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)\\
&\quad\leq \sum_{i=1}^{n}\big(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}+\frac{\nu_A(x_i)+\nu_B(x_i)}{2}+\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\big)\\
&\Rightarrow 0 \leq \frac{1}{n}\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)\leq 1\\
&\Rightarrow \frac{1}{2} \leq \big(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)}{2}\big)\leq 1\\
&\Rightarrow 0 \leq -\log\big(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)}{2}\big)\\
&\quad\leq 1\\
&\Rightarrow 0 \leq D\,(A||B)\leq 1.
\end{aligned}$$

P2. Let $A = B$, then it is obvious that $D\,(A||B) = 0$.
Now, consider

$$\begin{aligned}
&D\,(A||B)=0\\
&\Rightarrow -\log\big(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)}{2}\big)\\
&\quad=0\\
&\Rightarrow -\log\big(1+\frac{1}{n}\sum_{i=1}^{n}\big(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\big)\big)\\
&\quad=-\log 2\\
&\Rightarrow\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}=1\\
&\Rightarrow\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\\
&\quad=\frac{\mu_A(x_i)+\mu_B(x_i)}{2}+\frac{\nu_A(x_i)+\nu_B(x_i)}{2}+\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\\
&\Rightarrow\begin{pmatrix}\frac{\mu_A(x_i)+\mu_B(x_i)}{2}-\sqrt{\mu_A(x_i)\mu_B(x_i)}+\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\\ -\sqrt{\nu_A(x_i)\nu_B(x_i)}+\frac{\pi_A(x_i)+\pi_B(x_i)}{2}-\sqrt{\pi_A(x_i)\pi_B(x_i)}\end{pmatrix}=0\\
&\Rightarrow\frac{\big(\sqrt{\mu_A(x_i)}-\sqrt{\mu_B(x_i)}\big)^2}{2}+\frac{\big(\sqrt{\nu_A(x_i)}-\sqrt{\nu_B(x_i)}\big)^2}{2}+\frac{\big(\sqrt{\pi_A(x_i)}-\sqrt{\pi_B(x_i)}\big)^2}{2}\\
&\quad=0\\
&\Rightarrow\mu_A(x_i)=\mu_B(x_i),\,\nu_A(x_i)=\nu_B(x_i),\,\pi_A(x_i)=\pi_B(x_i).
\end{aligned}$$

Therefore, two sets coincide, *i.e.*, $A = B$.

P3. We have

$$\left(\sqrt{\min(\mu_A(x_i),\mu_C(x_i))}-\sqrt{\min(\mu_B(x_i),\mu_C(x_i))}\right)^2 \leq \left(\sqrt{\mu_A(x_i)}-\sqrt{\mu_B(x_i)}\right)^2$$
$$\Rightarrow \begin{pmatrix} \min(\mu_A(x_i),\mu_C(x_i))+\min(\mu_B(x_i),\mu_C(x_i)) \\ -2\left(\sqrt{\min(\mu_A(x_i),\mu_C(x_i))\min(\mu_B(x_i),\mu_C(x_i))}\right)\end{pmatrix} \leq \begin{pmatrix}\mu_A(x_i)+\mu_B(x_i) \\ -2\sqrt{\mu_A(x_i)\mu_B(x_i)}\end{pmatrix}. \tag{4.2}$$

Again, we have

$$\left(\sqrt{\max(\nu_A(x_i),\nu_C(x_i))}-\sqrt{\max(\nu_B(x_i),\nu_C(x_i))}\right)^2 \leq \left(\sqrt{\nu_A(x_i)}-\sqrt{\nu_B(x_i)}\right)^2$$
$$\Rightarrow \begin{pmatrix} \max(\nu_A(x_i),\nu_C(x_i))+\max(\nu_B(x_i),\nu_C(x_i)) \\ -2\left(\sqrt{\max(\nu_A(x_i),\nu_C(x_i))\max(\nu_B(x_i),\nu_C(x_i))}\right)\end{pmatrix} \leq \begin{pmatrix}\nu_A(x_i)+\nu_B(x_i) \\ -2\sqrt{\nu_A(x_i)\nu_B(x_i)}\end{pmatrix}. \tag{4.3}$$

We can also write

$$\begin{aligned}
&\begin{pmatrix} \sqrt{1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i))} \\ -\sqrt{1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i))}\end{pmatrix}^2 \\
&\leq \begin{pmatrix}\sqrt{1-\mu_A(x_i)-\nu_A(x_i)} \\ -\sqrt{1-\mu_B(x_i)-\nu_B(x_i)}\end{pmatrix}^2 \\
&\Rightarrow \begin{pmatrix} (1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i))) \\ +(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i))) \\ -2\begin{pmatrix}\sqrt{(1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i)))} \\ \times\sqrt{(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i)))}\end{pmatrix}\end{pmatrix} \\
&\leq \begin{pmatrix} 1-\mu_A(x_i)-\nu_A(x_i)+1-\mu_B(x_i)-\nu_B(x_i) \\ -2\left(\sqrt{(1-\mu_A(x_i)-\nu_A(x_i)).(1-\mu_B(x_i)-\nu_B(x_i))}\right)\end{pmatrix}.
\end{aligned} \tag{4.4}$$

Adding (4.2), (4.3) and (4.4) yields

$$\begin{pmatrix}
\min(\mu_A(x_i),\mu_C(x_i))+\min(\mu_B(x_i),\mu_C(x_i)) \\
-2\sqrt{\min(\mu_A(x_i),\mu_C(x_i)).\min(\mu_B(x_i),\mu_C(x_i))} \\
+\max(\nu_A(x_i),\nu_C(x_i))+\max(\nu_B(x_i),\nu_C(x_i)) \\
-2\sqrt{\max(\nu_A(x_i),\nu_C(x_i)).\max(\nu_B(x_i),\nu_C(x_i))} \\
+(1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i))) \\
+(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i))) \\
-2\begin{pmatrix}\sqrt{(1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i)))} \\ \times\sqrt{(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i)))}\end{pmatrix}
\end{pmatrix}$$

$$\leq \begin{pmatrix} \mu_A(x_i)+\mu_B(x_i)-2\sqrt{\mu_A(x_i)\mu_B(x_i)}+(1-\mu_B(x_i)-\nu_B(x_i)) \\ -2\left(\sqrt{\nu_A(x_i)\nu_B(x_i)}\right)+(1-\mu_A(x_i)-\nu_A(x_i))+\nu_A(x_i)+\nu_B(x_i) \\ -2\sqrt{(1-\mu_A(x_i)-\nu_A(x_i)).(1-\mu_B(x_i)-\nu_B(x_i))} \end{pmatrix}$$

$$\Rightarrow 2\begin{pmatrix} 1-\sqrt{\min(\mu_A(x_i),\mu_C(x_i)).\min(\mu_B(x_i),\mu_C(x_i))} \\ -\sqrt{\max(\nu_A(x_i),\nu_C(x_i)).\max(\nu_B(x_i),\nu_C(x_i))} \\ -\begin{pmatrix} \sqrt{(1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i)))} \\ \sqrt{(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i)))} \end{pmatrix} \end{pmatrix}$$

$$\leq 2\begin{pmatrix} 1-\sqrt{\mu_A(x_i)\mu_B(x_i)}-\sqrt{\nu_A(x_i)\nu_B(x_i)} \\ -\sqrt{(1-\mu_A(x_i)-\nu_A(x_i)).(1-\mu_B(x_i)-\nu_B(x_i))} \end{pmatrix}$$

$$\Rightarrow -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\begin{pmatrix} \sqrt{\min(\mu_A(x_i),\mu_C(x_i))\min(\mu_B(x_i),\mu_C(x_i))} \\ +\sqrt{\max(\nu_A(x_i),\nu_C(x_i))\max(\nu_B(x_i),\nu_C(x_i))} \\ +\begin{pmatrix} \sqrt{(1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i)))} \\ \sqrt{(1-\min(\mu_B(x_i),\mu_C(x_i))-\max(\nu_B(x_i),\nu_C(x_i)))} \end{pmatrix} \end{pmatrix}}{2}\right)$$

$$\leq -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\begin{pmatrix} \sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)} \\ +\sqrt{(1-\mu_A(x_i)-\nu_A(x_i))(1-\mu_B(x_i)-\nu_B(x_i))} \end{pmatrix}}{2}\right)$$

$$\Rightarrow D(A\cap C||B\cap C)\leq D(A||B) \text{ for every } C\in IFS(X).$$

P4. The proof is on similar lines as in P3.

P5. Consider

$$\begin{aligned} & D(A||B) \\ =& -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right) \\ =& -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right) \\ =& D\left(A^C||B^C\right). \end{aligned}$$

P6. We have

$$\begin{aligned} & D\left(A||B^C\right) \\ =& -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\nu_B(x_i)}+\sqrt{\nu_A(x_i)\mu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right) \\ =& -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\nu_A(x_i)\mu_B(x_i)}+\sqrt{\mu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right) \end{aligned}$$

$=D\left(A^C||B\right).$

P7. Let A be a crisp set, *i.e.*, $\mu_A(x_i)=0, \nu_A(x_i)=1$, then it is obvious that $D\left(A||A^C\right)=1$.
Now, consider

$$
\begin{aligned}
& D\left(A||A^C\right)=1 \\
\Rightarrow & -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^n\left(\sqrt{\mu_A(x_i)\nu_A(x_i)}+\sqrt{\nu_A(x_i)\mu_A(x_i)}+\sqrt{\pi_A(x_i)\pi_A(x_i)}\right)}{2}\right) \\
& =1 \\
\Rightarrow & \log\left(\frac{2}{1+\frac{1}{n}\sum_{i=1}^n\left(2\sqrt{\mu_A(x_i)\nu_A(x_i)}+\pi_A(x_i)\right)}\right)=\log 2 \\
\Rightarrow & \mu_A(x_i)+\nu_A(x_i)-2\sqrt{\mu_A(x_i)\nu_A(x_i)}=1 \\
\Rightarrow & \left(\sqrt{\mu_A(x_i)}-\sqrt{\nu_A(x_i)}\right)^2=1 \\
\Rightarrow & \mu_A(x_i)=0,\ \nu_A(x_i)=1 \text{ or } \mu_A(x_i)=1,\ \nu_A(x_i)=0, \textit{i.e.}, A \text{ is a crisp set.}
\end{aligned}
$$

Therefore, $D\left(A||A^C\right)=1$ if and only if A is a crisp set.
P8. Let$\mu_A(x_i)=\nu_A(x_i)$ in (4.1) for all $x_i\in X$, then it is obvious

$$
\begin{aligned}
D\left(A||A^C\right) &= -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^n\left(\sqrt{\mu_A(x_i)\nu_A(x_i)}+\sqrt{\nu_A(x_i)\mu_A(x_i)}+\sqrt{\pi_A(x_i)\pi_A(x_i)}\right)}{2}\right) \\
&=0.
\end{aligned}
$$

Now, if we consider

$$
\begin{aligned}
& D\left(A||A^C\right)=0 \\
\Rightarrow & -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^n\left(\sqrt{\mu_A(x_i)\nu_A(x_i)}+\sqrt{\nu_A(x_i)\mu_A(x_i)}+\sqrt{\pi_A(x_i)\pi_A(x_i)}\right)}{2}\right) \\
& =0 \\
\Rightarrow & -\log\left(1+\frac{1}{n}\sum_{i=1}^n\left(\sqrt{\mu_A(x_i)\nu_A(x_i)}+\sqrt{\nu_A(x_i)\mu_A(x_i)}+\sqrt{\pi_A(x_i)\pi_A(x_i)}\right)\right) \\
& =-\log 2 \\
\Rightarrow & 2\sqrt{\mu_A(x_i)\nu_A(x_i)}+\pi_A(x_i)=\mu_A(x_i)+\nu_A(x_i)+\pi_A(x_i) \\
\Rightarrow & \left(\sqrt{\mu_A(x_i)}-\sqrt{\nu_A(x_i)}\right)^2=0 \\
\Rightarrow & \mu_A(x_i)=\nu_A(x_i) \text{ for all } x_i\in X.
\end{aligned}
$$

Therefore, $D\left(A||A^C\right)=0$ if and only if $\mu_A(x_i)=\nu_A(x_i)$ for all $x_i\in X$.

P9. From (4.1), we have

$$D\left(A||A\bigcup B\right)$$
$$=-\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\begin{array}{l}\sqrt{\mu_A(x_i)\max\left(\mu_A(x_i),\mu_B(x_i)\right)}+\sqrt{\nu_A(x_i)\min\left(\nu_A(x_i),\nu_B(x_i)\right)}\\+\sqrt{(1-\mu_A(x_i)-\nu_A(x_i))\left(1-\max\left(\mu_A(x_i),\mu_B(x_i)\right)-\min\left(\nu_A(x_i),\nu_B(x_i)\right)\right)}\end{array}\right)}{2}\right), \tag{4.5}$$

$$D\left(A\bigcap B||B\right)$$
$$=-\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\begin{array}{l}\sqrt{\min\left(\mu_A(x_i),\mu_B(x_i)\right)\mu_B(x_i)}+\sqrt{\max\left(\nu_A(x_i),\nu_B(x_i)\right)\nu_B(x_i)}\\+\sqrt{\left(1-\min\left(\mu_A(x_i),\mu_B(x_i)\right)-\max\left(\nu_A(x_i),\nu_B(x_i)\right)\right)(1-\mu_B(x_i)-\nu_B(x_i))}\end{array}\right)}{2}\right). \tag{4.6}$$

For $A\subseteq B$, (4.5) and (4.6)give

$$D\left(A||A\cup B\right)=D\left(A\cap B||B\right)=D\left(A||B\right). \tag{4.7}$$

Again, for $B\subseteq A$, (4.5) and (4.6) give

$$D\left(A||A\cup B\right)=D\left(A\cap B||B\right)=0\leq D\left(A||B\right). \tag{4.8}$$

The proof follows from (4.7) and (4.8).
P10. We have

$$D\left(A\bigcap B||A\bigcup B\right)$$
$$=-\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\begin{array}{l}\sqrt{\mu_{A\bigcap B}(x_i)\mu_{A\bigcup B}(x_i)}+\sqrt{\nu_{A\bigcap B}(x_i)\nu_{A\bigcup B}(x_i)}\\+\sqrt{\left(1-\mu_{A\bigcap B}(x_i)-\nu_{A\bigcap B}(x_i)\right)\left(1-\mu_{A\bigcup B}(x_i)-\nu_{A\bigcup B}(x_i)\right)}\end{array}\right)}{2}\right)$$
$$=-\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left(\begin{array}{l}\sqrt{\min\left(\mu_A(x_i),\mu_B(x_i)\right).\max\left(\mu_A(x_i),\mu_B(x_i)\right)}\\+\sqrt{\max\left(\nu_A(x_i),\nu_B(x_i)\right).\min\left(\nu_A(x_i),\nu_B(x_i)\right)}\\+\sqrt{\begin{array}{l}\left(1-\min\left(\mu_A(x_i),\mu_B(x_i)\right)-\max\left(\nu_A(x_i),\nu_B(x_i)\right)\right)\\.\left(1-\max\left(\mu_A(x_i),\mu_B(x_i)\right)-\min\left(\nu_A(x_i),\nu_B(x_i)\right)\right)\end{array}}\end{array}\right)}{2}\right)$$
$$=-\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\left[\sqrt{\mu_A(x_i)\mu_B(x_i)}+\sqrt{\nu_A(x_i)\nu_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}\right]}{2}\right)$$
$$=D(A||B).$$

P11. For $A\subseteq B\subseteq C$, we have

$$\left(\sqrt{\mu_A(x_i)}-\sqrt{\mu_B(x_i)}\right)^2\leq\left(\sqrt{\mu_A(x_i)}-\sqrt{\mu_C(x_i)}\right)^2$$
$$\Rightarrow\mu_A(x_i)+\mu_B(x_i)-2\sqrt{\mu_A(x_i)\mu_B(x_i)}\leq\mu_A(x_i)+\mu_C(x_i)-2\sqrt{\mu_A(x_i)\mu_C(x_i)}, \tag{4.9}$$

$$\left(\sqrt{\nu_A(x_i)}-\sqrt{\nu_B(x_i)}\right)^2\leq\left(\sqrt{\nu_A(x_i)}-\sqrt{\nu_C(x_i)}\right)^2$$
$$\Rightarrow\nu_A(x_i)+\nu_B(x_i)-2\sqrt{\nu_A(x_i)\nu_B(x_i)}\leq\nu_A(x_i)+\nu_C(x_i)-2\sqrt{\nu_A(x_i)\nu_C(x_i)}, \tag{4.10}$$

$$\left(\sqrt{\pi_A(x_i)} - \sqrt{\pi_B(x_i)}\right)^2 \leq \left(\sqrt{\pi_A(x_i)} - \sqrt{\pi_C(x_i)}\right)^2$$
$$\Rightarrow \pi_A(x_i) + \pi_B(x_i) - 2\sqrt{\pi_A(x_i)\pi_B(x_i)} \leq \pi_A(x_i) + \pi_C(x_i) - 2\sqrt{\pi_A(x_i)\pi_C(x_i)}. \tag{4.11}$$

On adding (4.9), (4.10) and (4.11), we have

$$2\left(1 - \sqrt{\mu_A(x_i)\mu_B(x_i)} - \sqrt{\nu_A(x_i)\nu_B(x_i)} - \sqrt{\pi_A(x_i)\pi_B(x_i)}\right)$$
$$\leq 2\left(1 - \sqrt{\mu_A(x_i)\mu_C(x_i)} - \sqrt{\nu_A(x_i)\nu_C(x_i)} - \sqrt{\pi_A(x_i)\pi_C(x_i)}\right)$$
$$\Rightarrow \sqrt{\mu_A(x_i)\mu_B(x_i)} + \sqrt{\nu_A(x_i)\nu_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)}$$
$$\geq \sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\nu_A(x_i)\nu_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)}$$
$$\Rightarrow -\log\left(\frac{1 + \frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + \sqrt{\nu_A(x_i)\nu_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)}\right)}{2}\right)$$
$$\leq -\log\left(\frac{1 + \frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\nu_A(x_i)\nu_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)}\right)}{2}\right)$$
$$\Rightarrow D(A||B) \leq D(A||C).$$

P12. The proof is on similar lines as in P11.

This completes the proof. □

5. Applications in Medical Diagnosis

Most decisions in medical science have substantial uncertainties which deal with imprecision and fuzziness. There are a lot of diseases exist in medical science which share some common symptoms. So, it is very difficult for experts/physicians, which patient is actually suffering from which particular disease. The notion of divergence measure under intuitionistic fuzzy setting plays a decisive role in tackling these problems. K. C. Hung [5], De *et al.* [3], Szmidt& Kacprzyk [9–11] and Vlachos & Sergiadis [13] utilized the concept of IFS for reducing the uncertainty in medical diagnosis problems. We have used the approach of intuitionistic fuzzy divergence measure to deal with the problem of medical diagnosis.

We have considered the medical diagnosis problem as discussed in [2, 3, 5, 8–11, 13, 14]. The data consists of four patients $P = \{\text{Ane, Ben, Jac, Tom}\}$, five diagnosis $D = \{\text{viral fever, malaria, typhoid, stomach problem, chest pain}\}$ and five symptoms $S = \{\text{temperature, headache, stomach pain, chest pain}\}$. Table 1 represents the characteristic symptoms for the diagnoses concerned in which row indicates symptoms while column indicates diseases. Table 2 indicates the symptoms for each patient in which row indicate patients while column corresponds to various symptoms. Experts/physicians express their views about the disease of the patients with respect to symptoms in terms of membership degree, non-membership degree and hesitation degree in Table 1 and Table 2. In order to accomplish a proper diagnosis for each patient, we evaluate the proposed measure given by (4.1) between a diagnosis, symptoms and all patients. Finally, we assign to the i^{th} patient the diagnosis whose symptoms have the lowest amount of intuitionistic fuzzy divergence measure from patient's symptoms.

For some cases, experts give information about certain aspects of the disease and remain quiet for those unknown characteristics. It might not be feasible to make a decision for the experts/physicians on one inspection about the diseases of the patients. So, there is a need of effective approach, which will give certain information about the patient's condition and variation of symptoms and suggest diagnose accordingly. This can be done by reducing the hesitation margin, which result in an increment in both membership and non membership functions As a result, illustration of medical data by IFS, given by the triplet (μ, ν, π) will also undergo changes. However, we intend to reflect this change in the divergence measure which we are using for obtaining a proper diagnose and not in the medical data. We now replace the triplet (μ, ν, π) by $(\mu + \alpha\pi, \nu + \alpha\pi, \pi - 2\alpha\pi)$ in the measure (4.1), thereby obtaining a modified parametric symmetric divergence measure given by

$$D_\alpha (A||B) = -\log\left(\frac{1+\frac{1}{n}\sum_{i=1}^{n}\begin{pmatrix}\sqrt{(\mu_A(x_i)+\alpha\pi_A(x_i))\,(\mu_B(x_i)+\alpha\pi_B(x_i))}\\ +\sqrt{(\nu_A(x_i)+\alpha\pi_A(x_i))\,(\nu_B(x_i)+\alpha\pi_B(x_i))}\\ +\sqrt{(\pi_A(x_i)-2\alpha\pi_A(x_i))\,(\pi_B(x_i)-2\alpha\pi_B(x_i))}\end{pmatrix}}{2}\right). \tag{5.1}$$

Similarly, the parametric intuitionistic fuzzy divergence measure of (3.5) can be written as

$$D_{WY(\alpha)}(A||B) = \sum_{i=1}^{n}\left[\begin{array}{l}\begin{pmatrix}(\mu_A(x_i)+\alpha\pi_A(x_i))\ln\left(\frac{2\times(\mu_A(x_i)+\alpha\pi_A(x_i))}{(\mu_A(x_i)+\alpha\pi_A(x_i))+(\mu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\nu_A(x_i)+\alpha\pi_A(x_i))\ln\left(\frac{2\times(\nu_A(x_i)+\alpha\pi_A(x_i))}{(\nu_A(x_i)+\alpha\pi_A(x_i))+(\nu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\pi_A(x_i)-2\alpha\pi_A(x_i))\ln\left(\frac{2\times(\pi_A(x_i)-2\alpha\pi_A(x_i))}{(\pi_A(x_i)-2\alpha\pi_A(x_i))+(\pi_B(x_i)-2\alpha\pi_B(x_i))}\right)\end{pmatrix}\\ +\begin{pmatrix}(\mu_B(x_i)+\alpha\pi_B(x_i))\ln\left(\frac{2\times(\mu_B(x_i)+\alpha\pi_B(x_i))}{(\mu_A(x_i)+\alpha\pi_A(x_i))+(\mu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\nu_B(x_i)+\alpha\pi_B(x_i))\ln\left(\frac{2\times(\nu_B(x_i)+\alpha\pi_B(x_i))}{(\nu_A(x_i)+\alpha\pi_A(x_i))+(\nu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\pi_B(x_i)-2\alpha\pi_B(x_i))\ln\left(\frac{2\times(\pi_B(x_i)-2\alpha\pi_B(x_i))}{(\pi_A(x_i)-2\alpha\pi_A(x_i))+(\pi_B(x_i)-2\alpha\pi_B(x_i))}\right)\end{pmatrix}\end{array}\right]. \tag{5.2}$$

As shown in section 4, Vlachos & Sergiadis measure [13], $D_{VS}(A||B)$ assumes negative values and replacing the triplet (μ, ν, π) by $(\mu + \alpha\pi, \nu + \alpha\pi, \pi - 2\alpha\pi)$ will only worsen the situation. Replace (μ, ν, π) by $(\mu + \alpha\pi, \nu + (1-\alpha)\,\pi, 0)$ in (3.2), the resulting parametric divergence measure for IFS given by

$$D_{VS(\alpha)}(A||B) = \sum_{i=1}^{n}\begin{pmatrix}(\mu_A(x_i)+\alpha\pi_A(x_i))\ln\left(\frac{2\times(\mu_A(x_i)+\alpha\pi_A(x_i))}{(\mu_A(x_i)+\alpha\pi_A(x_i))+(\mu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\nu_A(x_i)+(1-\alpha)\pi_A(x_i))\ln\left(\frac{2\times(\nu_A(x_i)+(1-\alpha)\pi_A(x_i))}{(\nu_A(x_i)+(1-\alpha)\pi_A(x_i))+(\nu_B(x_i)+(1-\alpha)\pi_B(x_i))}\right)\\ +(\mu_B(x_i)+\alpha\pi_B(x_i))\ln\left(\frac{2\times(\mu_B(x_i)+\alpha\pi_B(x_i))}{(\mu_A(x_i)+\alpha\pi_A(x_i))+(\mu_B(x_i)+\alpha\pi_B(x_i))}\right)\\ +(\nu_B(x_i)+(1-\alpha)\pi_B(x_i))\ln\left(\frac{2\times(\nu_B(x_i)+(1-\alpha)\pi_B(x_i))}{(\nu_A(x_i)+(1-\alpha)\pi_A(x_i))+(\nu_B(x_i)+(1-\alpha)\pi_B(x_i))}\right)\end{pmatrix}, \tag{5.3}$$

will always be positive because the pair $(\mu+\alpha\pi, \nu+(1-\alpha)\,\pi, 0)$ forms a probability distribution and the measure (5.3) will satisfy the Shannon inequality. The same approach could be applied for other measures also; however we restrict ourselves to the three parametric divergence measures given by (5.1), (5.2) and (5.3) and utilized these measures for obtaining a proper diagnose for the data given in Tables 1 and 2.

The obtained result by the proposed measure (4.1) is presented in Table 3. By analyzing the variation of the parameter α, we evaluate all the feasible medical decision making results for the parametric divergence measures given by (5.1), (5.2) and (5.3), presented in Table 4-15. The symbol '*′ indicates the disease of the patient in Tables 3-15. From the tables, it can be seen that for smaller values of α, the differences between the lowest scores is very small, which results in so much uncertainty in prediction of disease and need an advanced diagnosis. However, from the results it is examined that with increase in value of α the difference between lowest scores also increases, which result in reduction in uncertainty. As we increase the value of α, we move towards the optimal solution and after some values of α, we get the same results.

From the evaluated results, it is very much clear that Ben and Tom suffer from stomach problem and viral fever, respectively. This diagnose does not change even after variation in parameter α. Therefore, we can say that stomach problem and viral fever are the correct diagnose for Ben and Tom. Ane suffers from Viral Fever in eleven out of thirteen approaches, whereas Jac suffers from typhoid in three out of the thirteen methods and this diagnose does not change with change in parameter α. So, we can say that Ane and Jac have more chances to suffer from viral fever and typhoid, respectively, *i.e.*, viral fever is the correct diagnose for Ane and typhoid is the correct diagnose for Jac. Finally, comparisons of the evaluated results with the existing results are shown in Table 16. The optimal decision is provided in Table 17. We reach at the discussion from the optimal results that Ane, Ben, Jac and Tom suffer from viral fever, stomach problem, typhoid and viral fever, respectively. The results show that the proposed parametric intuitionistic fuzzy divergence measure (PIFDM) is more efficient and comprehensive than the proposed divergence measure and the existing intuitionistic fuzzy divergence measures.

Table 1. Symptoms characteristics for the diagnosis

	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4, 0.0, 0.6)	(0.7, 0.0, 0.3)	(0.3, 0.3, 0.4)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
Headache	(0.3, 0.5,0.2)	(0.2, 0.6,0.2)	(0.6, 0.1,0.3)	(0.2, 0.4,0.4)	(0.0, 0.8,0.2)
Stomach pain	(0.1, 0.7,0.2)	(0.0, 0.9,0.1)	(0.2, 0.7,0.1)	(0.8, 0.0,0.2)	(0.2, 0.8,0.0)
Cough	(0.4, 0.3,0.3)	(0.7, 0.0,0.3)	(0.2, 0.6, 0.2)	(0.2, 0.7, 0.1)	(0.2, 0.8,0.0)
Chest pain	(0.1, 0.7,0.2)	(0.1, 0.8,0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7,0.1)	(0.8, 0.1,0.1)

Table 2. Symptoms characteristics for the patients

	Temperature	Headache	Stomach pain pain	Cough	Chest Pain
Ane	(0.8, 0.1, 0.1)	(0.6, 0.1, 0.3)	(0.2, 0.8, 0.0)	(0.6, 0.1, 0.3)	(0.1, 0.6, 0.3)
Ben	(0.0, 0.8, 0.2)	(0.4, 0.4, 0.2)	(0.6, 0.1, 0.3)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
Jac	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.0, 0.6, 0.4)	(0.2, 0.7, 0.1)	(0.0, 0.5, 0.5)
Tom	(0.6, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)

Table 3. Diagnosed results for the proposed divergence measure $D(A||B)$ given by (10)

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.06653*	0.07089	0.07665	0.19619	0.23044
Ben	0.15401	0.28623	0.08470	0.02556*	0.16945
Jac	0.08641*	0.14462	0.09889	0.21979	0.29012
Tom	0.03919*	0.08098	0.08202	0.12011	0.17527

Table 4. Diagnosed results for the proposed PIFDM given by (5.1) for $\alpha = 1/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.04865*	0.04868	0.06553	0.16462	0.19835
Ben	0.10968	0.20945	0.06852	0.01539*	0.13894
Jac	0.05796*	0.10453	0.06760	0.15939	0.21635
Tom	0.02885*	0.05716	0.07191	0.10052	0.14825

Table 5. Diagnosed results by the proposed PIFDM given by (5.1) for $\alpha = 2/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.03833*	0.03968	0.05550	0.14711	0.17917
Ben	0.09401	0.18315	0.05985	0.01214*	0.12188
Jac	0.04724*	0.08918	0.05131	0.13694	0.18566
Tom	0.02456*	0.04892	0.06281	0.09004	0.13183

Table 6. Diagnosed results by the proposed PIFDM given by (5.1) for $\alpha = 3/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.02971*	0.03281	0.04628	0.13289	0.16329
Ben	0.08303	0.16500	0.05283	0.00985*	0.10752
Jac	0.03919	0.07781	0.03786*	0.12078	0.16221
Tom	0.02118*	0.04293	0.05447	0.08163	0.11779

Table 7. Diagnosed results by the proposed PIFDM given by (5.1) for $\alpha = 4/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.02214*	0.02705	0.03772	0.12066	0.14946
Ben	0.07448	0.15101	0.04679	0.00804*	0.09476
Jac	0.03250	0.06849	0.02604*	0.10790	0.14255
Tom	0.01830*	0.03805	0.04675	0.07450	0.10524

Table 8. Diagnosed results by the proposed PIFDM given by (5.1) for $\alpha = 5/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.01532*	0.02201	0.02968	0.10985	0.13713
Ben	0.06747	0.13962	0.04145	0.00653*	0.08311
Jac	0.02671	0.06051	0.01535*	0.09713	0.12537
Tom	0.01575*	0.03386	0.03954	0.06827	0.09375

Table 9. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by(5.2) for $\alpha = 1/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.45976	0.43628*	0.55136	1.37680	1.64236
Ben	0.92426	1.54470	0.53124	0.10401*	0.95177
Jac	0.47121	0.74735	0.38076*	1.27401	1.48427
Tom	0.23591*	0.39509	0.51158	0.85478	1.04488

Table 10. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (5.2) for $\alpha = 2/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.36477	0.36396*	0.47743	1.26552	1.51648
Ben	0.85967	1.50240	0.55741	0.11792*	1.00938
Jac	0.46159	0.78966	0.40735*	1.20364	1.51190
Tom	0.23870*	0.43053	0.54248	0.81759	0.98180

Table 11. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (5.2) for $\alpha = 3/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.28521*	0.30538	0.41010	1.16782	1.40681
Ben	0.77465	1.39078	0.49864	0.09603*	0.91623
Jac	0.38483	0.70365	0.30948*	1.07886	1.35829
Tom	0.20574*	0.38079	0.48186	0.74788	0.92980

Table 12. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (5.2) for $\alpha = 4/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.21540*	0.25503	0.34806	1.08008	1.30927
Ben	0.70412	1.29702	0.44797	0.07843*	0.83334
Jac	0.32093	0.63023	0.22601*	0.97597	1.22859
Tom	0.17737*	0.33827	0.42602	0.68645	0.88613

Table 13. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (5.2) for $\alpha = 5/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.15264*	0.21040	0.29035	1.00015	1.22136
Ben	0.64385	1.21611	0.40325	0.06364*	0.75809
Jac	0.26535	0.56545	0.15219*	0.88796	1.11529
Tom	0.15222*	0.30080	0.37424	0.63156	0.84919

Table 14. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (23) for $\alpha = 1/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.20815*	0.21297	0.30010	0.92113	1.22340
Ben	0.55628	1.18120	0.33966	0.11625*	0.75246
Jac	0.35884	0.49761	0.26578*	1.21481	1.46664
Tom	0.19583*	0.26214	0.31911	0.64068	0.83245

Table 15. Diagnosed results for the measure $D_{WY(\alpha)}(A||B)$ given by (23) for $\alpha = 2/10$

	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Ane	0.18675*	0.20587	0.29154	0.93153	1.20827
Ben	0.56465	1.17054	0.34533	0.09624*	0.73846
Jac	0.31707	0.50075	0.21025*	1.09441	1.33324
Tom	0.18107*	0.26639	0.32796	0.63063	0.82592

Table 16. Comparison of Results

	De et. al [3]	**Szmidt & Kacprzyk [9]**	**Szmidt & Kacprzyk [10]**	**C. M Own [8] for p = 0**	**C. M Own [8] for p = 1**	**Boran [5]**	**Vlachos & Sergiadis [13]**
Ane	Malaria	Malaria	Viral Fever	Viral Fever	Viral Fever	Viral Fever	Viral Fever
Ben	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem
Jac	Malaria	Typhoid	Typhoid	Typhoid	Stomach Problem	Typhoid	Typhoid
Tom	Malaria	Viral Fever	Malaria	Viral Fever	Viral Fever	Viral Fever	Viral Fever
	Wei & Yei [15]	**Proposed measure** $D(A\|\|B)$ **given by (10)**	**Proposed PIFDM given by** (5.1) **for** $\alpha = 1/10, 2/10,$	**Proposed PIFDM given by** (5.1) **for** $\alpha = 3/10, 4/10, 5/10$	**Parametric measure** $D_{WY(\alpha)}(A\|\|B)$ **given by** (5.2) **for** $\alpha = 1/10, 2/10$	**Parametric measure** $D_{WY(\alpha)}(A\|\|B)$ **given by** (5.2) **for** $\alpha = 3/10, 4/10, 5/10$	**Parametric measure** $D_{WY(\alpha)}(A\|\|B)$ **given by** (5.3) **for** $\alpha = 1/10, 2/10$
Ane	Malaria	Viral Fever	Viral Fever	Viral Fever	Malaria	Viral Fever	Viral Fever
Ben	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem	Stomach Problem
Jac	Typhoid	Viral Fever	Viral Fever	Typhoid	Typhoid	Typhoid	Typhoid
Tom	Viral Fever	Viral Fever	Viral Fever	Viral Fever	Viral Fever	Viral Fever	Viral Fever

Table 17. Optimal Decision for medical diagnosis problem

	Patients			
	Ane	Ben	Jac	Tom
Optimal Decision	Viral Fever	Stomach Problem	Typhoid	Viral Fever

6. Conclusion

In the present work, we have emphasized on the problem of decision making in medical investigations and introduced a method in coping with this problem. We have defined a new divergence measure for (IFS) and proved some elegant properties apart from the axioms, which show the strength of the measure. We have scrutinized some existing measures of intuitionistic fuzzy divergence proposed by several researchers and demonstrated their counter-intuitive cases. We have incorporated the parameter α in the IFS and extend the proposed divergence measure to parametric intuitionistic fuzzy divergence measure (PIFDM). Taking the benefit of the parameter α, defined the parametric form of the Vlachos & Sergiadis measure [13] and Wei & Yei measure [14]. Finally, a comparison has done, which show the proposed PIFDM is more flexible, utilitarian and comprehensive than the proposed divergence measure and the existing intuitionistic fuzzy divergence measures. In future, we will extend our work to inter-valued intuitionistic fuzzy divergence measure and find out its other real life applications.

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