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LIE SYMMETRY ANALYSIS AND CONSERVATION LAWS FOR (2+1)-DIMENSIONAL COUPLED TIME-FRACTIONAL NONLINEAR SCHRÖDINGER EQUATIONS

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Abstract Lie symmetry analysis is used to solve coupled time-fractional nonlinear Schrödinger equations. Having established the Lie point symmetries of the original equations, they are reduced to nonlinear fractional ordinary differential equations. Exact solutions are found and then subjected to in-depth convergence analysis. Also, conservation laws for the coupled time-fractional nonlinear Schrödinger equations are derived systematically by leveraging the powerful Ibragimov method.

Keywords Lie symmetry analysis, coupled time-fractional nonlinear Schrödinger equations, exact solutions, conservation laws.

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1. Introduction

The nonlinear Schrödinger equation (NLSE) [4, 7, 23, 25] is a significant mathematical model in many fields, including fiber-optic communication, plasma physics, superfluid mechanics, and quantum mechanics. Herein, we study (2+1)-dimensional

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coupled time-fractional NLSEs [17] of the form

$$\begin{cases} i^C D_t^\alpha p = -\mu \Delta p - \gamma(|p|^2 + a|q|^2)p, & (x,y,t) \in \Omega \times (0,T], \\ i^C D_t^\alpha q = -\mu \Delta q - \gamma(|q|^2 + a|p|^2)q, & (x,y,t) \in \Omega \times (0,T], \end{cases} \tag{1.1}$$

where $i^2 = -1$, $0 < \alpha \leq 1$, $\Omega = (c,d) \times (h,l)$, the symbol ${}^C D_t^\alpha$ represents the Riemann-Liouville (RL) fractional derivative, p and q are unknown complex functions representing the amplitudes or envelopes of two wave packets, Δ denotes the Laplace operator, the positive value μ represents the group-velocity dispersion, $\gamma > 0$ signifies the self-focusing of pulses in a birefringent medium, and $a > 0$ denotes cross-phase modulation.

Because of its inherent nonlinearities, the NLSE often cannot be solved directly to obtain exact solutions by conventional methods such as the first-integral approach [11] or Darboux transformation [24]. Therefore, finding exact solutions to the NLSE is extremely challenging. In 2016, Eslami [10] used the Kudryashov approach to explore traveling-wave solutions for (1+1)-dimensional coupled fractional NLSEs; the Kudryashov approach offers accurate analysis of complex nonlinear equations, but its computational process is relatively complex, especially for high-order or more-complex equations. In 2023, Onder [22] combined the Kudryashov method with the Kudryashov auxiliary equations, thereby not only retaining the advantage of finding exact solutions by using the Kudryashov approach but also simplifying the solution process by introducing auxiliary equations, and thus obtaining soliton solutions for (1+1)-dimensional coupled fractional NLSEs more efficiently. However, that new method can require considerable mathematical skill to construct the auxiliary equations, and its adaptability to specific problems awaits further verification. At the same time, Ahmad et al. [1] found periodic rogue-wave solutions for (1+1)-dimensional coupled fractional NLSEs via the modified exponential function method. Akram et al. [5] used the extended $\left(\frac{G'}{G^2}\right)$-expansion approach and the modified simple equation method to generate soliton solutions for the space-time fractional nonlinear Schrödinger equation.

To derive exact solutions for nonlinear partial differential equations (PDEs), researchers have employed various methods including the $(\exp(-\phi(\epsilon)))$-expansion method [2,6], the generalized exponential rational function method, and the $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion method [3]. Additionally, Gao [12,13] has successfully applied similarity reductions for deriving exact solutions to nonlinear PDEs. Another powerful technique for finding exact solutions to PDEs is Lie symmetry analysis (LSA) [21]. The significance of Lie symmetry analysis lies in its ability to reduce the complexity of differential equations, thereby facilitating the identification of invariant solutions and providing insights into the underlying structures of the equations. In 1998, Buckwar and Luchko [8] used scale transformation groups to develop group-invariant solutions to the fractional diffusion-wave equation, which led to the application of LSA to fractional PDEs. Gazizov et al. [14] used LSA to solve specific fractional differential equations, and in recent years scholars have made widespread use of this method to investigate nonlinear fractional models with physical backgrounds [9,15,18–20,26,27].

Many scholars have concentrated their efforts on studying single time-fractional PDEs, whereas relatively little attention has been devoted to exact solutions of coupled time-fractional PDEs. Remarkably, there is still a scarcity of research into coupled equations that involve complex variables. Although the coupled time-fractional NLSEs discussed in this paper have attracted some interest, there remains significant potential for further exploration. Therefore, a deeper examination of this nonlinear model is essential. This paper makes a novel contribution by deriving exact solutions for the coupled NLSEs using LSA and Ibragimov method [16]. Additionally, it examines the symmetry reduction and conservation laws of the (2+1)-dimensional coupled time-fractional NLSEs.

This paper is organized as follows. In Section 2, we investigate symmetry reduction of the (2+1)-dimensional coupled time-fractional NLSEs. In Section 3, we derive a class of power-series solutions for (2+1)-dimensional coupled time-fractional NLSEs and then conduct convergence analysis. In Section 4, we use a new conservation theorem to construct conservation laws for (1.1), and finally we present our conclusions in Section 5.

2. Lie symmetry analysis for (2+1)-dimensional coupled time-fractional nonlinear Schrödinger equations

Here, we present the formulation of the RL fractional derivative, which is given by

$$ {}^{C}D_t^{\alpha}V(t,x) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\dfrac{\partial^m}{\partial t^m}\displaystyle\int_0^t (t-h)^{m-\alpha-1}V(h,x)dh, 0 \le m-1 < \alpha < m, m \in \mathbf{N}, \\ \dfrac{\partial^m V(t,x)}{\partial t^m}, \qquad \alpha = m \in \mathbf{N}, \end{cases} \tag{2.1} $$

with the Euler gamma function defined as $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$.

Definition 2.1. The Erdélyi–Kober (EK) fractional differential operator is defined as

$$ (\mathcal{P}_{\Omega}^{\tau,\alpha}\mathcal{F})(z) := \prod_{j=0}^{m-1}(\tau + j - \frac{1}{\Omega}z\frac{d}{dz})(\mathcal{K}_{\Omega}^{\tau+\alpha,m-\alpha}\mathcal{F})(z), \tag{2.2} $$

$$ m = \begin{cases} \alpha, & \alpha \in \mathbf{N}, \\ [\alpha]+1, & \alpha \notin \mathbf{N}, \end{cases} $$

where

$$ (\mathcal{K}_{\Omega}^{\tau,\alpha}\mathcal{F})(z) := \begin{cases} \mathcal{F}(z), & \alpha = 0, \\ \dfrac{1}{\Gamma(\alpha)}\displaystyle\int_1^{\infty}(u-1)^{\alpha-1}u^{-(\tau+\alpha)}\mathcal{F}(zu^{\frac{1}{\Omega}})du, & \alpha > 0 \end{cases} \tag{2.3} $$

is the EK fractional integral operator. Here, τ is related to α, and z is a function of the variables t, x, and y.

To facilitate our analysis, we define the complex variables

$$p(x,y,t) = u(x,y,t) + iv(x,y,t), \quad q(x,y,t) = s(x,y,t) + iw(x,y,t), \tag{2.4}$$

where $u(x,y,t)$, $v(x,y,t)$, $s(x,y,t)$, and $w(x,y,t)$ represent unknown real functions.

We substitute (2.4) into (1.1) and separate the real and imaginary parts to obtain the following equations:

$$\begin{cases} {}^C D_t^\alpha u + \mu\Delta v + \gamma[u^2+v^2+a(s^2+w^2)]v = 0, \\ {}^C D_t^\alpha v - \mu\Delta u - \gamma[u^2+v^2+a(s^2+w^2)]u = 0, \\ {}^C D_t^\alpha s + \mu\Delta w + \gamma[s^2+w^2+a(u^2+v^2)]w = 0, \\ {}^C D_t^\alpha w - \mu\Delta s - \gamma[s^2+w^2+a(u^2+v^2)]s = 0. \end{cases} \tag{2.5}$$

Consider the one-parameter Lie-group point transformations of (2.5), i.e.,

$$\begin{aligned} \widetilde{x} &= x + \varepsilon\xi(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{y} &= y + \varepsilon\rho(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{t} &= t + \varepsilon\tau(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{u} &= u + \varepsilon\eta^1(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{v} &= v + \varepsilon\eta^2(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{s} &= s + \varepsilon\eta^3(x,y,t,v,u,w,s) + O(\varepsilon^2), \\ \widetilde{w} &= w + \varepsilon\eta^4(x,y,t,v,u,w,s) + O(\varepsilon^2), \end{aligned} \tag{2.6}$$

where $\varepsilon \ll 1$ is the Lie group parameter, and ξ, ρ, τ, η^1, η^2, η^3, and η^4 are infinitesimals for the dependent and independent variables. There exists a set of vector fields in the relevant Lie algebra of symmetries, i.e.,

$$X = \xi\frac{\partial}{\partial x} + \tau\frac{\partial}{\partial t} + \rho\frac{\partial}{\partial y} + \eta^1\frac{\partial}{\partial u} + \eta^2\frac{\partial}{\partial v} + \eta^3\frac{\partial}{\partial s} + \eta^4\frac{\partial}{\partial w}. \tag{2.7}$$

A set of symmetric determining equations results from solving the invariant surface condition after the infinitesimal generators have been defined, i.e.,

$$\begin{cases} Pr^{(\alpha,2)}X({}^C D_t^\alpha u + \mu\Delta v + \gamma[u^2+v^2+a(s^2+w^2)]v|_{(2.5)} = 0, \\ Pr^{(\alpha,2)}X({}^C D_t^\alpha v - \mu\Delta u - \gamma[u^2+v^2+a(s^2+w^2)]u|_{(2.5)} = 0, \\ Pr^{(\alpha,2)}X({}^C D_t^\alpha s + \mu\Delta w + \gamma[s^2+w^2+a(u^2+v^2)]w|_{(2.5)} = 0, \\ Pr^{(\alpha,2)}X({}^C D_t^\alpha w - \mu\Delta s - \gamma[s^2+w^2+a(u^2+v^2)]s|_{(2.5)} = 0, \end{cases} \tag{2.8}$$

where the second-order prolongation operator of vector field X is represented by $Pr^{(\alpha,2)}X$, i.e.,

$$\begin{aligned} Pr^{(\alpha,2)}X = {} & \eta_1^{\alpha,t}\partial_{{}^C D_t^\alpha u} + \eta_2^{\alpha,t}\partial_{{}^C D_t^\alpha v} + \eta_3^{\alpha,t}\partial_{{}^C D_t^\alpha s} + \eta_4^{\alpha,t}\partial_{{}^C D_t^\alpha w} + \eta_1^{xx}\partial_{u_{xx}} \\ & + \eta_1^{yy}\partial_{u_{yy}} + \eta_2^{xx}\partial_{v_{xx}} + \eta_2^{yy}\partial_{v_{yy}} + \eta_3^{xx}\partial_{s_{xx}} + \eta_3^{yy}\partial_{s_{yy}} \\ & + \eta_4^{xx}\partial_{w_{xx}} + \eta_4^{yy}\partial_{w_{yy}} + X, \end{aligned} \tag{2.9}$$

where

$$\begin{aligned}\eta_1^x &= D_x(\eta^1 - \xi u_x - \rho u_y - \tau u_t) + \xi D_x(u_x) + \rho D_x(u_y) + \tau D_x(u_t),\\ \eta_1^{xx} &= D_x^2(\eta^1 - \xi u_x - \rho u_y - \tau u_t) + \xi D_x^2(u_x) + \rho D_x^2(u_y) + \tau D_x^2(u_t),\\ &\cdots .\end{aligned} \tag{2.10}$$

Here, D_x denotes the total differential operator in terms of x, i.e.,

$$D_x = \partial_x + u_x\partial_u + v_x\partial_v + s_x\partial_s + w_x\partial_w + u_{xx}\partial_{u_x} + v_{xx}\partial_{v_x} + s_{xx}\partial_{s_x} + w_{xx}\partial_{w_x} + \cdots . \tag{2.11}$$

The structure of the RL fractional derivative operator, characterized by its fixed lower limit in the integral, must remain invariant under the infinitesimal transformations described in (2.6). The invariant condition is

$$\tau(x, y, t, v, u, w, s)|_{t=0} = 0, \tag{2.12}$$

and the α-th extended infinitesimals involving the RL fractional derivative with (2.12) are

$$\begin{aligned}\eta_j^{\alpha,t} =& {}^C D_t^\alpha \eta^j + (\eta^j_{u^j} - \alpha D_t\tau)\,{}^C D_t^\alpha u^j - u^j\,{}^C D_t^\alpha \eta^j_{u^j} + \lambda^j\\ &+ \sum_{n=1}^{+\infty}\left[\begin{pmatrix}\alpha\\ n\end{pmatrix}\frac{\partial^n \eta^j_{u^j}}{\partial t^n} - \begin{pmatrix}\alpha\\ n+1\end{pmatrix} D_t^{n+1}(\tau)\right]{}^C D_t^{\alpha-n}(u^j)\\ &- \sum_{n=1}^{+\infty}\begin{pmatrix}\alpha\\ n\end{pmatrix} D_t^n(\xi)\,{}^C D_t^{\alpha-n}(u^j_x),\, j = 1, 2, 3, 4,\end{aligned} \tag{2.13}$$

where

$$\begin{aligned}\lambda^j =& \sum_{n=2}^{+\infty}\sum_{m=2}^{n}\sum_{k=2}^{m}\sum_{r=0}^{k-1}\binom{\alpha}{n}\binom{n}{m}\binom{k}{r}\frac{1}{k!}\frac{t^{n-\alpha}}{\Gamma(n+1-\alpha)}\\ &\times [-u^j]^r \frac{\partial^m}{\partial t^m}[-u^j]^{k-r}\frac{\partial^{n-m+k}\eta^j}{\partial t^{n-m}\partial[u^j]^k}, \qquad j = 1, 2, 3, 4\end{aligned}$$

and $\binom{\alpha}{n} = \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1-n)\Gamma(n+1)}$.

Equations (2.8) and (2.9) provide us with the following:

$$\begin{aligned}&\eta_1^{\alpha,t} + \mu(\eta_2^{xx} + \eta_2^{yy}) + \gamma[2uv\eta^1 + \eta^2(u^2 + 3v^2 + a(s^2 + w^2)) + 2a(s\eta^3 + w\eta^4)v] = 0,\\ &\eta_2^{\alpha,t} - \mu(\eta_1^{xx} + \eta_1^{yy}) - \gamma[\eta^1(3u^2 + v^2 + a(s^2 + w^2)) + 2uv\eta^2 + 2a(s\eta^3 + w\eta^4)u] = 0,\\ &\eta_3^{\alpha,t} + \mu(\eta_4^{xx} + \eta_4^{yy}) + \gamma[\eta^4(3w^2 + s^2 + a(u^2 + v^2)) + 2sw\eta^3 + 2a(u\eta^1 + v\eta^2)w] = 0,\\ &\eta_4^{\alpha,t} - \mu(\eta_3^{xx} + \eta_3^{yy}) - \gamma[\eta^3(3s^2 + w^2 + a(u^2 + v^2)) + 2ws\eta^4 + 2a(u\eta^1 + v\eta^2)s] = 0,\end{aligned} \tag{2.14}$$

and adding (2.10) and (2.13) to (2.14) produces the following defining equations for

(1.1):

$$\begin{cases} \xi_y = \xi_t = \xi_u = \xi_v = \xi_s = \xi_w = \tau_x = \tau_y = \tau_u = \tau_v = \tau_s = \tau_w = 0, \\ \eta^1_v = \eta^1_s = \eta^1_w = \eta^2_u = \eta^2_s = \eta^2_w = \eta^3_v = \eta^3_u = \eta^3_w = \eta^4_u = \eta^4_v = \eta^4_s = 0, \\ \alpha\tau_t - 2\xi_x = 0, \quad \xi_x = \rho_y, \quad \rho_x = \rho_t = \rho_u = \rho_v = \rho_s = \rho_w = 0, \\ \alpha\tau_t vw - \eta^3_s vw + 2\eta^2 w + \eta^4 v = 0, \quad \alpha\tau_t uw - \eta^3_s uw + 2\eta^1 w + \eta^4 u = 0, \\ \xi_x = \rho_y, \quad \eta^3_s = \eta^4_w, \quad 3\eta^4 - \eta^3_s w + \alpha\tau_t w = 0, \\ \binom{\alpha}{n}\dfrac{\partial^n \eta^1_u}{\partial t^n} - \binom{\alpha}{n+1} D^{n+1}_t(\tau) = 0, \\ \binom{\alpha}{n}\dfrac{\partial^n \eta^2_v}{\partial t^n} - \binom{\alpha}{n+1} D^{n+1}_t(\tau) = 0, \\ \binom{\alpha}{n}\dfrac{\partial^n \eta^3_s}{\partial t^n} - \binom{\alpha}{n+1} D^{n+1}_t(\tau) = 0, \\ \binom{\alpha}{n}\dfrac{\partial^n \eta^4_w}{\partial t^n} - \binom{\alpha}{n+1} D^{n+1}_t(\tau) = 0, \quad n = 1, 2, \cdots . \end{cases} \tag{2.15}$$

The vector fields of the one-parameter Lie group for the Lie point symmetry of (2.5) are calculated from the solutions of the aforementioned equations, i.e.,

$$X_1 = \alpha x\frac{\partial}{\partial x} + \alpha y\frac{\partial}{\partial y} + 2t\frac{\partial}{\partial t} - \alpha u\frac{\partial}{\partial u} - \alpha v\frac{\partial}{\partial v} - \alpha s\frac{\partial}{\partial s} - \alpha w\frac{\partial}{\partial w},\ X_2 = \frac{\partial}{\partial x},\ X_3 = \frac{\partial}{\partial y}. \tag{2.16}$$

However, because vector fields X_2 and X_3 do not provide physically meaningful results, we consider only the case of X_1, whose characteristic equation is

$$\frac{dx}{\alpha x} = \frac{dy}{\alpha y} = \frac{dt}{2t} = \frac{du}{-\alpha u} = \frac{dv}{-\alpha v} = \frac{ds}{-\alpha s} = \frac{dw}{-\alpha w}. \tag{2.17}$$

Also, the solutions that remain unchanged under the symmetry-group transformations are

$$z = (x-y)t^{-\frac{\alpha}{2}},\ u = f_1(z)t^{-\frac{\alpha}{2}},\ v = f_2(z)t^{-\frac{\alpha}{2}},\ s = f_3(z)t^{-\frac{\alpha}{2}},\ w = f_4(z)t^{-\frac{\alpha}{2}}. \tag{2.18}$$

We use (2.5) to derive the subsequent pertinent outcomes by using the similarity variables alongside the group-invariant solutions.

Theorem 2.1. *The transformations in* (2.18) *reduce the (2+1)-dimensional coupled time-fractional NLSEs in* (2.5) *to fractional ordinary differential equations, i.e.,*

$$\begin{cases} (\mathcal{P}^{1-\frac{3}{2}\alpha,\alpha}_{\frac{2}{\alpha}} f_1)(z) + 2\mu f_2'' + \gamma[f_1^2 f_2 + f_2^3 + a(f_2 f_3^2 + f_2 f_4^2)] = 0, \\ (\mathcal{P}^{1-\frac{3}{2}\alpha,\alpha}_{\frac{2}{\alpha}} f_2)(z) - 2\mu f_1'' - \gamma[f_1 f_2^2 + f_1^3 + a(f_1 f_3^2 + f_1 f_4^2)] = 0, \\ (\mathcal{P}^{1-\frac{3}{2}\alpha,\alpha}_{\frac{2}{\alpha}} f_3)(z) + 2\mu f_4'' + \gamma[f_3^2 f_4 + f_4^3 + a(f_1^2 f_4 + f_2^2 f_4)] = 0, \\ (\mathcal{P}^{1-\frac{3}{2}\alpha,\alpha}_{\frac{2}{\alpha}} f_4)(z) - 2\mu f_3'' - \gamma[f_3 f_4^2 + f_3^3 + a(f_1^2 f_3 + f_2^2 f_3)] = 0 \end{cases} \tag{2.19}$$

with the EK fractional differential operator.

Proof. For $n-1<\alpha<n$, where n is a positive integer, a specific transformation reduces the equations to a simpler form, i.e.,

$$^{C}D_t^{\alpha}u=\frac{\partial^n}{\partial t^n}\left[\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-h)^{n-1-\alpha}h^{-\frac{\alpha}{2}}f_1((x-y)h^{-\frac{\alpha}{2}})dh\right]. \tag{2.20}$$

Upon setting $r=\frac{t}{h}$, we obtain the differential relation $dh=-\frac{t}{r^2}dr$. Consequently, (2.20) is transformed into the following form:

$$\begin{aligned}^{C}D_t^{\alpha}u&=\frac{\partial^n}{\partial t^n}\left[\frac{1}{\Gamma(n-\alpha)}\int_0^t(\frac{t}{r})^{n-\alpha-1}(r-1)^{n-\alpha-1}t^{-\frac{\alpha}{2}}r^{\frac{\alpha}{2}}f_1(zr^{\frac{\alpha}{2}})(-\frac{t}{r^2}dr)\right]\\&=\frac{\partial^n}{\partial t^n}\left[\frac{t^{n-\frac{3\alpha}{2}}}{\Gamma(n-\alpha)}\int_1^{\infty}r^{-(n+1-\frac{3\alpha}{2})}(r-1)^{n-1-\alpha}f_1(zr^{\frac{\alpha}{2}})dr\right]\\&=\frac{\partial^n}{\partial t^n}\left[t^{n-\frac{3\alpha}{2}}(\mathcal{K}_{\frac{2}{\alpha}}^{1-\frac{\alpha}{2},n-\alpha}f_1)(z)\right].\end{aligned} \tag{2.21}$$

We simplify the right-hand side of (2.21) by applying the connection $z=t^{-\frac{\alpha}{2}}(x-y)$ with $\Phi\in C^1(0,\infty)$, and the expression becomes

$$t\frac{\partial}{\partial t}\Phi(z)=t\frac{\partial z}{\partial t}\frac{\partial\Phi(z)}{\partial z}=-\frac{\alpha}{2}z\frac{\partial\Phi(z)}{\partial z}. \tag{2.22}$$

From (2.22), we obtain

$$\begin{aligned}^{C}D_t^{\alpha}u&=\frac{\partial^n}{\partial t^n}\left[t^{n-\frac{3\alpha}{2}}(\mathcal{K}_{\frac{2}{\alpha}}^{1-\frac{\alpha}{2},n-\alpha}f_1)(z)\right]\\&=\frac{\partial^{n-1}}{\partial t^{n-1}}\left[\frac{\partial}{\partial t}(t^{n-\frac{3\alpha}{2}}(\mathcal{K}_{\frac{2}{\alpha}}^{1-\frac{\alpha}{2},n-\alpha}f_1)(z))\right]\\&=\frac{\partial^{n-1}}{\partial t^{n-1}}\left[t^{n-\frac{3\alpha}{2}-1}(n-\frac{3\alpha}{2}-\frac{\alpha}{2}z\frac{\partial}{\partial z})(\mathcal{K}_{\frac{2}{\alpha}}^{1-\frac{\alpha}{2},n-\alpha}f_1)(z)\right].\end{aligned} \tag{2.23}$$

After applying the above steps $n-1$ times, we obtain the following key result:

$$^{C}D_t^{\alpha}u=\cdots=t^{-\frac{3\alpha}{2}}\prod_{j=1}^{n}\left(1-\frac{3\alpha}{2}+j-\frac{\alpha}{2}z\frac{\partial}{\partial z}\right)(\mathcal{K}_{\frac{2}{\alpha}}^{1-\frac{\alpha}{2},n-\alpha}f_1)(z). \tag{2.24}$$

From the definition of the EK fractional differential operator, (2.24) becomes

$$^{C}D_t^{\alpha}u=t^{-\frac{3\alpha}{2}}(\mathcal{P}_{\frac{2}{\alpha}}^{1-\frac{3\alpha}{2},\alpha}f_1)(z). \tag{2.25}$$

The same procedure is used to transform the remaining equations in (2.5), resulting in

$$\begin{aligned}^{C}D_t^{\alpha}v&=t^{-\frac{3\alpha}{2}}(\mathcal{P}_{\frac{2}{\alpha}}^{1-\frac{3\alpha}{2},\alpha}f_2)(z),\\^{C}D_t^{\alpha}s&=t^{-\frac{3\alpha}{2}}(\mathcal{P}_{\frac{2}{\alpha}}^{1-\frac{3\alpha}{2},\alpha}f_3)(z),\\^{C}D_t^{\alpha}w&=t^{-\frac{3\alpha}{2}}(\mathcal{P}_{\frac{2}{\alpha}}^{1-\frac{3\alpha}{2},\alpha}f_4)(z).\end{aligned} \tag{2.26}$$

By substituting (2.18), (2.25), and (2.26) into (2.5), we validate (2.19). After much effort, we have arrived at the crux of this theorem, which involves transforming the governing equations into nonlinear fractional differential equations to facilitate their solutions, as outlined in the next section. □

3. Exact solutions

In this section, we derive several exact solutions to (2.19) by using the power-series method [19,27], then we demonstrate their convergence. We assume that the exact solutions to (2.19) take the following form:

$$\begin{aligned} f_1(z) &= \sum_{n=0}^{\infty} a_n z^n, \ f_2(z) = \sum_{n=0}^{\infty} b_n z^n, \\ f_3(z) &= \sum_{n=0}^{\infty} c_n z^n, \ f_4(z) = \sum_{n=0}^{\infty} d_n z^n. \end{aligned} \tag{3.1}$$

Then upon substituting (3.1) into (2.19), we derive the following results:

$$\begin{cases} \sum_{n=0}^{\infty} \frac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})} a_n z^n + 2\mu \sum_{n=0}^{\infty} (n+2)(n+1) b_{n+2} z^n + \gamma \Bigg[(\sum_{n=0}^{\infty} a_n z^n)^2 \times \\ (\sum_{n=0}^{\infty} b_n z^n) + (\sum_{n=0}^{\infty} b_n z^n)^3 + a(\sum_{n=0}^{\infty} b_n z^n)((\sum_{n=0}^{\infty} c_n z^n)^2 + (\sum_{n=0}^{\infty} d_n z^n)^2) \Bigg] = 0, \\ \sum_{n=0}^{\infty} \frac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})} b_n z^n - 2\mu \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} z^n - \gamma \Bigg[(\sum_{n=0}^{\infty} b_n z^n)^2 \times \\ (\sum_{n=0}^{\infty} a_n z^n) + (\sum_{n=0}^{\infty} a_n z^n)^3 + a(\sum_{n=0}^{\infty} a_n z^n)((\sum_{n=0}^{\infty} c_n z^n)^2 + (\sum_{n=0}^{\infty} d_n z^n)^2) \Bigg] = 0, \\ \sum_{n=0}^{\infty} \frac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})} c_n z^n + 2\mu \sum_{n=0}^{\infty} (n+2)(n+1) d_{n+2} z^n + \gamma \Bigg[(\sum_{n=0}^{\infty} c_n z^n)^2 \times \\ (\sum_{n=0}^{\infty} d_n z^n) + (\sum_{n=0}^{\infty} d_n z^n)^3 + a(\sum_{n=0}^{\infty} d_n z^n)((\sum_{n=0}^{\infty} a_n z^n)^2 + (\sum_{n=0}^{\infty} b_n z^n)^2) \Bigg] = 0, \\ \sum_{n=0}^{\infty} \frac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})} d_n z^n - 2\mu \sum_{n=0}^{\infty} (n+2)(n+1) c_{n+2} z^n - \gamma \Bigg[(\sum_{n=0}^{\infty} d_n z^n)^2 \times \\ (\sum_{n=0}^{\infty} c_n z^n) + (\sum_{n=0}^{\infty} c_n z^n)^3 + a(\sum_{n=0}^{\infty} c_n z^n)((\sum_{n=0}^{\infty} a_n z^n)^2 + (\sum_{n=0}^{\infty} b_n z^n)^2) \Bigg] = 0. \end{cases} \tag{3.2}$$

Comparing the coefficients in (3.2) for $n = 0$ yields the following:

$$\begin{cases} a_2 = \frac{1}{4\mu} \left[\frac{\Gamma(2-\frac{\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2})} b_0 - \gamma(b_0^2 a_0 + a_0^3 + a a_0(c_0^2 + d_0^2)) \right], \\ b_2 = -\frac{1}{4\mu} \left[\frac{\Gamma(2-\frac{\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2})} a_0 + \gamma(a_0^2 b_0 + b_0^3 + a b_0(c_0^2 + d_0^2)) \right], \\ c_2 = \frac{1}{4\mu} \left[\frac{\Gamma(2-\frac{\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2})} d_0 - \gamma(d_0^2 c_0 + c_0^3 + a c_0(a_0^2 + b_0^2)) \right], \\ d_2 = -\frac{1}{4\mu} \left[\frac{\Gamma(2-\frac{\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2})} c_0 + \gamma(c_0^2 d_0 + d_0^3 + a d_0(a_0^2 + b_0^2)) \right]. \end{cases} \tag{3.3}$$

For $n \geq 1$, we derive the following:

$$\begin{cases} a_{n+2} = \dfrac{1}{2\mu(n+1)(n+2)}\left[\dfrac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})}b_n - \gamma\left(\sum_{k=0}^{n}\sum_{j=0}^{k}(a_j a_{k-j} a_{n-k} \right.\right. \\ \qquad \left.\left. + a_j b_{k-j} b_{n-k}) + a\sum_{k=0}^{n}\sum_{j=0}^{k}(a_j c_{k-j} c_{n-k} + a_j d_{k-j} d_{n-k})\right)\right], \\ b_{n+2} = -\dfrac{1}{2\mu(n+1)(n+2)}\left[\dfrac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})}a_n + \gamma\left(\sum_{k=0}^{n}\sum_{j=0}^{k}(a_j a_{k-j} b_{n-k} \right.\right. \\ \qquad \left.\left. + b_j b_{k-j} b_{n-k}) + a\sum_{k=0}^{n}\sum_{j=0}^{k}(b_j c_{k-j} c_{n-k} + b_j d_{k-j} d_{n-k})\right)\right], \\ c_{n+2} = \dfrac{1}{2\mu(n+1)(n+2)}\left[\dfrac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})}d_n - \gamma\left(\sum_{k=0}^{n}\sum_{j=0}^{k}(c_j c_{k-j} c_{n-k} \right.\right. \\ \qquad \left.\left. + c_j d_{k-j} d_{n-k}) + a\sum_{k=0}^{n}\sum_{j=0}^{k}(a_j a_{k-j} c_{n-k} + b_j b_{k-j} c_{n-k})\right)\right], \\ d_{n+2} = -\dfrac{1}{2\mu(n+1)(n+2)}\left[\dfrac{\Gamma(2-\frac{\alpha}{2}+\frac{n\alpha}{2})}{\Gamma(2-\frac{3\alpha}{2}+\frac{n\alpha}{2})}c_n + \gamma\left(\sum_{k=0}^{n}\sum_{j=0}^{k}(c_j c_{k-j} d_{n-k} \right.\right. \\ \qquad \left.\left. + d_j d_{k-j} d_{n-k}) + a\sum_{k=0}^{n}\sum_{j=0}^{k}(a_j a_{k-j} d_{n-k} + b_j b_{k-j} d_{n-k})\right)\right]. \end{cases} \tag{3.4}$$

By applying (2.18), (3.3), and (3.4), exact solutions to (1.1) have been derived rigorously, i.e.,

$$\begin{aligned} p(x,y,t) &= \sum_{n=0}^{\infty}(a_n + ib_n)(x-y)^n t^{-\frac{(n+1)\alpha}{2}}, \\ q(x,y,t) &= \sum_{n=0}^{\infty}(c_n + id_n)(x-y)^n t^{-\frac{(n+1)\alpha}{2}}. \end{aligned} \tag{3.5}$$

To contextualize this study further, we investigate in depth the convergence of the explicit power-series solutions in (3.5). Regarding the expressions in (3.4), the following are readily apparent:

$$
\begin{aligned}
|a_{n+2}| \le |b_n| + \sum_{k=0}^{n}\sum_{j=0}^{k}(&|a_j||a_{k-j}||a_{n-k}| + |a_j||b_{k-j}||b_{n-k}| + |a_j||c_{k-j}||c_{n-k}| \\
&+ |a_j||d_{k-j}||d_{n-k}|), \\
|b_{n+2}| \le |a_n| + \sum_{k=0}^{n}\sum_{j=0}^{k}(&|a_j||a_{k-j}||b_{n-k}| + |b_j||b_{k-j}||b_{n-k}| + |b_j||c_{k-j}||c_{n-k}| \\
&+ |b_j||d_{k-j}||d_{n-k}|), \\
|c_{n+2}| \le |d_n| + \sum_{k=0}^{n}\sum_{j=0}^{k}(&|c_j||c_{k-j}||c_{n-k}| + |c_j||d_{k-j}||d_{n-k}| + |a_j||a_{k-j}||c_{n-k}| \\
&+ |b_j||b_{k-j}||c_{n-k}|), \\
|d_{n+2}| \le |c_n| + \sum_{k=0}^{n}\sum_{j=0}^{k}(&|c_j||c_{k-j}||d_{n-k}| + |d_j||d_{k-j}||d_{n-k}| + |a_j||a_{k-j}||d_{n-k}| \\
&+ |b_j||b_{k-j}||d_{n-k}|).
\end{aligned}
\tag{3.6}
$$

Next, we introduce the following four power series that are pivotal in our further analysis:

$$
R(z) = \sum_{n=0}^{\infty} r_n z^n,\ M(z) = \sum_{n=0}^{\infty} m_n z^n,\ H(z) = \sum_{n=0}^{\infty} h_n z^n,\ Q(z) = \sum_{n=0}^{\infty} q_n z^n. \tag{3.7}
$$

By defining $r_j = |a_j|$, $m_j = |b_j|$, $h_j = |c_j|$, and $q_j = |d_j|$ for all indices $j = 0, 1, 2, \cdots$, we have the following:

$$
\begin{aligned}
r_{n+2} &= m_n + \sum_{k=0}^{n}\sum_{j=0}^{k}(r_j r_{k-j} r_{n-k} + r_j m_{k-j} m_{n-k} + r_j h_{k-j} h_{n-k} + r_j q_{k-j} q_{n-k}), \\
m_{n+2} &= r_n + \sum_{k=0}^{n}\sum_{j=0}^{k}(r_j r_{k-j} m_{n-k} + m_j m_{k-j} m_{n-k} + m_j h_{k-j} h_{n-k} + m_j q_{k-j} q_{n-k}), \\
h_{n+2} &= q_n + \sum_{k=0}^{n}\sum_{j=0}^{k}(h_j h_{k-j} h_{n-k} + h_j q_{k-j} q_{n-k} + r_j r_{k-j} h_{n-k} + m_j m_{k-j} h_{n-k}), \\
q_{n+2} &= h_n + \sum_{k=0}^{n}\sum_{j=0}^{k}(h_j h_{k-j} q_{n-k} + q_j q_{k-j} q_{n-k} + r_j r_{k-j} q_{n-k} + m_j m_{k-j} q_{n-k}),
\end{aligned}
\tag{3.8}
$$

where $n = 0, 1, 2, \cdots$. Clearly, we have $|a_n| \le r_n$, $|b_n| \le m_n$, $|c_n| \le h_n$, and $|d_n| \le q_n$ for $n = 0, 1, 2, \cdots$, i.e., the majorant series of (3.1) are those in (3.7). From calculations, the following equations hold:

$$
\begin{aligned}
R(z) &= r_0 + r_1 z + (M + R^3 + RM^2 + RH^2 + RQ^2)z^2, \\
M(z) &= m_0 + m_1 z + (R + M^3 + R^2 M + H^2 M + Q^2 M)z^2, \\
H(z) &= h_0 + h_1 z + (Q + H^3 + Q^2 H + R^2 H + M^2 H)z^2, \\
Q(z) &= q_0 + q_1 z + (H + Q^3 + H^2 Q + R^2 Q + M^2 Q)z^2.
\end{aligned}
\tag{3.9}
$$

Next, we show that $R(z)$, $M(z)$, $H(z)$, and $Q(z)$ each have a positive radius of convergence. Regarding the implicit-function equations of the independent variable z, we show their convergence properties, i.e.,

$$\begin{aligned}
F_1(z,R,M,H,Q) &= R - r_0 - r_1 z - (M + R^3 + RM^2 + RH^2 + RQ^2)z^2,\\
F_2(z,R,M,H,Q) &= M - m_0 - m_1 z - (R + M^3 + R^2M + H^2M + Q^2M)z^2,\\
F_3(z,R,M,H,Q) &= H - h_0 - h_1 z - (Q + H^3 + Q^2H + R^2H + M^2H)z^2,\\
F_4(z,R,M,H,Q) &= Q - q_0 - q_1 z - (H + Q^3 + H^2Q + R^2Q + M^2Q)z^2.
\end{aligned} \tag{3.10}$$

The functions $F_j(z,R,M,H,Q)$ for $j = 1,2,3,4$ are analytic in the neighborhood of $(0, r_0, m_0, h_0, q_0)$ and each satisfies $F_j(0, r_0, m_0, h_0, q_0) = 0$ for the respective j. Also, the Jacobian is

$$\frac{\partial(F_1,F_2,F_3,F_4)}{\partial(R,M,H,Q)}|_{(0,r_0,m_0,h_0,q_0)} = 1 \neq 0. \tag{3.11}$$

Therefore, the convergence of exact solutions to the present (2+1)-dimensional coupled time-fractional NLSEs has been proven via the implicit function theorem.

4. Conservation laws for system (1.1)

Because of their capacity to (i) supply conserved quantities for each produced solution, (ii) demonstrate integrability, and (iii) establish the existence and uniqueness of solutions, conservation laws are essential for fractional PDEs. In this section, we construct conservation laws for (2.5) via the Ibragimov theorem [16].

Consider a vector $T = (T^x, T^y, T^t)$ that satisfies the following conservation equation:

$$[D_y(T^y) + D_t(T^t) + D_x(T^x)]|_{(2.5)} = 0, \tag{4.1}$$

where $T^y = T^y(x,y,t,v,u,...)$, $T^t = T^t(x,y,t,v,u,...)$, and $T^x = T^x(x,y,t,v,u,...)$ are referred to as conserved vectors of (2.5). According to the Ibragimov theorem, the formal Lagrangian of (2.5) is expressed as

$$\begin{aligned}
L = {} & \mathcal{A}(x,y,t)({}^C D_t^\alpha u + \mu\Delta v + \gamma[u^2 + v^2 + a(s^2 + w^2)]v)\\
& + \mathcal{B}(x,y,t)({}^C D_t^\alpha v - \mu\Delta u - \gamma[u^2 + v^2 + a(s^2 + w^2)]u)\\
& + \mathcal{G}(x,y,t)({}^C D_t^\alpha s + \mu\Delta w + \gamma[s^2 + w^2 + a(u^2 + v^2)]w)\\
& + \mathcal{K}(x,y,t)({}^C D_t^\alpha w - \mu\Delta s - \gamma[s^2 + w^2 + a(u^2 + v^2)]s),
\end{aligned} \tag{4.2}$$

where $\mathcal{A}(x,y,t)$, $\mathcal{B}(x,y,t)$, $\mathcal{G}(x,y,t)$, and $\mathcal{K}(x,y,t)$ are sufficiently smooth functions.

The adjoint Euler–Lagrange equations for (2.5) are

$$\frac{\delta L}{\delta u} = 0, \qquad \frac{\delta L}{\delta v} = 0, \qquad \frac{\delta L}{\delta s} = 0, \qquad \frac{\delta L}{\delta w} = 0, \tag{4.3}$$

which define the Euler–Lagrange operator as

$$\frac{\delta}{\delta V}=\frac{\partial}{\partial V}+({}^{C}D_t^{\alpha})^*\frac{\partial}{\partial^{C}D_t^{\alpha}V}-D_x\frac{\partial}{\partial V_x}+D_x^2\frac{\partial}{\partial V_{xx}}-D_y\frac{\partial}{\partial V_y}+D_y^2\frac{\partial}{\partial V_{yy}}, \quad (4.4)$$

where $({}^{C}D_t^{\alpha})^*$ is the adjoint operator to ${}^{C}D_t^{\alpha}$ and is represented as

$$({}^{C}D_t^{\alpha})^*=(-1)^n I_t^{n-\alpha}(D_t^n)={}^{C}D_t^{\alpha}. \quad (4.5)$$

Here, $I_c^{n-\alpha}$ is the right-sided operator of fractional integration of order $n-\alpha$, which is expressed as

$$I_t^{n-\alpha}R(x,t)=\frac{1}{\Gamma(n-\alpha)}\int_t^C\frac{R(\tau,x)}{(\tau-t)^{1+\alpha-n}}d\tau. \quad (4.6)$$

After a simple calculation involving (4.3), we obtain the following adjoint equations of (2.5):

$$\begin{aligned}
&({}^{C}D_t^{\alpha})^*\mathcal{A}+2\gamma\mathcal{A}uv-\gamma\mathcal{B}[3u^2+v^2+a(s^2+w^2)]+2\gamma a\mathcal{G}uw\\
&\qquad-2\gamma a\mathcal{K}us-\mu(\mathcal{B}_{xx}+\mathcal{B}_{yy})=0,\\
&({}^{C}D_t^{\alpha})^*\mathcal{B}+\gamma\mathcal{A}[u^2+3v^2+a(s^2+w^2)]-2\gamma\mathcal{B}uv+2\gamma a\mathcal{G}vw\\
&\qquad-2\gamma a\mathcal{K}vs+\mu(\mathcal{A}_{xx}+\mathcal{A}_{yy})=0,\\
&({}^{C}D_t^{\alpha})^*\mathcal{G}+2a\gamma\mathcal{A}sv-2a\gamma\mathcal{B}su-\gamma\mathcal{K}[3s^2+w^2+a(u^2+v^2)]\\
&\qquad+2\gamma\mathcal{G}sw-\mu(\mathcal{K}_{xx}+\mathcal{K}_{yy})=0,\\
&({}^{C}D_t^{\alpha})^*\mathcal{K}+2a\gamma\mathcal{A}vw-2a\gamma\mathcal{B}uw+\gamma\mathcal{G}[3w^2+s^2+a(u^2+v^2)]\\
&\qquad-2\gamma\mathcal{K}sw+\mu(\mathcal{G}_{xx}+\mathcal{G}_{yy})=0.
\end{aligned} \quad (4.7)$$

Equation (2.5) allows each Lie point to symmetrically form conservation laws $D_j(T^j)$, where the following formulas are used to construct the components T^j:

$$\begin{aligned}
T^x&=\xi L+V^j\left[\frac{\partial L}{\partial u_x^j}-D_x(\frac{\partial L}{\partial u_{xx}^j})\right]+D_x(V^j)\left(\frac{\partial L}{\partial u_{xx}^j}\right),\\
T^y&=\rho L+V^j\left[\frac{\partial L}{\partial u_y^j}-D_y(\frac{\partial L}{\partial u_{yy}^j})\right]+D_y(V^j)\left(\frac{\partial L}{\partial u_{yy}^j}\right),\\
T^t&=\tau L+D_t^{\alpha-1}(V^j)\frac{\partial L}{\partial^{C}D_t^{\alpha}u^j}+I(V^j,D_t\frac{\partial L}{\partial^{C}D_t^{\alpha}u^j}),
\end{aligned} \quad (4.8)$$

where $V^j=\eta^j-\xi u_x^j-\rho u_y^j-\tau u_t^j$ and I is defined as

$$I(f_1,f_2)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\int_t^T\frac{f_1(\phi,x,y)f_2(\psi,x,y)}{(\psi-\phi)^{-\alpha}}d\phi d\psi. \quad (4.9)$$

Now, we use the fundamental definitions in (4.2), (4.8), and (4.9) to find the conservation laws for (2.5). By doing so, we derive the following components of the conservation laws for (2.5) and so obtain the conserved vectors of vector field X_1:

$$\begin{aligned}
T^t = {}& 2t[\mathcal{A}({}^C D_t^\alpha u + \mu\Delta v) + \mathcal{B}({}^C D_t^\alpha v - \mu\Delta u) + \mathcal{G}({}^C D_t^\alpha s + \mu\Delta w) + \mathcal{K}({}^C D_t^\alpha w - \mu\Delta s)] \\
& + 2t\mathcal{A}\gamma(u^2 + v^2 + a(s^2 + w^2))v - 2t\mathcal{B}\gamma(u^2 + v^2 + a(s^2 + w^2))u \\
& + 2t\mathcal{G}\gamma(s^2 + w^2 + a(u^2 + v^2))w - 2t\mathcal{K}\gamma(s^2 + w^2 + a(u^2 + v^2))s \\
& + \mathcal{A}D_t^{\alpha-1}(-\alpha u - \alpha x u_x - 2tu_t - \alpha y u_y) + I(-\alpha u - \alpha x u_x - 2tu_t - \alpha y u_y, \mathcal{A}_t) \\
& + \mathcal{B}D_t^{\alpha-1}(-\alpha v - \alpha x v_x - 2tv_t - \alpha y v_y) + I(-\alpha v - \alpha x v_x - 2tv_t - \alpha y v_y, \mathcal{B}_t) \\
& + \mathcal{G}D_t^{\alpha-1}(-\alpha s - \alpha x s_x - 2ts_t - \alpha y s_y) + I(-\alpha s - \alpha x s_x - 2ts_t - \alpha y s_y, \mathcal{G}_t) \\
& + \mathcal{K}D_t^{\alpha-1}(-\alpha w - \alpha x w_x - 2tw_t - \alpha y w_y) + I(-\alpha w - \alpha x w_x - 2tw_t - \alpha y w_y, \mathcal{K}_t),
\end{aligned}$$

$$\begin{aligned}
T^x = {}& \alpha x[\mathcal{A}({}^C D_t^\alpha u + \mu\Delta v) + \mathcal{B}({}^C D_t^\alpha v - \mu\Delta u) + \mathcal{G}({}^C D_t^\alpha s + \mu\Delta w) + \mathcal{K}({}^C D_t^\alpha w - \mu\Delta s)] \\
& + \alpha x\mathcal{A}\gamma(u^2 + v^2 + a(s^2 + w^2))v - \alpha x\mathcal{B}\gamma(u^2 + v^2 + a(s^2 + w^2))u \\
& + \alpha x\mathcal{G}\gamma(s^2 + w^2 + a(u^2 + v^2))w - \alpha x\mathcal{K}\gamma(s^2 + w^2 + a(u^2 + v^2))s \\
& - \mu\mathcal{B}_x(\alpha u + \alpha x u_x + 2tu_t + \alpha y u_y) + \mu\mathcal{B}(2\alpha u_x + \alpha x u_{xx} + 2tu_{xt} + \alpha y u_{xy}) \\
& + \mu\mathcal{A}_x(\alpha v + \alpha x v_x + 2tv_t + \alpha y v_y) - \mu\mathcal{A}(2\alpha v_x + \alpha x v_{xx} + 2tv_{xt} + \alpha y v_{xy}) \\
& - \mu\mathcal{K}_x(\alpha s + \alpha x s_x + 2ts_t + \alpha y s_y) + \mu\mathcal{K}(2\alpha s_x + \alpha x s_{xx} + 2ts_{xt} + \alpha y s_{xy}) \\
& + \mu\mathcal{G}_x(\alpha w + \alpha x w_x + 2tw_t + \alpha y w_y) - \mu\mathcal{G}(2\alpha w_x + \alpha x w_{xx} + 2tw_{xt} + \alpha y w_{xy}),
\end{aligned}$$

$$\begin{aligned}
T^y = {}& \alpha y[\mathcal{A}({}^C D_t^\alpha u + \mu\Delta v) + \mathcal{B}({}^C D_t^\alpha v - \mu\Delta u) + \mathcal{G}({}^C D_t^\alpha s + \mu\Delta w) + \mathcal{K}({}^C D_t^\alpha w - \mu\Delta s)] \\
& + \alpha y\mathcal{A}\gamma(u^2 + v^2 + a(s^2 + w^2))v - \alpha y\mathcal{B}\gamma(u^2 + v^2 + a(s^2 + w^2))u \\
& + \alpha y\mathcal{G}\gamma(s^2 + w^2 + a(u^2 + v^2)w - \alpha y\mathcal{K}\gamma(s^2 + w^2 + a(u^2 + v^2)))s \\
& - \mu\mathcal{B}_y(\alpha u + \alpha x u_x + 2tu_t + \alpha y u_y) + \mu\mathcal{B}(2\alpha u_y + \alpha x u_{xy} + 2tu_{yt} + \alpha y u_{yy}) \\
& + \mu\mathcal{A}_y(\alpha v + \alpha x v_x + 2tv_t + \alpha y v_y) - \mu\mathcal{A}(2\alpha v_y + \alpha x v_{xy} + 2tv_{yt} + \alpha y v_{yy}) \\
& - \mu\mathcal{K}_y(\alpha s + \alpha x s_x + 2ts_t + \alpha y s_y) + \mu\mathcal{K}(2\alpha s_y + \alpha x s_{xy} + 2ts_{yt} + \alpha y s_{yy}) \\
& + \mu\mathcal{G}_y(\alpha w + \alpha x w_x + 2tw_t + \alpha y w_y) - \mu\mathcal{G}(2\alpha w_y + \alpha x w_{xy} + 2tw_{yt} + \alpha y w_{yy}).
\end{aligned}$$

For vector field X_2, we have the following conserved vectors:

$$\begin{aligned}
T^t = {}& -\mathcal{A}{}^C D_t^{\alpha-1}(u_x) + I(-u_x, \mathcal{A}_t) - \mathcal{B}{}^C D_t^{\alpha-1}(v_x) + I(-v_x, \mathcal{B}_t) - \mathcal{G}{}^C D_t^{\alpha-1}(s_x) \\
& + I(-s_x, \mathcal{G}_t) - \mathcal{K}{}^C D_t^{\alpha-1}(w_x) + I(-w_x, \mathcal{K}_t), \\
T^x = {}& \mathcal{A}{}^C D_t^\alpha u + \mathcal{B}{}^C D_t^\alpha v + \mathcal{G}{}^C D_t^\alpha s + \mathcal{K}{}^C D_t^\alpha w - \mu\mathcal{B}u_{yy} + \mu\mathcal{A}v_{yy} - \mu\mathcal{K}s_{yy} + \mu\mathcal{G}w_{yy} \\
& - \mu\mathcal{B}_x u_x + \mu\mathcal{A}_x v_x - \mu\mathcal{K}_x s_x + \mu\mathcal{G}_x w_x + \mathcal{A}\gamma[u^2 + v^2 + a(s^2 + w^2)]v \\
& - \mathcal{B}\gamma[u^2 + v^2 + a(s^2 + w^2)]u + \mathcal{G}\gamma[s^2 + w^2 + a(u^2 + v^2)]w \\
& - \mathcal{K}\gamma[s^2 + w^2 + a(u^2 + v^2)]s, \\
T^y = {}& -\mu\mathcal{B}_y u_x + \mu\mathcal{B}u_{xy} + \mu\mathcal{A}_y v_x - \mu\mathcal{A}v_{xy} - \mu\mathcal{K}_y s_x + \mu\mathcal{K}s_{xy} + \mu\mathcal{G}_y w_x - \mu\mathcal{G}w_{xy}.
\end{aligned}$$

After rigorous calculations, we have the following expressions for the conserved vectors of vector field X_3:

$$\begin{aligned}
T^t &= -\mathcal{A}^C D_t^{\alpha-1}(u_y) + I(-u_y, \mathcal{A}_t) - \mathcal{B}^C D_t^{\alpha-1}(v_y) + I(-v_y, \mathcal{B}_t) - \mathcal{G}^C D_t^{\alpha-1}(s_y) \\
&\quad + I(-s_y, \mathcal{G}_t) - \mathcal{K}^C D_t^{\alpha-1}(w_y) + I(-w_y, \mathcal{K}_t), \\
T^x &= -\mu\mathcal{B}_x u_y + \mu\mathcal{B}u_{xy} + \mu\mathcal{A}_x v_y - \mu\mathcal{A}v_{xy} - \mu\mathcal{K}_x s_y + \mu\mathcal{K}s_{xy} + \mu\mathcal{G}_x w_y - \mu\mathcal{G}w_{xy}, \\
T^y &= \mathcal{A}^C D_t^{\alpha} u + \mathcal{B}^C D_t^{\alpha} v + \mathcal{G}^C D_t^{\alpha} s + \mathcal{K}^C D_t^{\alpha} w - \mu\mathcal{B}u_{xx} + \mu\mathcal{A}v_{xx} + \mu\mathcal{G}w_{xx} - \mu\mathcal{K}s_{xx} \\
&\quad - \mu\mathcal{B}_y u_y + \mu\mathcal{A}_y v_y - \mu\mathcal{K}_y s_y + \mu\mathcal{G}_y w_y + \mathcal{A}\gamma[u^2 + v^2 + a(s^2 + w^2)]v \\
&\quad - \mathcal{B}\gamma[u^2 + v^2 + a(s^2 + w^2)]u + \mathcal{G}\gamma[s^2 + w^2 + a(u^2 + v^2)]w \\
&\quad - \mathcal{K}\gamma[s^2 + w^2 + a(u^2 + v^2)]s.
\end{aligned}$$

5. Conclusions

Herein, LSA was used to investigate (2+1)-dimensional coupled time-fractional NLSEs that involve the RL fractional derivative. Under Lie point symmetries, (2.5) was simplified to fractional nonlinear ordinary differential equations with new independent variables. Furthermore, using power-series theory, we constructed exact solutions to (1.1) and then subjected them to thorough convergence analysis. Finally, we derived conservation laws for (1.1). In future work, we will use LSA to tackle more-complex space–time fractional PDEs.

Author contributions

All authors participated equally and have read and approved the final version of this paper.

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