Extinction and persistence in a logistic model with birth and harvesting impulses*

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Abstract. This paper deals with a diffusive logistic model with birth and harvesting impulses, where birth pulses are for increase of population in short time because of birth, and harvesting pulses are used to describe decrease of population by regular harvesting or interventions. Firstly, the principal eigenvalue depending the impulsive rates, which is regarded as a threshold value, is introduced and characterized. Secondly, the asymptotic behavior of population is fully investigated and the sufficient conditions for the solution to be extinct or persist are given. Our results show that the increase brought about by birth, the decrease caused by harvest, and the intervention timing all have an impact on the persistence of species.

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1 Introduction

This paper characterizes the dynamics of the diffusive logistic model with impulses

$$\begin{cases} u_t = d\Delta u + a(t,x)u - b(t,x)u^p, & t \in (n^+, (n+\tau)] \bigcup ((n+\tau)^+, (n+1)],\ x \in \Omega, \\ u(t,x) = 0, & t > 0,\ x \in \partial\Omega, \\ u(0,x) = u_0(x), & x \in \overline{\Omega}, \\ u(n^+,x) = (1+\alpha)u(n,x), & x \in \overline{\Omega}, \\ u((n+\tau)^+,x) = (1-\beta)u(n+\tau,x), & x \in \overline{\Omega},\ n = 0,1,2,\cdots, \end{cases} \tag{1.1}$$

where Ω is a bounded and connected domain of $\mathbb{R}^N$ $(N \geq 1)$ with smooth boundary $\partial\Omega$. p, τ are constants satisfying $p > 1$ and $0 < \tau < 1$. $t \in (n^+, (n+\tau)]$ is expressed that the equation holds for $t \in (n, (n+\tau)]$, and take its value $u(n^+, x)$ instead of $u(n,x)$ at the initial time of the time interval $(n, (n+\tau)]$ for $n = 0,1,2,\cdots$, so is $t \in ((n+\tau)^+, (n+1)]$. $u(t,x)$

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is the density of species at time t and in space x and the positive constant d is the diffusion rate. The function $a(t,x)(\in C^{\theta/2,\theta}([0,\tau]\times\overline{\Omega})$ for some $0<\theta\le 1)$ is a periodic function of time with period 1, and denotes intrinsic growth rate of population. It can be negative, which means that the resources on position x at time t are not conducive to survival. The positive function $b(t,x)(\in C^{\theta/2,\theta}([0,\tau]\times\overline{\Omega}))$ is a periodic function of time with period 1, and $0<b^m\le b(t,x)\le b^M$ in $[0,\tau]\times\overline{\Omega}$. Initial function $u_0(x)$ satisfies $u_0(x)\in C^2(\overline{\Omega})$, $u_0(x)\ge,\not\equiv 0$ for $x\in\overline{\Omega}$ and $u_0(x)=0$ for $x\in\partial\Omega$.

The function $(1+\alpha)u$ with $\alpha>0$ represents birth pulse, and an impulse occurs at every time $t=n$ $(n=0,1,2,\cdots)$, while $(1-\beta)u$ with $0<\beta<1$ is the impulsive function representing harvesting control, and the impulse occurs at every time $t=n+\tau$ $(n=0,1,2,\cdots)$. The species grow and diffuse within the successive stages $(n,(n+\tau)]$ and $((n+\tau),(n+1)]$.

Besides growth, death, disperse [17], we are more interested in the distribution and dynamics of species influenced by impulsive perturbation [2, 10, 13, 16, 20]. Especially, problem (1.1) has been discussed in [10] numerically.

On condition that $\alpha=\beta=0$, which means there is no impulse, problem (1.1) has attracted much attention, see [1, 3, 5, 14] and references therein.

This paper is organized as follows. Section 2 contains global existence and uniqueness of the solution, and the principal eigenvalue for a periodic eigenvalue problem with impulse is investigated in Sections 3. Section 4 is devoted to dynamics of the solution to the problem (1.1).

2 Existence, uniqueness and estimates

The global existence and uniqueness of solution to problem (1.1) with impulses can be obtained by the bootstrap method.

For $n=0$, birth pulse takes place at time $t=0$. Then the solution $u(t,x)$ satisfies problem (1.1) with a new initial value $u(0^+,x)$ over time interval $(0^+,\tau]$. Recalling that $u_0(x)\in C^2(\overline{\Omega})$, we can deduce that the new initial value $u(0^+,x)=(1+\alpha)u_0(x)\in C^2(\overline{\Omega})$. Hence, it follows from the classical theory of partial differential equation [12], we have the existence and uniqueness of solution $u(t,x)$ to problem (1.1) for $t\in(0^+,\tau]$, $u(t,x)\in C^{1,2}((0,\tau]\times\overline{\Omega})$ and

$$u(t,x)\le \max\{(1+\alpha)u_0^M,\ (a^M/b^m)^{1/(p-1)}\}\le M_1:=(1+\alpha)\max\{u_0^M,\ (a^M/b^m)^{1/(p-1)}\}$$

since that $u(t,x)\le\overline{u}(t)$ for $(t,x)\in(0,\tau]\times\overline{\Omega}$ and $\overline{u}(t)$ satisfies

$$\begin{cases} \overline{u}_t=a^M\overline{u}-b^m\overline{u}^p, & t\in(0,\tau],\\ \overline{u}(0)=(1+\alpha)u_0^M, \end{cases}$$

where for any continue function $f(x)$ in $\overline{\Omega}$, we denote $f^m=\min_{x\in\overline{\Omega}} f$ and $f^M=\max_{x\in\overline{\Omega}} f$.

Similarly, $u(\tau^+,x)=(1-\beta)u(\tau,x)$ becomes a new initial value for $t\in(\tau^+,1]$, which also belongs to $C^2(\overline{\Omega})$. Then, $u(t,x)\in C^{1,2}((\tau,1]\times\overline{\Omega})$ exists uniquely. Moreover,

$$u(t,x)\le M_1^*:=\max\{(1-\beta)M_1,\ (a^M/b^m)^{1/(p-1)}\}\le M_1,\ (t,x)\in(\tau,1]\times\overline{\Omega})$$

since that $u(t,x) \le \overline{w}(t)$ for $(t,x) \in (\tau, 1] \times \overline{\Omega}$ and $\overline{w}(t)$ satisfies

$$\begin{cases} \overline{w}_t = a^M \overline{w} - b^m \overline{w}^p, & t \in (\tau, 1], \\ \overline{w}(0) = (1-\beta) M_1. \end{cases}$$

Taking $n = 1, 2, \cdots$ and by the same procedures, we can find that, for any n, problem (1.1) admits a unique solution $u(t,x)$ for $t \in [0, n]$, and

$$u(t,x) \le M_n := (1+\alpha)^n \max\{u_0^M,\ (a^M/b^m)^{1/(p-1)}\},\ (t,x) \in [0,n] \times \overline{\Omega}.$$

Therefore, we conclude the following global existence and uniqueness of solution.

Theorem 2.1 *Problem* (1.1) *admits a unique solution $u(t,x)$ for all $t > 0$. Moreover,*

$$u(t,x) \in PC^{1,2}((0,+\infty) \times \overline{\Omega}) := \bigcap_{n=0}^{\infty} [C^{1,2}((n, n+\tau] \times \overline{\Omega}) \bigcap C^{1,2}((n+\tau, n+1] \times \overline{\Omega})]$$

and $u(t,x) \le M_{[t]+1}$ for $(t,x) \in [0,\infty) \times \overline{\Omega}$.

3 The principal eigenvalue

As in [3, 19], the long-time behavior of problem (1.1) is related to its corresponding periodic problem

$$\begin{cases} U_t = d\Delta U + a(t,x)U - b(t,x)U^p, & t \in (0^+, \tau] \bigcup (\tau^+, 1],\ x \in \Omega, \\ U(t,x) = 0, & 0 < t \le 1,\ x \in \partial\Omega, \\ U(0,x) = U(1,x), & x \in \overline{\Omega}, \\ U(0^+,x) = (1+\alpha)U(0,x), & x \in \overline{\Omega}, \\ U(\tau^+,x) = (1-\beta)U(\tau,x), & x \in \overline{\Omega}, \end{cases} \tag{3.1}$$

and the existence of the positive solution to (3.1) depends on the principal eigenvalue $\mu_1(d, a(t,x), \alpha, \beta)$ of the following periodic eigenvalue problem

$$\begin{cases} \phi_t - d\Delta\phi - a(t,x)\phi = \mu_1\phi, & t \in (0^+, \tau] \bigcup (\tau^+, 1],\ x \in \Omega, \\ \phi(t,x) = 0, & t \in [0,1],\ x \in \partial\Omega, \\ \phi(0,x) = \phi(1,x), & x \in \overline{\Omega}, \\ \phi(0^+,x) = (1+\alpha)\phi(0,x), & x \in \overline{\Omega}, \\ \phi(\tau^+,x) = (1-\beta)\phi(\tau,x), & x \in \overline{\Omega}. \end{cases} \tag{3.2}$$

The existence of μ_1 can be guaranteed by using Krein-Rutman theorem [5, 6] on a Banach space involving impulse ([21]). We give a sketch here.

To overcome the difficulties induced by different impulses, we consider the following equivalent eigenvalue problem

$$\begin{cases} \xi_t - d\Delta\xi - a(t,x)\xi = \mu_1\xi, & t\in(0,\tau],\ x\in\Omega,\\ \eta_t - d\Delta\eta - a(t,x)\xi = \mu_1\eta, & t\in(\tau,1],\ x\in\Omega,\\ \xi(t,x)=0, & t\in[0,\tau],\ x\in\partial\Omega,\\ \eta(t,x)=0, & t\in[\tau,1],\ x\in\partial\Omega,\\ \xi(0,x)=(1+\alpha)\eta(1,x),\ \xi(t,x)=\xi(\tau,x) & t\in[\tau,1],\, x\in\overline{\Omega},\\ \eta(\tau,x)=(1-\beta)\xi(\tau,x),\ \eta(t,x)=\eta(\tau,x) & t\in[0,\tau],\, x\in\overline{\Omega}. \end{cases} \tag{3.3}$$

In fact, we can take $\phi(t,x)=\xi(t,x)$ for $t\in(0,\tau]$, $\phi(t,x)=\eta(t,x)$ for $t\in(\tau,1]$, and $\phi(0^+,x)=\xi(0,x)$, $\phi(0,x)=\eta(1,x)$, $\phi(\tau^+,x)=\eta(\tau,x)$ and $\phi(\tau,x)=\xi(\tau,x)$ for $x\in\overline{\Omega}$.

Now let W be a Banach space,

$$\begin{aligned} W = \ & D_0^{0,1}([0,1]\times\overline{\Omega}) := \{(\xi,\eta)\in[C^{0,1}([0,1]\times\overline{\Omega})]^2 : \xi=\eta=0\ \forall(t,x)\in[0,1]\times\partial\Omega,\\ & \xi(t,x)=\xi(\tau,x), t\in[\tau,1],\ \eta(t,x)=\eta(\tau,x),\ t\in[0,\tau],\ x\in\overline{\Omega},\\ & \xi(0,x)=(1+\alpha)\eta(1,x),\ \ \eta(\tau,x)=(1-\beta)\xi(\tau,x)\ \forall x\in\overline{\Omega}\} \end{aligned}$$

with the positive cone

$$W^+ := \text{closure}\{(\xi,\eta)\in W:\ \xi(t,x),\eta(t,x)\gg 0\ \forall(t,x)\in[0,1]\times\partial\Omega\},$$

and its interior

$$\text{Int}(W^+) = \{(\xi,\eta)\in W:\ \xi(t,x),\eta(t,x)\gg 0\ \forall(t,x)\in[0,1]\times\partial\Omega\}$$

being nonempty, where ν is outward unit normal vector of $\partial\Omega$ and $\xi\gg 0$ means that $\xi(t,x)>0$ for all $(t,x)\in[0,1]\times\Omega$ and $\frac{\partial\xi}{\partial\nu}(t,x)<0$ for all $(t,x)\in[0,1]\times\partial\Omega$, we normally call it strongly positive function.

Let $M^* = 1+\max_{[0,\tau]\times\overline{\Omega}}|a(t,x)| + \ln(1/(1-\beta))$. For any given $(\xi,\eta)\in W$, the linear problem

$$\begin{cases} w_t - d\Delta w + M^*w - a(t,x)w = \xi, & t\in(0,\tau],\ x\in\Omega,\\ z_t - d\Delta z + M^*z - a(t,x)z = \eta, & t\in(\tau,1],\ x\in\Omega,\\ w(t,x)=0, & t\in[0,\tau],\ x\in\partial\Omega,\\ z(t,x)=0, & t\in[\tau,1],\ x\in\partial\Omega,\\ w(0,x)=(1+\alpha)z(1,x),\ w(t,x)=w(\tau,x) & t\in[\tau,1],\, x\in\overline{\Omega},\\ z(\tau,x)=(1-\beta)w(\tau,x),\ z(t,x)=z(\tau,x) & t\in[0,\tau],\, x\in\overline{\Omega} \end{cases} \tag{3.4}$$

admits a unique solution (w,z) satisfying $w,z\in C^{(1+\alpha)/2,1+\alpha}([0,1]\times\overline{\Omega})\bigcap W$ with any $0<\alpha<1$ by the classical theory of partial differential equation [12].

Now, define a operator $\mathcal{A}(\xi,\eta)=(w,z)$. Since that the imbedding $C^{(1+\alpha)/2,1+\alpha}\hookrightarrow C^{0,1}$ is compact, $\mathcal{A}$ is a linear compact operator. Moreover, $\mathcal{A}$ is strongly positive with respect to W by the strong maximum principle and Hopf's boundary lemma. Therefore, it follows from Krein-Rutman theorem that there exist a unique $\sigma_1 := r(\mathcal{A})>0$ and a function $(w,z)\in\text{Int}(W^+)$ such that $\mathcal{A}(w,z)=\sigma_1(w,z)$, then $\mu_1 := 1/\sigma_1 - M^*$ is the principal eigenvalue of (3.3) and the corresponding eigenfunctions $\xi(t,x)$ and $\eta(t,x)$ are strongly positive, that is, $\xi(t,x), \eta(t,x) > 0$ in $[0,1]\times\Omega$ and $\frac{\partial\xi}{\partial\nu}(t,x), \frac{\partial\eta}{\partial\nu}(t,x) < 0$ for $(t,x) \in [0,1]\times\partial\Omega$.

Coming back to problem (3.2) with impulse, we have the existence of the principal eigenvalue by equivalence.

Theorem 3.1 *Assume that* $a(x,t) := a(t)$. *Then the principal eigenvalue of problem* (1.1) *can be precisely expressed as*

$$\mu_1 = -\ln[(1+\alpha)(1-\beta)] + d\lambda_1 - \int_0^1 a(t)dt, \tag{3.5}$$

where $\lambda_1(>0)$ *is the principal eigenvalue of* $-\Delta$ *in* Ω *with homogeneous Dirichlet boundary condition.*

Proof: Let

$$\phi(t,x) = f(t)\psi(x),$$

where $\psi(x)$ is the corresponding eigenfunction of λ_1, which satisfies the eigenvalue problem

$$\begin{cases} -\Delta\psi = \lambda_1\psi, & x \in \Omega, \\ \psi(x) = 0, & x \in \partial\Omega. \end{cases} \tag{3.6}$$

Substituting $\phi(t,x) = f(t)\psi(y)$ into the reaction diffusion equation in (3.2) yields

$$\frac{f'(t)}{f(t)} + d\lambda_1 = a(t) + \mu_1,$$

then integrating both sides from $t \in (0^+,\tau]\bigcup(\tau^+,1]$, yield

$$\int_{0^+}^{\tau} \frac{f'(t)}{f(t)}dt + \int_{\tau^+}^{1} \frac{f'(t)}{f(t)}dt + d\lambda_1 = \int_0^1 a(t)dt + \mu_1.$$

Recalling that

$$\begin{cases} f(0) = f(1), \\ f(0^+) = (1+\alpha)f(0), \\ f(\tau^+) = (1-\beta)f(\tau), \end{cases}$$

we obtain $\mu_1 = -\ln[(1+\alpha)(1-\beta)] + d\lambda_1 - \int_0^1 a(t)dt$. □

Theorem 3.2 *Assume that* $a(x,t) := a(t)$. *If period* 1 *is replaced by* T, *that is,* $\phi(0,x) = \phi(T,x)$ *hold for* $x \in \overline{\Omega}$ *in Theorem* 3.1, *then*

$$\mu_1 = \frac{-\ln[(1+\alpha)(1-\beta)]}{T} + d\lambda_1 - \frac{\int_0^T a(t)dt}{T}.$$

Proof: If period 1 is replaced by T, then the corresponding periodic problem (3.2) is written as

$$\begin{cases} \phi_t - d\Delta\phi - a(t)\phi = \mu_1\phi, & t \in (0^+, \tau] \bigcup (\tau^+, T],\ x \in \Omega, \\ \phi(t,x) = 0, & t \in [0,T],\ x \in \partial\Omega, \\ \phi(0,x) = \phi(T,x), & x \in \overline{\Omega}, \\ \phi(0^+,x) = (1+\alpha)\phi(0,x), & x \in \overline{\Omega}, \\ \phi(\tau^+,x) = (1-\beta)\phi(\tau,x), & x \in \overline{\Omega}. \end{cases}$$

By the same method in Theorem 2.1, we obtain

$$\int_{0^+}^{\tau} \frac{f'(t)}{f(t)} dt + \int_{\tau^+}^{T} \frac{f'(t)}{f(t)} dt + d\lambda_1 T = \int_0^T a(t)dt + \mu_1 T,$$

so $\mu_1 = \frac{-\ln[(1+\alpha)(1-\beta)]}{T} + d\lambda_1 - \frac{\int_0^T a(t)dt}{T}$. □

Before considering the properties of the principal eigenvalue $\mu_1(d, a(t,x), \alpha, \beta)$ in the problem (3.2), we first introduce the auxiliary problem

$$\begin{cases} -\phi_t^* - d\Delta\phi^* - a(t,x)\phi^* = \lambda_1\phi^*, & t \in (0^+, \tau] \bigcup (\tau^+, 1],\ x \in \Omega, \\ \phi^*(t,x) = 0, & t \in [0,1],\ x \in \partial\Omega, \\ \phi^*(0^+,x) = \dfrac{1}{1+\alpha}\phi^*(0,x), & x \in \overline{\Omega}, \\ \phi^*(\tau^+,x) = \dfrac{1}{1-\beta}\phi^*(\tau,x), & x \in \overline{\Omega}, \\ \phi^*(0,x) = \phi^*(1,x), & x \in \overline{\Omega}. \end{cases} \quad (3.7)$$

Lemma 3.3 *The principal eigenvalues λ_1 in (3.7) and μ_1 in (3.2) are the same, that is, $\lambda_1 = \mu_1$.*

Proof: Multiplying the first equation in (3.7) by ϕ and the first equation in (3.2) by ϕ^*, respectively, we obtain

$$\begin{cases} \phi_t\phi^* - d\Delta\phi\phi^* = a(t,x)\phi\phi^* + \mu_1\phi\phi^*, \\ -\phi_t^*\phi - d\Delta\phi^*\phi = a(t,x)\phi\phi^* + \lambda_1\phi\phi^*, \end{cases}$$

then abstracting these two equations gives

$$\phi_t\phi^* + \phi_t^*\phi - d(\Delta\phi\phi^* - \Delta\phi^*\phi) = (\mu_1 - \lambda_1)\phi\phi^*. \quad (3.8)$$

Since

$$\int_\Omega (\Delta\phi\phi^* - \Delta\phi^*\phi)dx = 0$$

and

$$\begin{aligned} & (\textstyle\int_{0^+}^{\tau} + \int_{\tau^+}^{1})(\phi_t\phi^* + \phi_t^*\phi)dt \\ = \ & \phi(\tau,x)\phi^*(\tau,x) - \phi(0^+,x)\phi^*(0^+,x) + \phi(1,x)\phi^*(1,x) - \phi(\tau^+,x)\phi^*(\tau^+,x) \\ = \ & \phi(\tau,x)\phi^*(\tau,x) - (1+\alpha)\tfrac{1}{1+\alpha}\phi(0,x)\phi^*(0,x) + \phi(0,x)\phi^*(0,x) - (1-\beta)\tfrac{1}{1-\beta}\phi(\tau,x)\phi^*(\tau,x) \\ = \ & 0, \end{aligned}$$

so integrating both sides of the equations in (3.8) over $(t,x) \in ((0^+,\tau]\bigcup(\tau^+,1])\times\Omega$, yields

$$(\mu_1-\lambda_1)(\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\int_{\Omega}\phi\phi^* dxdt=0,$$

which ends the proof since $(\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\int_{\Omega}\phi\phi^* dxdt>0$. □

Theorem 3.4 *The following statements hold:*

(i) $\mu_1(d,a(t,x),\alpha,\beta)$ is nonincreasing with respect to $a(t,x)$ for any given d, α and β;

(ii) $\mu_1(d,a(t,x),\alpha,\beta)$ is strictly monotonic decreasing with respect to α for any given d, $a(t,x)$ and β;

(iii) $\mu_1(d,a(t,x),\alpha,\beta)$ is strictly monotonic increasing with respect to β for any given d, $a(t,x)$and α;

(iv) $\mu_1(d,a(t,x),\alpha,\beta)$ is nondecreasing with respect to d for any given $a(t,x)$, α and β.

Proof: (i) It can be observed directly from the first equation in (3.2) that $\mu_1(a(t,x),\alpha,\beta,d)$ is nonincreasing with respect to $a(t,x)$.

We next prove (ii). ϕ and μ_1 are smooth functions of $\alpha\in(0,+\infty)$, $\beta\in(0,1)$ and $d\in(0,+\infty)$ by standard result about perturbation [7]. So differentiating both sides of equations in problem (3.2) with respect to α yields

$$\begin{cases}\phi'_t-d\Delta\phi'-a(t,x)\phi'=\mu'_1\phi+\mu_1\phi', & t\in(0^+,\tau]\bigcup(\tau^+,1],\ x\in\Omega,\\ \phi'(t,x)=0, & t\in[0,1],\ x\in\partial\Omega,\\ \phi'(0^+,x)=\phi(0,x)+(1+\alpha)\phi'(0,x), & x\in\overline{\Omega},\\ \phi'(\tau^+,x)=(1-\beta)\phi'(\tau,x), & x\in\overline{\Omega},\\ \phi'(0,x)=\phi'(1,x), & x\in\overline{\Omega}.\end{cases}\tag{3.9}$$

Multiplying the first equation in (3.9) by ϕ^* asserts

$$\phi'_t\phi^*-d\Delta\phi'\phi^*-a(t,x)\phi'\phi^*=\mu'_1\phi\phi^*+\mu_1\phi'\phi^*.\tag{3.10}$$

Recalling the periodicity and impulsive conditions of $\phi(t,x)$, $\phi^*(t,x)$ and $\phi'(t,x)$, one easily checks that

$$\begin{aligned}&(\textstyle\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\phi'_t\phi^*dt\\ =\ &\phi'(\tau,x)\phi^*(\tau,x)-\phi'(0^+,x)\phi^*(0^+,x)+\phi'(1,x)\phi^*(1,x)-\phi'(\tau^+,x)\phi^*(\tau^+,x)\\ -\ &(\textstyle\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\phi'\phi^*_t dt\\ =\ &\phi'(\tau,x)\phi^*(\tau,x)-[\phi(0,x)+(1+\alpha)\phi'(0,x)]\frac{1}{1+\alpha}\phi^*(0,x)+\phi'(0,x)\phi^*(0,x)\\ -\ &(1-\beta)\frac{1}{1-\beta}\phi'(\tau,x)\phi^*(\tau,x)-(\textstyle\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\phi'\phi^*_t dt\\ =\ &\frac{-1}{1+\alpha}\phi(0,x)\phi^*(0,x)-(\textstyle\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\phi'\phi^*_t dt.\end{aligned}$$

So integrating both sides of equations in (3.10) over $(t,x)\in(0^+,\tau]\bigcup(\tau^+,1]\times\Omega$, we get

$$\begin{aligned}&\frac{-1}{1+\alpha}\textstyle\int_{\Omega}\phi(0,x)\phi^*(0,x)dx-(\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\int_{\Omega}(\phi'\phi^*_t-d\phi'\Delta\phi^*)dxdt\\ =\ &(\textstyle\int_{0^+}^{\tau}+\int_{\tau^+}^{1})\int_{\Omega}(a(t,x)\phi'\phi^*+\mu'_1\phi\phi^*+\mu_1\phi'\phi^*)dxdt,\end{aligned}$$

which, together with $-\phi_t^* - d\Delta\phi^* - a(t,x)\phi^* = \mu_1\phi^*$ in (3.7), yields

$$\frac{-1}{1+\alpha}\int_\Omega \phi(0,x)\phi^*(0,x)dx = \mu_1'(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\phi^* dxdt,$$

thus

$$\mu_1' = \frac{\frac{-1}{1+\alpha}\int_\Omega \phi(0,x)\phi^*(0,x)dx}{(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\phi^* dxdt} < 0.$$

(iii) can be derived by the same procedure in (ii) and we give the sketches here. Differentiating both sides of equations in problem (3.2) with respect to β yields

$$\begin{cases} \dot{\phi}_t - d\Delta\dot{\phi} - a(t,x)\dot{\phi} = \dot{\mu}_1\phi + \mu_1\dot{\phi}, & t \in (0^+,\tau]\bigcup(\tau^+,1],\ x \in \Omega, \\ \dot{\phi}(t,x) = 0, & t \in [0,1],\ x \in \partial\Omega, \\ \dot{\phi}(0^+,x) = (1+\alpha)\dot{\phi}(0,x), & x \in \overline{\Omega}, \\ \dot{\phi}(\tau^+,x) = -\phi(\tau,x) + (1-\beta)\dot{\phi}(\tau,x), & x \in \overline{\Omega}. \\ \dot{\phi}(0,x) = \dot{\phi}(1,x), & x \in \overline{\Omega}. \end{cases} \tag{3.11}$$

By careful calculations similarly as in (ii), we finally obtain

$$\dot{\mu}_1 = \frac{\frac{1}{1-\beta}\int_\Omega \phi(0,x)\phi^*(0,x)dx}{(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\phi^* dxdt} > 0$$

since $0 < \beta < 1$.

We finally prove (iv). Differentiating both sides of equations in problem (3.2) with respect to d yields

$$\begin{cases} \phi_t' - d\Delta\phi' - \Delta\phi - a(t,x)\phi' = \mu_1'\phi + \mu_1\phi', & t \in (0^+,\tau]\bigcup(\tau^+,1],\ x \in \Omega, \\ \phi'(t,x) = 0, & t \in [0,1],\ x \in \partial\Omega, \\ \phi'(0^+,x) = (1+\alpha)\phi'(0,x), & x \in \overline{\Omega}, \\ \phi'(\tau^+,x) = (1-\beta)\phi'(\tau,x), & x \in \overline{\Omega}, \\ \phi'(0,x) = \phi'(1,x), & x \in \overline{\Omega}. \end{cases} \tag{3.12}$$

Since

$$(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\phi_t'\phi^* dt = -(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\phi'\phi_t^* dt$$

and

$$\int_\Omega \Delta\phi\phi^* dx = \int_\Omega \Delta\phi^*\phi dx,$$

so multiplying the first equation in (3.12) by ϕ^* and then integrating both sides of this equation over $(t,x) \in ((0^+,\tau]\bigcup(\tau^+,1]) \times \Omega$ to conclude

$$(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega(-\phi'\phi_t^* - \phi\Delta\phi^* - d\phi'\Delta\phi^* - a(t,x)\phi'\phi^*)dxdt = (\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega(\mu_1'\phi\phi^* + \mu_1\phi'\phi^*)dxdt.$$

Recalling (3.7), one easily checks

$$\mu_1' = \frac{-(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\Delta\phi^* dtdx}{(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\phi^* dxdt} = \frac{(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \frac{\partial\phi^*}{\partial\eta}\frac{\partial\phi}{\partial\eta}dxdt}{(\int_{0^+}^{\tau} + \int_{\tau^+}^{1})\int_\Omega \phi\phi^* dxdt} > 0,$$

where η is the outward unit vector of $\partial\Omega$. Therefore, $\mu_1(d, a(t,x), \alpha, \beta)$ is nondecreasing with respect to d for any given $a(t,x)$, α and β.

□

4 The dynamical behavior of the solution

Theorem 4.1 *If $\mu_1(d, a(t,x), \alpha, \beta) > 0$, then the solution $u(t,x)$ to problem* (1.1) *satisfies* $\lim_{t\to\infty} u(t,x) = 0$ *uniformly for $x \in \overline{\Omega}$.*

Proof: Constructing

$$\widetilde{u}(t,x) = Me^{-\mu_1 t}\phi(t,x),$$

where $\phi(t,x)\,(\leq 1)$ is a positive eigenfunction of problem (3.2) corresponding to μ_1, and M to be chosen later.

For $t \in (n^+, (n+\tau)] \bigcup ((n+\tau)^+, (n+1)]$ and $x \in \Omega$, careful calculations yield

$$\begin{aligned} & \tilde{u}_t - d\triangle\tilde{u} - a(t,x)\tilde{u} + b(t,x)\tilde{u}^p \\ = \; & Me^{-\mu_1 t}[-\mu_1\phi + \phi_t - d\triangle\phi - a(t,x)\phi + b(t,x)(Me^{-\mu_1 t})^{(p-1)}\phi^p] \\ = \; & M^p b(t,x)(e^{-\mu_1 t})^p\phi^p > 0. \end{aligned}$$

Impulsive conditions hold, that is, for $x \in \overline{\Omega}$,

$$\tilde{u}(n^+, x) = (1+\alpha)\tilde{u}(n,x)$$

and

$$\tilde{u}((n+\tau)^+, x) = (1-\beta)\tilde{u}(n+\tau, x).$$

Also, a big enough M can be chosen such that $\tilde{u}(0,x) = M\phi(0,x) \geq u_0(x)$ in $x \in \overline{\Omega}$. It follows from the comparison principle [18] and [16, Lemma 3.1] that

$$u(t,x) \leq \tilde{u}(t,x), \;\; x \in \overline{\Omega}, \;\; t > 0.$$

Since $\lim_{t\to\infty} \tilde{u}(t,x) = 0$ by $\mu_1 > 0$, it is clear that $\lim_{t\to\infty} u(t,x) = 0$ uniformly for $x \in \overline{\Omega}$. □

Theorem 4.2 *Assume that $\mu_1(d, a(t,x), \alpha, \beta) < 0$, for each $u_0 \in C(\overline{\Omega})$ such that $u_0 \geq 0$ and $u_0 \not\equiv 0$, the following assertions hold:*
(i) Periodic problem (3.1) *admits a unique solution $U(t,x)$;*
(ii) the solution $u(t,x)$ to (1.1) *satisfies* $\lim_{m\to\infty} u(t+m, x) = U(t,x)$ *for any $t \geq 0$ and uniformly for $x \in \overline{\Omega}$, where $U(t,x)$ is the unique solution defined in (i).*

Proof: (i) The main method is to find an upper and lower solution $\tilde{U}$ and $\hat{U}$ to periodic problem (3.1), respectively.

By Theorem 3.1, there exists a positive constant $K := |\ln(1+\alpha)(1-\beta)| + d\lambda_1 + a^M$ such that

$$\phi_t - d\Delta\phi = a^M\phi - K\phi + \mu_1^{\triangle}\phi$$

with $\mu_1^{\triangle} := \mu_1^{\triangle}(d, a^M - K, \alpha, \beta) > 0$. Next, define

$$\Omega_\varepsilon := \{x \in \Omega : \text{ dist}\,(x, \partial\Omega) > \varepsilon\}, \;\; \chi_{\Omega_\varepsilon}(x) = 1 \,\text{for}\, x \in \Omega_\varepsilon \,\text{and}\, \chi_{\Omega_\varepsilon}(x) = 0 \,\text{for}\, x \in \Omega/\Omega_\varepsilon\},$$

and $(\varphi, \mu_{1,\varepsilon}^{\triangle})$ be the eigenfunction pair with $\max_{(t,x)\in[0,1]\times\overline{\Omega}}\varphi(t,x)=1$, satisfying

$$\varphi_t - d\Delta\varphi = (a^M - K\chi_{\Omega_\varepsilon})\varphi + \mu_{1,\varepsilon}^{\triangle}\varphi.$$

Since $\mu_1^{\triangle} > 0$, we can choose ε sufficiently small such that $\mu_{1,\varepsilon}^{\triangle} := \mu_{1,\varepsilon}^{\triangle}(d, a^M - K\chi_{\Omega_\varepsilon}, \alpha, \beta) > 0$.

Now, we define

$$\tilde{U} = M\varphi,$$

where M is sufficiently big and chosen to be later. We otain

$$\begin{aligned}
&\tilde{U}_t - d\Delta\tilde{U} - a(t,x)\tilde{U} + b(t,x)\tilde{U}^p\\
=&M\varphi_t - dM\Delta\varphi - Ma(t,x)\varphi + b(t,x)(M\varphi)^p\\
>&\tilde{U}[\mu_{1,\varepsilon}^{\triangle} - K\chi_{\Omega_\varepsilon} + b^m(M\varphi)^{(p-1)}]\\
>&\tilde{U}[-K\chi_{\Omega_\varepsilon} + b^m(M\varphi)^{(p-1)}]\\
\geq&0
\end{aligned}$$

provided that $M \geq K/(b^m \min_{[0,1]\times\overline{\Omega}_\varepsilon}[\varphi^{(p-1)}(t,x)])^{1-p}$.

Moreover, we have

$$\tilde{U}(0^+,x) - (1+\alpha)\tilde{U}(0,x) = 0,\ \tilde{U}(\tau^+,x) - (1-\beta)\tilde{U}(\tau,x) = 0,$$

so $\tilde{U}$ is the upper solution, which satisfies

$$\begin{cases}
\tilde{U}_t \geq d\Delta\tilde{U} + a(t,x)\tilde{U} - b(t,x)\tilde{U}^p, & t\in(0^+,\tau]\bigcup(\tau^+,1],\ x\in\Omega,\\
\tilde{U}(t,x) = 0, & t>0, x\in\partial\Omega,\\
\tilde{U}(0,x) = \tilde{U}(1,x), & x\in\Omega,\\
\tilde{U}(0^+,x) - (1+\alpha)\tilde{U}(0,x) = 0, & x\in\overline{\Omega},\\
\tilde{U}(\tau^+,x) - (1-\beta)\tilde{U}(\tau,x) = 0, & x\in\overline{\Omega}.
\end{cases}$$

In what follows, let us consider a lower solution. Define

$$\hat{U}(t,x) = \begin{cases}
\varepsilon\phi(0,x), & t=0,\ x\in\overline{\Omega},\\
\varepsilon\dfrac{\rho_1}{1-\beta}\phi(0^+,x), & t=0^+,\ x\in\overline{\Omega},\\
\varepsilon\dfrac{\rho_1}{1+\alpha}\phi(\tau^+,x), & t=\tau^+,\ x\in\overline{\Omega},\\
\varepsilon\dfrac{\rho_1}{1-\beta}e^{\frac{-\mu_1}{2}t}\phi(t,x), & t\in(0^+,\tau]\bigcup(\tau^+,1],\ x\in\overline{\Omega},
\end{cases}$$

where ϕ is positive eigenfunction related to the principal eigenvalue μ_1 of problem (3.2) with $\max_{(t,x)\in[0,1]\times\overline{\Omega}}\phi(t,x)=1$, where ε is sufficiently small positive constant. To make sure that $\hat{U}(0,x) = \hat{U}(1,x)$, we have $\rho_1 = (1-\beta)e^{\frac{\mu_1}{2}}$, and $0<\rho_1<1-\beta$.

It can be derived that

$$\begin{aligned}
&\hat{U}_t - d\Delta\hat{U} - a(t,x)\hat{U} + b(t,x)\hat{U}^p\\
\leq&\hat{U}[\frac{\mu_1}{2} + b^M(\varepsilon\frac{\rho_1}{1-\beta}e^{\frac{-\mu_1}{2}t}\phi)^{(p-1)}]\\
\leq&0.
\end{aligned}$$

Moreover, impulsive conditions satisfy

$$\begin{array}{rl} & \hat{U}(0^+,x)-(1+\alpha)\hat{U}(0,x) \\ = & \varepsilon\frac{\rho_1}{1-\beta}\phi(0^+,x)-(1+\alpha)\varepsilon\phi(0,x) \\ = & \varepsilon\frac{\rho_1}{1-\beta}(1+\alpha)\phi(0,x)-(1+\alpha)\varepsilon\phi(0,x) \\ < & \varepsilon(1+\alpha)\phi(0,x)-(1+\alpha)\varepsilon\phi(0,x) \\ = & 0 \end{array}$$

and

$$\begin{array}{rl} & \hat{U}(\tau^+,x)-(1-\beta)\hat{U}(\tau,x) \\ = & \varepsilon\frac{\rho_1}{1+\alpha}\phi(\tau^+,x)-(1-\beta)\varepsilon\frac{\rho_1}{1-\beta}e^{\frac{-\mu_1}{2}\tau}\phi(\tau,x) \\ = & \varepsilon\rho_1(1-\beta)\phi(\tau,x)[\frac{1}{1+\alpha}-\frac{1}{1-\beta}e^{\frac{-\mu_1}{2}\tau}] \\ \leq & 0 \end{array}$$

since $\frac{1}{1-\beta}e^{\frac{-\mu_1}{2}\tau}>\frac{1}{1-\beta}>\frac{1}{1+\alpha}$. Therefore, $\hat{U}(t,x)$ is a lower solution to periodic problem (3.1).

By defining $\bar{U}^{(0)}=\tilde{U}$ and $\underline{U}^{(0)}:=\hat{U}$, we have

$$\underline{U}^{(0)}(t,x)\leq U(t,x)\leq\bar{U}^{(0)}(t,x),\ t\geq 0,\ x\in\overline{\Omega}.$$

We now construct two iteration sequences for $\{\bar{U}^{(n)}\}$ and $\{\underline{U}^{(n)}\}$ satisfying

$$\begin{cases} \bar{U}_t^{(n)}-d\Delta\bar{U}^{(n)}+K\bar{U}^{(n)}=K\bar{U}^{(n-1)}+a(t,x)\bar{U}^{(n-1)} & \\ -b(t,x)(\bar{U}^{(n-1)})^p, & t\in(0^+,\tau]\bigcup(\tau^+,1],\ x\in\Omega, \\ \underline{U}_t^{(n)}-d\Delta\underline{U}^{(n)}+K\underline{U}^{(n)}=K\underline{U}^{(n-1)}+a(t,x)\underline{U}^{(n-1)} & \\ -b(t,x)(\underline{U}^{(n-1)})^p, & t\in(0^+,\tau]\bigcup(\tau^+,1],\ x\in\Omega, \\ \bar{U}^{(n)}(t,x)=\underline{U}^{(n)}(t,x)=0, & t\in[0,1],\ x\in\partial\Omega \end{cases} \tag{4.1}$$

with periodic conditions

$$\bar{U}^{(n)}(0,x)=\bar{U}^{(n-1)}(1,x),\ \underline{U}^{(n)}(0,x)=\underline{U}^{(n-1)}(1,x),\ x\in\overline{\Omega}, \tag{4.2}$$

and impulsive conditions

$$\bar{U}^{(n)}(0^+,x)=(1+\alpha)\bar{U}^{(n-1)}(1,x),\ \underline{U}^{(n)}(0^+,x)=(1+\alpha)\underline{U}^{(n-1)}(1,x),\ x\in\Omega \tag{4.3}$$

and

$$\bar{U}^{(n)}(\tau^+,x)=(1-\beta)\bar{U}^{(n-1)}(\tau+1,x),\ \underline{U}^{(n)}(\tau^+,x)=(1-\beta)\underline{U}^{(n-1)}(\tau+1,x),\ x\in\Omega,$$

where $K=\max_{[0,1]\times\overline{\Omega}}[-a(t,x)+pb(t,x)\tilde{U}^{p-1}]$ ensuring the monotonicity of the function $Kz+a(t,x)z-b(t,x)z^p$ with z.

Since

$$\hat{U}\leq\underline{U}^{(k)}\leq\underline{U}^{(k+1)}\leq\bar{U}^{(k+1)}\leq\bar{U}^{(k)}\leq\tilde{U},$$

we obtain

$$\lim_{k\to\infty}\bar{U}^{(k)}=\bar{U}^*\text{ and }\lim_{k\to\infty}\underline{U}^{(k)}=\underline{U}^*,$$

by limiting in (4.1), we can see that $\bar{U}^*$ and $\underline{U}^*$ are two periodic solutions to periodic problem (3.1), satisfying

$$\hat{U} \le \underline{U}^{(k)} \le \underline{U}^{(k+1)} \le \underline{U}^* \le \bar{U}^* \le \bar{U}^{(k+1)} \le \bar{U}^{(k)} \le \tilde{U}.$$

For the uniqueness of the periodic solution, suppose that U_1 and U_2 are two solutions of problem (3.1) and define

$$S = \{s \in [0,1], sU_1 \le U_2,\ t \in [0,1],\ x \in \overline{\Omega}\}.$$

By using the fact that $f(u)/u$ is strictly decreasing with respect to v in $[0, \max\limits_{[0,1]\times\overline{\Omega}} U_2]$, where $f(v) = a(t,x)u - b(t,x)u^p$, we can prove that $1 \in S$ by contradiction similarly as in [16, Theorem 3.4], therefore $U_1 \le U_2$. On the hand, we have $U_2 \le U_1$ and we arrive the uniqueness.

(ii) It follows from the initial iteration in (i) that

$$\underline{U}^{(0)}(t,x) \le u(t,x) \le \bar{U}^{(0)}(t,x),\ t \ge 0,\ x \in \overline{\Omega}.$$

Also,

$$\underline{U}^{(1)}(0,x) = \underline{U}^{(0)}(1,x) \le u(1,x) \le \bar{U}^{(0)}(1,x) = \bar{U}^{(1)}(0,x)$$

for $x \in \overline{\Omega}$.

It is clear by the iteration process that

$$\underline{U}^{(1)}(0^+,x) = (1+\alpha)\underline{U}^{(0)}(1,x) \le (1+\alpha)u(1,x) = u(1^+,x) \le (1+\alpha)\bar{U}^{(0)}(1,x) = \bar{U}^{(1)}(0^+,x),$$

and

$$\begin{aligned} & \underline{U}^{(1)}(\tau^+,x) = (1-\beta)\underline{U}^{(0)}(\tau+1,x) \le (1-\beta)u(\tau+1,x) \\ = & \ u(\tau^+ + 1,x) \le (1-\beta)\bar{U}^{(0)}(\tau+1,x) \\ = & \ \bar{U}^{(1)}(\tau^+,x) \end{aligned}$$

for $x \in \overline{\Omega}$. Thus, $\underline{U}^{(1)}(t,x) \le u(t+1,x) \le \bar{U}^{(1)}(t,x)$ holds for $t \in (0^+,\tau]\bigcup(\tau^+,1]$ and $x \in \overline{\Omega}$ by comparison argument, and induction asserts that $\underline{U}^{(1)}(t,x) \le u(t+1,x) \le \bar{U}^{(1)}(t,x)$ for $t \ge 0$ and $x \in \overline{\Omega}$. Similarly, we can conclude that for any m,

$$\underline{U}^{(m)}(t,x) \le u(t+m,x) \le \bar{U}^{(m)}(t,x),\ t \ge 0,\ x \in \overline{\Omega}$$

by iteration. Therefore, $\lim\limits_{m\to\infty} \underline{U}^{(m)}$ and $\lim\limits_{m\to\infty} \bar{U}^{(m)}$ exist. Also, $\lim\limits_{m\to\infty} \underline{U}^{(m)}(t,x) = \lim\limits_{m\to\infty} \bar{U}^{(m)}(t,x) = U(t,x)$ by the uniqueness of the solution to problem (3.1), which ends the proof. □

Combining Theorems 3.2, 4.1 and 4.2 gives the following corollary.

Corollary 4.3 *Assume that $a(x,t) := a(t)$ and the period 1 is replaced by T. Let $u(t,x)$ be the solution to problem* (1.1).

(i) If $d\lambda_1 T > \ln[(1+\alpha)(1-\beta)] + \int_0^T a(t)dt$, then $\lim\limits_{t\to\infty} u(t,x) = 0$ uniformly for $x \in \bar{\Omega}$;

(ii) if $d\lambda_1 T < \ln[(1+\alpha)(1-\beta)] + \int_0^T a(t)dt$, then $\lim_{m\to\infty} u(t+mT,x) = U(t,x)$ for any $t \ge 0$ and uniformly for $x \in \overline{\Omega}$, where $U(t,x)$ is the unique solution of the periodic problem (3.1).

Our result shows that large diffusion rate (d), impulsive timing (T) and harvesting rate (β) all are unfavorable to the survival of species, while large birth rate (α) and habitat (which means that λ_1 is small) are beneficial for the survival of species.

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