Global stability of periodic solution for a 3-species nonautonomous ratio-dependent diffusive predator-prey system

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Abstract: A 3-species nonautonomous ratio-dependent reaction-diffusive predator-prey system is considered in this article. Firstly, by utilizing a comparison principle and fixed point theorem, the existence of solution of the system which is space homogenous strictly positive and periodic is obtained. And the obtained conditions ensuring the existence of solution can be very easily verified. At the same time, we develop some new analysis techniques as a byproduct. Furthermore, with the help of the upper and lower solutions (UALS) approach for the parabolic partial differential equations and Lyapunov theory, we aim at the globally asymptotically stability problems of the solutions, and some judgment criteria are achieved. Finally, we give some numerical simulations results which validate the theoretical findings of this article.

Keywords: Nonautonomous; Reaction-diffusion; Predator-prey; Ratio-dependent; Global stability

1. Introduction

More and more attention has been paid to stability analysis theory of predator-prey models[1-10] since 1920s when Lotka [11] and Volterra [12] proposed the classical Lotka-Volterra predator-prey model. The "functional response" is thought as the core question in these models, which describes the rate at which predators consume prey. In 1989, Arditi and Ginzburg [13] incorporated predator dependence into functional responses, where they regarded the response function as a function of ratio. Then, in 1999, Conser et al. [14] showed that it's more appropriate to consider ratio-dependent terms into predator-prey model by using some basic but different principles. In 2000, authors in [15] constructed a kind of average Lyapunov function to study a food web model, combined with the knowledge of saturated equilibria, the problems of permanent coexistence and extinction are studied of species. In 2009, M. Haque [16] considered a predator-prey models with interacting populations and ratio-dependent, and obtained the stability of the system. In 2013, Gao and Li [17] studied a predator-prey ratio-dependent system which has a strong Allee effect in prey and has a Bogdanov-Takens bifurcation related with a catastrophic crash of the predator population. In 2015, Agrawal and Saleem [18] considered a predator-prey system with three different populations and ratio-dependent and proved that for the suitable parameters, the model has chaotic attractors. In 2018, Mandal [19] researched a stochastically forced predator-prey ratio-dependent system especially with Allee effect for prey population and demonstrated that the model has the stable interior equilibrium point or limit cycle for the coexistence of both species. In 2020, Jiang et al. [20] considered a predator-prey model with ratio-dependent, the qualitative behaviors are investigated. By utilizing he comparison principle, the global asymptotical stability are studied for the boundary equilibrium, and some sufficient conditions without delays and diffusion effect were obtained. More recently, in 2023, Yu et al. [21] investigated a novel predator-prey ratio-dependent model which has additional food supply and obtained rich dynamic properties of the system. It is noteworthy that none of the above ratio-dependent predator-prey systems contain diffusion terms. Due to the fact that animals always involuntarily gather towards food and water sources, the new model obtained by adding diffusion terms to the above model can more truly depict the objective laws of interactions between species. However, the methods mentioned in the above literature cannot be directly used to study this type of new models.

On the other hand, in mathematical ecology, the classic predator prey models only reveals the population changes caused by predation when the densities of predators and prey are independent of space. It ignores the fact that populations are generally not evenly distributed, as well as that prey and predator instinctively find ways to survive. Prey species (herbivores) usually gather in areas with rich water and grass, while predators (carnivores) typically lurk in areas where prey is frequently found. Above factors are concerned with the diffusion process, which may be fairly complex due to the different aggregation of predator and prey leading to different species mobility. Such mobility can be affected by the aggregation of the different species (cross diffusion) or that of same species (diffusion). Therefore, studying reaction-diffusion population models has important theoretical and practical significance. In recent years, predator-prey models with diffusion have received widespread attention. In 2013, Ko and Ahn [22-23] studied a reaction-diffusion ratio-dependent system with two competing predator species and one prey and achieved the global attractor and persistence of the equations. In 2015, Yang et al. [24] studied a reaction-diffusive food chain model with homogeneous Robin boundary conditions and obtained the existence and uniqueness for coexistence states as well as the existence of the global attractor by using the fixed point index theory. In 2017, Wang [25] investigated the dynamical behavior of a predator-prey diffusive model and obtained some conditions ensuring the existence of non-constant equilibrium solutions and periodic orbits with the help of coincidence degree theory and bifurcation approach. In 2018, Wang and Zhang [26] investigated a reaction-diffusive Leslie-Gower prey-predator model with double free boundaries and proved the existence, uniqueness and regularity of global solution for the model. In 2020, Wu and Zhao [27] studied the existence and stability of the equilibrium solution for predator-prey diffusive model by constructing generalized Jacobian matrix. In 2021, Tian ang Guo [28] studied a reaction-diffusive predator-prey model with Allee effect and constant stocking rate for predator and obtained some sufficient conditions ensuring the asymptotical stability of a spatially homogeneous steady-state solution. In 2022, Yan and Zhang [29] studied a predator-prey diffusion system with B-D response function and obtained some conditions to ensure the stability and instability conditions of the positive equilibrium solutions for the model. It is worth noting that the above systems are all autonomous. However, it is very difficult to study the reaction diffusion ecosystem of more than three species using the eigenvalue methods mentioned in the above literature, and it is even more difficult to research nonautonomous reaction diffusion model.

Additionally, most natural environments (such as seasonal effects of weather, food supplies, mating habits and so forth) are dynamically evolutional such that the birth rate, death rate, and interaction of a population are not invariable and the parameters in an real ecosystem model should be a function of time rather than a constant. Therefore, studying nonautonomous ecosystems is more meaningful than studying corresponding autonomous ecosystems. In 2015, Li and She [30] studied a nonautonomous density-dependent predator-prey model and obtained some sufficient condition of the permanence for the model and the uniqueness of positive periodic solutions. In 2017, Jiang et al. [31] studied a nonautonomous food web with B-D functional response and obtained the existence of positive periodic solution for the model by using Leray-Schauder degree theory. In 2019, Wang et al. [32] studied a nonautonomous predator-prey model with feedback controls and prey diffusion and established some easily verifiable sufficient conditions which guarantee the permanence and globally stability of positive solution for the system by using the delayed differential inequalities and Lyapunov stability theory. In 2020, Tripathi et al. [33] studied a nonautonomous predator-prey model with Crowley-Martin functional response and achieved some sufficient conditions to ensure the permanence and globally attractivity of periodic solution for the system. In 2021, Wu et al. [34] studied a nonautonomous predator-prey model with a prey refuge and Holling type II schemes and obtained some sufficient conditions that ensure the permanence and global stability of the system by using the Lyapunov stability theory and comparison theorem of differential equations. In 2022, Sk et al. [35] researched a nonautonomous 3-species predator-prey system and obtained some sufficient conditions that ensure the stability and instability of periodic solution for the model. In 2023, Guo and Ma [36] studied a nonautonomous periodic predator-prey model with fear effect and general functional responses and achieved some sufficient conditions to ensure the existence of positive periodic solutions for the model by employing the coincidence degree theory. It is worth noting that the above systems are all nonautonomous predator-prey model without diffusion. However, it is very difficult to study the nonautonomous reaction-diffusion ecosystem using the methods obtained in the above literature.

The analysis of ecosystem stability has always been an important topic that biologists and mathematicians are committed to researching. However, as we know, the stability analysis for a nonautonomous predator-prey reaction-diffusion model with multi-species and ratio-dependent functional responses is rather difficult because the interaction in different species is more complex and diverse. Based on this, the researches on this field are still open. More and more experts and scholars focus on attention to reaction-diffusion models especially with 3-species recently, but their researches primarily concerned with the competition and mutualism systems without or with delay (cf. [37-43]) as well as the prey-predator systems without ratio-dependent functional responses (cf [44-47]). As is well known, the methods for studying reaction-diffusion competition and mutualism models are difficult to directly apply to studying predator-prey models, especially nonautonomous multi-species reaction-diffusion predator-prey models. At the same time, the introduction of ratio-dependence functions also greatly increases the difficulty of model research.

Based on the above analysis and inspired by the previous works, in this work, we focus on the following 3-species nonautonomous ratio-dependent predator-prey reaction-diffusion model

$$\begin{cases}\partial u_1(x,t)/\partial t-d_1(t)\Delta u_1(x,t)=u_1(x,t)[r_1(t)-a_{11}(t)u_1(x,t)-\dfrac{a_{12}(t)u_2(x,t)}{b_{12}(t)u_2(x,t)+u_1(x,t)}-\dfrac{a_{13}(t)u_3(x,t)}{b_{13}(t)u_3(x,t)+u_1(x,t)}],\\ \partial u_2(x,t)/\partial t-d_2(t)\Delta u_2(x,t)=u_2(x,t)[-r_2(t)+\dfrac{a_{21}(t)u_1(x,t)}{b_{12}(t)u_2(x,t)+u_1(x,t)}-a_{23}(t)u_3(x,t)],\\ \partial u_3(x,t)/\partial t-d_3(t)\Delta u_3(x,t)=u_3(x,t)[-r_3(t)+\dfrac{a_{31}(t)u_1(x,t)}{b_{13}(t)u_3(x,t)+u_1(x,t)}-a_{32}(t)u_2(x,t)],\end{cases} \tag{1.1}$$

with the following boundary and initial conditions

$$\partial u_i(x,t)/\partial n=0,\ (x,t)\in\partial\Omega\times R^+,\ u_i(x,0)=u_{i0}(x)>0,\ x\in\Omega,\ i=1,2,3. \tag{1.2}$$

where Ω is a bounded smooth domain in R^n with boundary $\partial\Omega$, Δ is a Laplace operator on Ω , $\partial/\partial n$ is the outward normal derivation on $\partial\Omega$, $u_i(x,t)$ denotes the density of i-th populations at the time of t and point $x=(x_1,x_2,\cdots,x_n)$. From Table 1.1, it can be seen that the biological significance of the parameters in model (1.1).

Table 1.1 The biological significance of parameters in model (1.1)

Parameter	Definition	Parameter	Definition
$d_i(t),(i=2,3)$	The diffusivity rates	$a_{1i}(t),(i=2,3)$	The capturing rates of the predators
$r_1(t)$	The intrinsic growth rate	$a_{11}(t)$	The interaction within prey species
$r_i(t),(i=2,3)$	The death rates	$a_{23}(t),\ a_{32}(t)$	The interaction between two predator species
$a_{i1}(t),(i=2,3)$	The conversion rates	$b_{1i}(t),(i=2,3)$	The interference within predator species

The coefficients of the reaction-diffusion predator-prey model (1.1) are positive and continuous, ω- periodic functions. The systems (1.1)-(1.2) describe the interaction between predator and prey species which is based on ratio-dependent functions. And it is an extended model of the famous Lotka-Volterra predator-prey model which have one prey and two competing predators, whose reduction systems have been intensively investigated. Especially, Wang et al. [48] researched the model (1.1) with feedback controls and without diffusion. In this paper, the strictly positive space homogenous periodic solution are studied, and the global asymptotic stability of the new system are given in which we only need a set of easily verified conditions. These results show the permanence of the nonautonomous predator-prey ratio-dependent reaction-diffusion system, the instability of the semitrivial solutions and trivial solutions.

The article organization are showed as follows. In Section 2, we will give some preliminary results and definitions. In Section 3, we will investigate the existence of the strictly positive space homogenous periodic solution of the predator-prey model. In Section 4, we pay more attention to the globally asymptotically stability of the strictly positive periodic solution. In Section 5, we will give some numerical simulation to support the theoretical findings of this article. Lastly, we will give a conclusion to summarize the important contributions of this article.

Remark 1: The innovations and achievements of this article are listed as follows: (1) By introducing ratio-dependent functional responses and variable coefficient into the known population models, a new Lotka-Volterra predator-prey model that can more truly depict the interaction among populations is proposed. (2) By considering of comparison principle and fixed point method, in this process, some new theories and methods have been creatively developed, the existence of the strictly positive space homogenous periodic solution of the new predator-prey system are obtained in which only a set of simplify verified conditions are needed. (3) By constructing a novel Lyapunov functions and utilizing the approach of UALS for the parabolic partial differential equations, the globally asymptotically stability of the space homogenous strictly positive periodic solution are studied in which some sufficient conditions are obtained. (4) Compared with the results in [22, 23, 25, 27, 29, 48], the results obtained in this article are more general, and provides more convenience for the further long-term application of Lotka-Volterra predator-prey model.

2. Preliminary

Some definition and preliminary results are showed in this section.

Definition 2.1 Suppose that $\tilde{U}(x,t)\equiv(\tilde{u}_1(x,t),\tilde{u}_2(x,t),\tilde{u}_3(x,t))$, $\hat{U}(x,t)=(\hat{u}_1(x,t),\hat{u}_2(x,t),\hat{u}_3(x,t))$, if $\tilde{U}(x,t)\geq\hat{U}(x,t)$ and for $(x,t)\in\Omega\times R^+$

$$\partial\tilde{u}_1(x,t)/\partial t-d_1(t)\Delta\tilde{u}_1(x,t)\geq\tilde{u}_1(x,t)[r_1(t)-a_{11}(t)\tilde{u}_1(x,t)-\frac{a_{12}(t)\hat{u}_2(x,t)}{b_{12}(t)\hat{u}_2(x,t)+\tilde{u}_1(x,t)}-\frac{a_{13}(t)\hat{u}_3(x,t)}{b_{13}(t)\hat{u}_3(x,t)+\tilde{u}_1(x,t)}],$$

$$\partial\tilde{u}_2(x,t)/\partial t-d_2(t)\Delta\tilde{u}_2(x,t)\geq\tilde{u}_2(x,t)[-r_2(t)+\frac{a_{21}(t)\tilde{u}_1(x,t)}{b_{12}(t)\tilde{u}_2(x,t)+\tilde{u}_1(x,t)}-a_{23}(t)\hat{u}_3(x,t)],$$

$$\partial\tilde{u}_3(x,t)/\partial t-d_3(t)\Delta\tilde{u}_3(x,t)\geq\tilde{u}_3(x,t)[-r_3(t)+\frac{a_{31}(t)\tilde{u}_1(x,t)}{b_{13}(t)\tilde{u}_3(x,t)+\tilde{u}_1(x,t)}-a_{32}(t)\hat{u}_2(x,t)],$$

$$\partial\hat{u}_1(x,t)/\partial t-d_1(t)\Delta\hat{u}_1(x,t)\leq\hat{u}_1(x,t)[r_1(t)-a_{11}(t)\hat{u}_1(x,t)-\frac{a_{12}(t)\tilde{u}_2(x,t)}{b_{12}(t)\tilde{u}_2(x,t)+\hat{u}_1(x,t)}-\frac{a_{13}(t)\tilde{u}_3(x,t)}{b_{13}(t)\tilde{u}_3(x,t)+\hat{u}_1(x,t)}],$$

$$\partial\hat{u}_2(x,t)/\partial t-d_2(t)\Delta\hat{u}_2(x,t)\leq\hat{u}_2(x,t)[-r_2(t)+\frac{a_{21}(t)\hat{u}_1(x,t)}{b_{12}(t)\hat{u}_2(x,t)+\hat{u}_1(x,t)}-a_{23}(t)\tilde{u}_3(x,t)],$$

$$\partial\hat{u}_3(x,t)/\partial t-d_3(t)\Delta\hat{u}_3(x,t)\leq\hat{u}_3(x,t)[-r_3(t)+\frac{a_{31}(t)\hat{u}_1(x,t)}{b_{13}(t)\hat{u}_3(x,t)+\hat{u}_1(x,t)}-a_{32}(t)\tilde{u}_2(x,t)],$$

and

$$\partial\tilde{u}_i(x,t)/\partial n\geq 0,\partial\hat{u}_i(x,t)/\partial n\leq 0,(x,t)\in\partial\Omega\times R^+,\tilde{u}_i(x,0)\geq u_{i0}(x),\hat{u}_i(x,0)\leq u_{i0}(x),x\in\bar{\Omega},i=1,2,3,$$

Then $\tilde{U}(x,t)$ and $\hat{U}(x,t)$ are called a pair of ordered UALS for model (1.1)-(1.2) .

Lemma 2.1 ([49]) If $\tilde{U}(x,t)$ and $\hat{U}(x,t)$ are a pair of ordered UALS for models (1.1)-(2.2), then models (1.1)-(1.2) have a unique solution $U(x,t)$. Furthermore, it follows that $\tilde{U}(x,t)\geq U(x,t)\geq\hat{U}(x,t)$.

Lemma 2.2 ([50]) Suppose that the function $\varphi(x):R^+\to R$ is uniformly continuous, and the limit $\lim_{x\to\infty}\int_0^x\varphi(s)ds$ exists and is finite, then $\lim_{x\to+\infty}\varphi(x)=0$.

Lemma 2.3 ([51]) Suppose that $V\subset R_n$ is compact and convex and the mapping $\varphi:V\to V$ is continuous, then there exists $x^*\in V$ such that $\varphi(x^*)=x^*$.

3. Existence of the spatial homogeneity periodic solution

Suppose that $\varphi(x)$ is ω - periodic function in R^+, we denote

$$\varphi^m=\sup\{\varphi(x),x\in R^+\},\ \varphi^l=\inf\{\varphi(x),x\in R^+\}.$$

Next, we study the following ODE corresponding to model (1.1)

$$\begin{cases} \frac{du_1(t)}{dt} = u_1(t)[r_1(t) - a_{11}(t)u_1(t) - \frac{a_{12}(t)u_2(t)}{b_{12}(t)u_2(t)+u_1(t)} - \frac{a_{13}(t)u_3(t)}{b_{13}(t)u_3(t)+u_1(t)}], \\ \frac{du_2(t)}{dt} = u_2(t)[-r_2(t) + \frac{a_{21}(t)u_1(t)}{b_{12}(t)u_2(t)+u_1(t)} - a_{23}(t)u_3(t)], \\ \frac{du_3(t)}{dt} = u_3(t)[-r_3(t) + \frac{a_{31}(t)u_1(t)}{b_{13}(t)u_3(t)+u_1(t)} - a_{32}(t)u_2(t)]. \end{cases} \tag{3.1}$$

For the ODE (3.1), we let

$$M_1^* = \frac{r_1^m}{a_{11}^l},\ m_1^* = \frac{r_1^l b_{12}^l b_{13}^l - a_{12}^m b_{13}^l - a_{13}^m b_{12}^l}{a_{11}^m b_{12}^l b_{13}^l},\ M_2^* = M_1 \frac{a_{21}^m - r_2^l}{r_2^l b_{12}^l},\ M_3^* = M_1 \frac{a_{31}^m - r_3^l}{r_3^l b_{13}^l},$$

$$m_2^* = \frac{m_1(-a_{23}^m M_3 + a_{21}^l - r_2^m)}{r_2^m b_{12}^m + a_{23}^m b_{12}^m M_3},\ m_3^* = \frac{m_1(-a_{32}^m M_2 + a_{31}^l - r_3^m)}{r_3^m b_{13}^m + a_{32}^m b_{13}^m M_2}.$$

Definition 3.1 Suppose that there exist seven positive real numbers $M_i, m_i, (i=1,2,3)$ and T, such that $M_i \ge u_i(t) \ge m_i$, as $t > T$ for each positive solution $(u_1(t), u_2(t), u_3(t))$ of the ODE (3.1) with the positive initials, then ODE (3.1) is called permanent.

Theorem 3.1 If it holds that

(H_1) $r_1^l b_{12}^l b_{13}^l - a_{12}^m b_{13}^l - a_{13}^m b_{12}^l > 0$, (H_2) $a_{21}^l - a_{23}^m M_3 - r_2^m > 0$, (H_3) $a_{31}^l - a_{32}^m M_2 - r_3^m > 0$.

Then the ODE (3.1) is permanent.

Proof. When the ODE (3.1) satisfies the conditions $(H_1)-(H_3)$, we choose six appropriate positive numbers $M_i, m_i, (i=1,2,3)$ to satisfy the following inequality

$$M_i > M_i^* > m_i^* > m_i > 0. \tag{3.2}$$

According to the first equation of ODE (3.1), it follows that

$$\frac{du_1(t)}{dt} \le u_1(t)[r_1(t) - a_{11}(t)u_1(t)] \le u_1(t)[r_1^m - a_{11}^l u_1(t)] = a_{11}^l u_1(t)[-u_1(t) + \frac{r_1^m}{a_{11}^l}]$$
$$= a_{11}^l u_1(t)[-u_1(t) + M_1^*] < a_{11}^l u_1(t)[-u_1(t) + M_1],$$

Based on the comparison theorem of ODE, it follows that

(1) When $M_1 > u_1(t_0) > 0$, if $t \ge t_0$, then $M_1 \ge u_1(t)$.

(2) When $M_1 \le u_1(t_0)$, for a enough large t, we have $M_1 \ge u_1(t)$. Otherwise, if $u_1(t) > M_1$, then there is $\alpha > 0$ such that $u_1(t) \ge M_1^* + \alpha$. Furthermore, one has

$$\frac{du_1(t)}{dt}\Big|_{u_1(t)>M_1} \le u_1(t)[r_1(t) - a_{11}(t)u_1(t)] \le a_{11}^l u_1(t)[M_1^* - u_1(t)] < -a_{11}^l \alpha u_1(t),$$

thus, it holds that

$$u_1(t) < u_1(t_0)\exp(-a_{11}^l \alpha\, t) \to 0 \quad \text{as} \quad t \to +\infty.$$

The above inequality contradicts $u_1(t) > M_1$, so we can choose a adequacy large $T_1 \ge t_0 \ge 0$ such that

$$M_1 \ge u_1(t) \quad \text{when} \quad t > T_1. \tag{3.3}$$

According to the second equation of ODE (3.1), and using (3.3), we can get

$$\frac{du_2(t)}{dt} \le u_2(t)[-r_2^l + \frac{a_{21}^m M_1}{b_{12}(t)u_2(t)+M_1}] = u_2(t)[\frac{-r_2^l(b_{12}^l u_2(t)+M_1)+a_{21}^m M_1}{b_{12}^l u_2(t)+M_1}]$$
$$= u_2(t)\frac{r_2^l b_{12}^l}{b_{12}^l u_2(t)+M_1}[-u_2(t)+\frac{M_1(a_{21}^m - r_2^l)}{r_2^l b_{12}^l}] = u_2(t)\frac{r_2^l b_{12}^l}{b_{12}^l u_2(t)+M_1}[-u_2(t)+M_2^*]$$
$$< u_2(t)\frac{r_2^l b_{12}^l}{b_{12}^l u_2(t)+M_1}[-u_2(t)+M_2].$$

According to the same analysis method as above, one has

(3) When $M_2 > u_2(t_0) > 0$, if $t \ge t_0$, then $M_2 \ge u_3(t)$,

(4) When $M_2 \le u_2(t_0)$, for a adequacy large t, we have $M_2 \ge u_2(t)$.

Therefore, we can choose a adequacy large $T_2 \ge t_0 \ge 0$ such that

$$M_2 \ge u_2(t) \quad \text{when} \quad t > T_2. \tag{3.4}$$

Likewise, on the basis of the third equation of ODE (3.1), and using (3.3), it follows that there is a adequacy large $T_3 \ge t_0 \ge 0$ such that

$$M_3 \ge u_3(t) \quad \text{when} \quad t > T_3. \tag{3.5}$$

Next, we prove that $u_1(t), u_2(t), u_3(t)$ have positive lower bound. According to the first equation of ODE (3.1), we can obtain that

$$\frac{du_1(t)}{dt} \ge u_1(t)[r_1^l - a_{11}^m u_1(t) - \frac{a_{12}^m}{b_{12}^l} - \frac{a_{13}^m}{b_{13}^l}]$$
$$= u_1(t)a_{11}^m[-u_1(t) + \frac{r_1^l b_{12}^l b_{13}^l - a_{12}^m b_{13}^l - a_{13}^m b_{12}^l}{a_{11}^m b_{12}^l b_{13}^l}]$$
$$= u_1(t)a_{11}^m[-u_1(t)+m_1^*] > u_1(t)a_{11}^m[-u_1(t)+m_1].$$

Based on the comparison theorem of ODE, we can obtain that

(5) When $u_1(t_0) > m_1$, if $t \ge t_0$, then $u_1(t) \ge m_1$,

(6) When $0 < u_1(t_0) \le m_1$, for a enough large t, we have $m_1 \le u_1(t)$. Otherwise, if $u_1(t) < m_1$, then there exists $\beta > 0$ such that $m_1^* - \beta \ge u_1(t)$. Furthermore, one has

$$\frac{du_1(t)}{dt}\Big|_{u_1(t)<m_1} \ge a_{11}^m u_1(t)[-u_1(t)+m_1^*] > a_{11}^m \beta u_1(t),$$

thus, it follows that $u_1(t) > u_1(t_0)\exp(a_{11}^m \beta t) \to +\infty$ as $t \to +\infty$. The above inequality contradicts $u_1(t) < m_1$, so we can choose a adequacy large $T_1' \ge t_0 \ge 0$ such that

$$m_1 \le u_1(t) \quad \text{when} \quad t > T_1'. \tag{3.6}$$

According to the second equation of ODE (3.1), and invoking (3.5) and (3.6), we can obtain that

$$\frac{du_2(t)}{dt} \ge u_2(t)[-r_2^m + \frac{a_{21}^l m_1}{b_{12}^m u_2(t)+m_1} - a_{23}^m M_3]$$
$$= u_2(t)[\frac{-(r_2^m + a_{23}^m M_3)(b_{12}^m u_2(t)+m_1)+a_{21}^l m_1}{b_{12}^m u_2(t)+m_1}]$$

$$=u_2(t)\frac{(r_2^m+a_{23}^mM_3)b_{12}^m}{b_{12}^mu_2(t)+m_1}[-u_2(t)+\frac{a_{21}^lm_1-(r_2^m+a_{23}^mM_3)m_1}{(r_2^m+a_{23}^mM_3)b_{12}^m}]$$

$$=u_2(t)\frac{(r_2^m+a_{23}^mM_3)b_{12}^m}{b_{12}^mu_2(t)+m_1}[-u_2(t)+m_2^*]>u_2(t)\frac{(r_2^m+a_{23}^mM_3)b_{12}^m}{b_{12}^mu_2(t)+m_1}[-u_2(t)+m_2].$$

By the similar analysis method above and the comparison theorem of ODE, it holds that

(7) When $u_2(t_0)>m_2$, if $t\ge t_0$, then $u_2(t)\ge m_2$,

(8) When $0<u_2(t_0)\le m_2$, if there exists a sufficiently large t, we have $m_2\le u_2(t)$.

Therefore, we can choose a adequacy large $T_2'\ge t_0\ge 0$ such that

$$m_2\le u_2(t) \text{ when } t>T_2'. \tag{3.7}$$

Analogously, it follows that there is a adequacy large $T_3'\ge t_0\ge 0$ such that

$$0<m_3\le u_3(t),\ m_3<m_3^*=\frac{m_1(-a_{32}^mM_2+a_{31}^l-r_3^m)}{r_3^mb_{13}^m+a_{32}^mb_{13}^mM_2} \text{ when } t>T_3'. \tag{3.8}$$

From (3.3)-(3.8), and set $T=\max_{1\le i\le 3}\{T_i,T_i'\}$, then we have $M_i\ge u_i(t)\ge m_i$ as $T<t$ for each positive solution $(u_1(t),u_2(t),u_3(t))$ of ODE (3.1) with any positive initial values. The proof of **Theorem 3.1** is completed.

Theorem 3.2 If the model (1.1) satisfy the assumptions $(H_1)-(H_3)$, there is a strictly positive spatial homogeneity ω-periodic solution $U(t)=\left(u_1^*(t),u_2^*(t),u_3^*(t)\right)$ for the model (1.1).

Proof. Based on the existence and uniqueness theorem of solutions of ODE, we can define a Poincaré mapping $\varphi:R_+^3\to R_+^3$ in the following form

$$\varphi(U_0)=U(t,\omega,t_0,U_0),$$

where $U(t,\omega,t_0,U_0)=(u_1(t),u_2(t),u_3(t))$ be a positive solution of ODE (3.1) subject to the initial conditions $U_0=(u_1(t_0),u_2(t_0),u_3(t_0))$. And define

$$S=\left\{(u_1,u_2,u_3)\in R_+^3\middle|\ m_i\le u_i\le M_i, i=1,2,3\right\},$$

then it is quite clear that that $S\subset R_+^3$ is a convex and compact set. By the **Theorem 3.1** and the continuity of solution of ODE (3.1) with regard to the initial values, it is not difficult to know that the mapping φ is a continuous mapping from S to S. Furthermore, from **Lemma 2.3** we can obtain that ODE (3.1) has a positive ω-periodic solution $(u_1^*(t),u_2^*(t),u_3^*(t)), t\in R^+$. It is easy to know that $(u_1^*(t),u_2^*(t),u_3^*(t))$ is the spatial homogeneity ω-periodic solution for system (1.1). This finishes the proof of **Theorem 3.2**.

4. Stability of the spatial homogeneity periodic solution

In present section, we obtain the globally asymptotically stability of the spatial homogeneity ω-periodic solution of model (1.1) by invoking the new method of UALS for the parabolic partial differential equations and Lyapunov stability theory, some easily verifiable sufficient conditions are given.

Theorem 4.1 Suppose that the ω-periodic model (1.1) satisfies assumptions $(H_1)-(H_3)$ and the following assumptions

$$(H_4)\quad a_{11}^l-\frac{(a_{12}^m+b_{12}^m a_{21}^m)M_2}{(b_{12}^l m_2+m_1)^2}-\frac{(a_{13}^m+b_{13}^m a_{31}^m)M_3}{(b_{13}^l m_3+m_1)^2}>0;$$

$$(H_5)\quad -a_{32}^m-\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}+\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2}>0;$$

$$(H_6)\quad -a_{23}^m-\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}+\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}>0;$$

then there is a spatial homogeneity strictly positive ω-periodic solution $(u_1^*(t),u_2^*(t),u_3^*(t))$. And the ω-periodic solution is globally asymptotically stable, i.e., the solution $(u_1(x,t),u_2(x,t),u_3(x,t))$ of models (1.1)-(1.2) with any initial values fulfills

$$\lim_{t\to\infty}\left(u_i(x,t)-u_i^*(t)\right)=0,\ \text{uniformly for } x\in\bar{\Omega},\ i=1,2,3. \tag{4.1}$$

Proof. By means of **Theorem 3.2**, we have obtained the existence results, next we pay more attention to the stability. Let $l_i=\min_{x\in\bar\Omega}u_{i0}(x)$, $r_i=\max_{x\in\bar\Omega}u_{i0}(x), i=1,2,3,$ then $0<l_i\le u_{i0}(x)\le r_i$. Let $(\tilde u_1(t),\tilde u_2(t),\tilde u_3(t))$ and $(\hat u_1(t),\hat u_2(t),\hat u_3(t))$ are the solutions for ODE (3.1) subject to initial values $(\tilde u_1(0),\tilde u_2(0),\tilde u_3(0))=(r_1,r_2,r_3)$ and $(\hat u_1(0),\hat u_2(0),\hat u_3(0))=(l_1,l_2,l_3)$ respectively, then there exist a pair of ordered UALS $(\tilde u_1(t),\tilde u_2(t),\tilde u_3(t))$ and $(\hat u_1(t),\hat u_2(t),\hat u_3(t))$ for (1.1)-(1.2). Therefore, from **Lemma 2.1** systems (1.1)-(1.2) have a unique solution $(u_1(x,t),u_2(x,t),u_3(x,t)),\ (x,t)\in\bar\Omega\times R^+$, which satisfies

$$(\hat u_1(t),\hat u_2(t),\hat u_3(t))\le(u_1(x,t),u_2(x,t),u_3(x,t))\le(\tilde u_1(t),\tilde u_2(t),\tilde u_3(t)).$$

If we can prove

$$\lim_{t\to\infty}(\tilde u_i(t)-u_i^*(t))=\lim_{t\to\infty}(\hat u_i(t)-u_i^*(t))=0,(i=1,2,3), \tag{4.2}$$

then (4.1) is established. So, if we want to achieve (4.2), we have to prove the solution $(u_1(t),u_2(t),u_3(t))$ for ODE (3.1) with any positive initial value $(u_1(0),u_2(0),u_3(0))=(u_{10},u_{20},u_{30})$ satisfies

$$\lim_{t\to\infty}\left(u_i(t)-u_i^*(t)\right)=0, i=1,2,3. \tag{4.3}$$

By means of **Theorem 3.1**, there exist seven positive real numbers M_i, m_i and T such that

$$m_i\le u_i(t)\le M_i \quad \text{when} \quad t>T.$$

Set Lyapunov function

$$V(t)=\sum_{i=1}^{3}\left|\ln u_i(t)-\ln u^*_i(t)\right|, t>0.$$

Suppose that $D^+V(t)$ is the right derivation on function $V(t)$, it follows that

$$D^+V(t)=\sum_{i=1}^{3}D^+[\left|\ln u_i(t)-\ln u_i^*(t)\right|]=\sum_{i=1}^{3}\operatorname{sgn}\{u_i(t)-u_i^*(t)\}\left(\frac{1}{u_i(t)}\frac{du_i(t)}{dt}-\frac{1}{u_i^*(t)}\frac{du_i^*(t)}{dt}\right)$$

$$=\operatorname{sgn}\{u_1(t)-u_1^*(t)\}[-a_{11}(t)(u_1(t)-u_1^*(t))-a_{12}(t)(\frac{u_2(t)}{(b_{12}(t)u_2(t)+u_1(t))}$$

$$-\frac{u_2^*(t)}{(b_{12}(t)u_2^*(t)+u_1^*(t))})-a_{13}(t)(\frac{u_3(t)}{(b_{13}(t)u_3(t)+u_1(t))}-\frac{u_3^*(t)}{(b_{13}(t)u_3^*(t)+u_1^*(t))})]$$

$$+\operatorname{sgn}\{u_2(t)-u_2^*(t)\}[-a_{23}(t)(u_3(t)-u_3^*(t))$$

$$+a_{21}(t)(\frac{u_1(t)}{b_{12}(t)u_2(t)+u_1(t)}-\frac{u_1^*(t)}{b_{12}(t)u_2^*(t)+u_1^*(t)})]+\operatorname{sgn}\{u_3(t)-u_3^*(t)\}$$

$$[-a_{32}(u_2(t)-u_2^*(t))+a_{31}(t)(\frac{u_1(t)}{b_{13}(t)u_3(t)+u_1(t)}-\frac{u_1^*(t)}{b_{13}(t)u_3^*(t)+u_1^*(t)})]$$

$$=\operatorname{sgn}\{u_1(t)-u_1^*(t)\}[-a_{11}(t)(u_1(t)-u_1^*(t))-$$

$$a_{12}(t)\frac{u_1^*(t)(u_2(t)-u_2^*(t))-u_2^*(t)(u_1(t)-u_1^*(t))}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}$$

$$-a_{13}(t)\frac{u_1^*(t)(u_3(t)-u_3^*(t))-u_3^*(t)(u_1(t)-u_1^*(t))}{(b_{13}(t)u_3(t)+u_1(t))(b_{13}(t)u_3^*(t)+u_1(t))}]$$

$$+\operatorname{sgn}\{u_2(t)-u_2^*(t)\}[a_{21}(t)b_{12}(t)\frac{u_2^*(t)(u_1(t)-u_1^*(t))-u_1^*(t)(u_2(t)-u_2^*(t))}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}$$

$$-a_{23}(t)(u_3(t)-u_3^*(t))]$$

$$+\operatorname{sgn}\{u_3(t)-u_3^*(t)\}[a_{31}(t)b_{13}(t)\frac{u_3^*(t)(u_1(t)-u_1^*(t))-u_1^*(t)(u_3(t)-u_3^*(t))}{(b_{13}(t)u_3(t)+u_1(t))(b_{13}(t)u_3^*(t)+u_1^*(t))}$$

$$-a_{32}(t)(u_2(t)-u_2^*(t))]$$

$$\le\left|u_1(t)-u_1^*(t)\right|[-a_{11}(t)+\frac{(a_{12}(t)+a_{21}(t)b_{12}(t))u_2^*(t)}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}+$$

$$\frac{(a_{13}(t)+a_{31}(t)b_{13}(t))u_3^*(t)}{(b_{13}(t)u_3(t)+u_1(t))(b_{13}(t)u_3^*(t)+u_1^*(t))}]$$

$$+\left|u_2(t)-u_2^*(t)\right|[a_{32}(t)+\frac{(a_{12}(t)-a_{21}(t)b_{12}(t))u_1^*(t)}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}]$$

$$+\left|u_3(t)-u_3^*(t)\right|[a_{23}(t)+\frac{(a_{13}(t)-a_{31}(t)b_{13}(t))u_1^*(t)}{(b_{13}(t)u_2(t)+u_1(t))(b_{13}(t)u_2^*(t)+u_1^*(t))}]$$

$$=\left|u_1(t)-u_1^*(t)\right|[-a_{11}(t)+\frac{(a_{12}(t)+a_{21}(t)b_{12}(t))u_2^*(t)}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}+$$

$$\frac{(a_{13}(t)+a_{31}(t)b_{13}(t))u_3^*(t)}{(b_{13}(t)u_3(t)+u_1(t))(b_{13}(t)u_3^*(t)+u_1^*(t))}]$$

$$+\left|u_2(t)-u_2^*(t)\right|[a_{32}(t)+\frac{a_{12}(t)u_1^*(t)}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}$$

$$-\frac{a_{21}(t)b_{12}(t)u_1^*(t)}{(b_{12}(t)u_2(t)+u_1(t))(b_{12}(t)u_2^*(t)+u_1^*(t))}]$$

$$+\left|u_3(t)-u_3^*(t)\right|[a_{23}(t)+\frac{a_{13}(t)u_1^*(t)}{(b_{13}(t)u_2(t)+u_1(t))(b_{13}(t)u_2^*(t)+u_1^*(t))}]$$

$$-\frac{a_{31}(t)b_{13}(t)u_1^*(t)}{(b_{13}(t)u_2(t)+u_1(t))(b_{13}(t)u_2^*(t)+u_1^*(t))}]$$

$$\le -|u_1(t)-u_1^*(t)|[a_{11}^l-\frac{(a_{12}^m+a_{21}^m b_{12}^m)M_2}{(b_{12}^l m_2+m_1)^2}-\frac{(a_{13}^m+a_{31}^m b_{13}^m)M_3}{(b_{13}^l m_3+m_1)^2}]$$

$$-|u_2(t)-u_2^*(t)|[-a_{32}^m-\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}+\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2}]$$

$$-|u_3(t)-u_3^*(t)|[-a_{23}^m-\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}+\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}].$$

In view of conditions $(H_4)-(H_6)$, one has

$$\alpha=\min\{a_{11}^l-\frac{(a_{12}^m+a_{21}^m b_{12}^m)M_2}{(b_{12}^l m_2+m_1)^2}-\frac{(a_{13}^m+a_{31}^m b_{13}^m)M_3}{(b_{13}^l m_3+m_1)^2},$$
$$-a_{32}^m-\frac{a_{12}^m M_1}{(b_{12}^l m_2+m_1)^2}+\frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2+M_1)^2},$$
$$-a_{23}^m-\frac{a_{13}^m M_1}{(b_{13}^l m_3+m_1)^2}+\frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3+M_1)^2}\}>0.$$

Thus,

$$D^+V(t)\le -\alpha\sum_{i=1}^{3}|u_i(t)-u_i^*(t)|. \tag{4.4}$$

Integrating (4.4) from T to t, $t_0\le T$, we have

$$V(t)+\alpha\int_T^t(\sum_{i=1}^{3}|u_i(t)-u_i^*(t)|)ds\le V(T)<+\infty.$$

Therefore,

$$\int_T^t(\sum_{i=1}^{3}|u_i(t)-u_i^*(t)|)ds\le\frac{V(T)}{\alpha}<+\infty. \tag{4.5}$$

By (4.5), we have

$$\sum_{i=1}^{3}|u_i(t)-u_i^*(t)|\in L^1(T,+\infty).$$

Because of the permanence of ODE (3.1), $\sum_{i=1}^{3}|u_i(t)-u_i^*(t)|$ is uniformity continuous. With help of **Lemma 2.2**, it follows that

$$\lim_{t\to+\infty}|u_i(t)-u_i^*(t)|=0,\ (i=1,2,3).$$

This ends the proof of **Theorem 4.1**.

5. Numerical example

An example is given to validate the results achieved in this article. To prove the correctness of the **Theorem 4.1**, we choose the 2-periodic function as the coefficients of ODE (1.1) and discuss the following 3-species reaction-diffusion 2-periodic model with ratio-dependent functions. Based on the assumptions $(H_1)-(H_6)$ of **Theorem 4.1**, with the help of some calculations we choose some special values of parameters shown in models (5.1)-(5.2). It should be noted that, the selection of above parameters is not unique.

$$\begin{cases} \frac{\partial u_1(x,t)}{\partial t} - \frac{\partial^2 u_1(x,t)}{\partial x^2} = u_1(x,t)[(21+\cos\pi t)-(13+\sin\pi t)u_1(x,t) - \frac{(0.075+0.025\sin\pi t)u_2(x,t)}{(0.95+0.05\sin\pi t)u_2(x,t)+u_1(x,t)} \\ \qquad -\frac{(0.065+0.035\sin\pi t)u_3(x,t)}{(0.97+0.07\sin\pi t)u_3(x,t)+u_1(x,t)}], t>0, x\in(0,2\pi), \\ \frac{\partial u_2(x,t)}{\partial t} - \frac{\partial^2 u_2(x,t)}{\partial x^2} = u_2(x,t)[-(3.25+0.25\cos\pi t) + \frac{(4.9+0.1\sin\pi t)u_1(x,t)}{(0.95+0.05\sin\pi t)u_2(x,t)+u_1(x,t)} \\ \qquad -(0.12+0.1\sin\pi t)u_3(x,t)], t>0, x\in(0,2\pi), \\ \frac{\partial u_3(x,t)}{\partial t} - \frac{\partial^2 u_3(x,t)}{\partial x^2} = u_3(x,t)[-(3.1+0.1\cos\pi t) + \frac{(4.8+0.1\sin\pi t)u_1(x,t)}{(0.97+0.07\sin\pi t)u_3(x,t)+u_1(x,t)} \\ \qquad -(0.13+0.1\sin\pi t)u_2(x,t)], t>0, x\in(0,2\pi), \end{cases} \tag{5.1}$$

with the following initial values and Neumman boundary conditions

$$u_1(x,0)=1.6, u_2(x,0)=0.8, u_3(x,0)=0.85, x\in(0,2\pi), \quad \frac{\partial u_1}{\partial n}=\frac{\partial u_2}{\partial n}=\frac{\partial u_3}{\partial n}=0, t>0, x=0,2\pi. \tag{5.2}$$

By calculating, we have

$$M_1^* \approx 1.8333,\ M_1=1.8334,\ m_1^* \approx 1.4127,\ m_1=1.4126,$$
$$M_2^* \approx 1.3581,\ M_2=1.3582,\ M_3^* \approx 1.2902,\ M_3=1.2903,$$
$$m_2^* \approx 0.3794,\ m_2=0.3793,\ m_3^* \approx 0.4593,\ m_3=0.4592,$$
$$r_1^l b_{12}^l b_{13}^l - a_{12}^m b_{13}^l - a_{13}^m b_{12}^l = 16.02,$$
$$a_{21}^l - a_{23}^m M_3 - r_2^m \approx 1.0161,\ a_{31}^l - a_{32}^m M_2 - r_3^m \approx 1.1876,$$
$$a_{11}^l - \frac{(a_{12}^m + b_{12}^m a_{21}^m)M_2}{(b_{12}^l m_2 + m_1)^2} - \frac{(a_{13}^m + b_{13}^m a_{31}^m)M_3}{(b_{13}^l m_3 + m_1)^2} \approx 7.7374,$$
$$-a_{32}^m - \frac{a_{12}^m M_1}{(b_{12}^l m_2 + m_1)^2} + \frac{a_{21}^l b_{12}^l m_1}{(b_{12}^m M_2 + M_1)^2} \approx 0.3095,$$
$$-a_{23}^m - \frac{a_{13}^m M_1}{(b_{13}^l m_3 + m_1)^2} + \frac{a_{31}^l b_{13}^l m_1}{(b_{13}^m M_3 + M_1)^2} \approx 0.3176.$$

It is quite clear that that models (5.1)-(5.2) satisfy the assumptions of **Theorem 4.1**. From **Theorem 4.1** it is easy to know that the system (5.1) has a spatial homogeneity strictly positive 2-periodic solution $(u_1^*(t), u_2^*(t), u_3^*(t))$. Moreover, the solution $(u_1(x,t), u_2(x,t), u_3(x,t))$ of models (5.1)-(5.2) fulfills

$$\lim_{t\to\infty}\left(u_i(x,t)-u_i^*(t)\right)=0, \text{ uniformly for } x\in(0,2\pi),\ i=1,2,3.$$

By employing the finite differences method and the MATLAB 7.1 software package, we can obtain some numerical solutions for the systems (5.1)-(5.2) which are shown in **Figure 5.1** to **Figure 5.3**. From **Figures 5.1-5.3**, it is not difficult to find that the systems (5.1)-(5.2) have a strictly positive globally asymptotically stable spatial homogeneity 2- periodic solution.

Studying the conditions under which ecosystems are in equilibrium and how to artificially control them has always been an important topic worthy of in-depth research. From theoretical results (Theorem 4.1) and numerical simulations (Figures 5.1-5.3) obtained in this paper, it can be found that the 3-species reaction-diffusion nonautonomous system (1.1)-(1.2) can be in equilibrium when the prey grows rapidly enough and the two predator's capture rates are high enough. To be precise, in models (5.1)-(5.2), the densities of prey and predator will oscillate periodically with a period of 2 and distribute homogeneously in space when the time is long enough.

6. Conclusion

This article shows the great strength of UALS approach for nonlinear nonautonomous reaction-diffusion equations. It’s widely used for solving the problems for nonlinear differential equations in chemistry, engineering and mathematical physics etc. The technique constructing Lyapunov function and a pair of ordered UALS provides a novel approach for reference to deal with the nonlinear differential equation.

The problem of periodic solution for a 3-species nonautonomous reaction-diffusion predator-prey system which have ratio-dependent functional responses is studied. The existence and stability of the space homogenous strictly positive periodic solution are obtained for the nonautonomous nonlinear reaction-diffusion equations only for some easily verifiable criterions. These criterions improve and generalize some previous results. It is especially worth mention that it’s flexible for applications due to the sufficient conditions obtained in this article are very simple. It should be noted that in this study, we do not considered the delays in the model. However, in ecosystems, time delays are widespread and can affect the stability of the system. Consequently, our next goal is to study the multi-species nonautonomous diffusion ecosystem with time delays.

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Authors' contributions

L. Jia, J. Huang and C. Wang contributed equally to each part of this article.

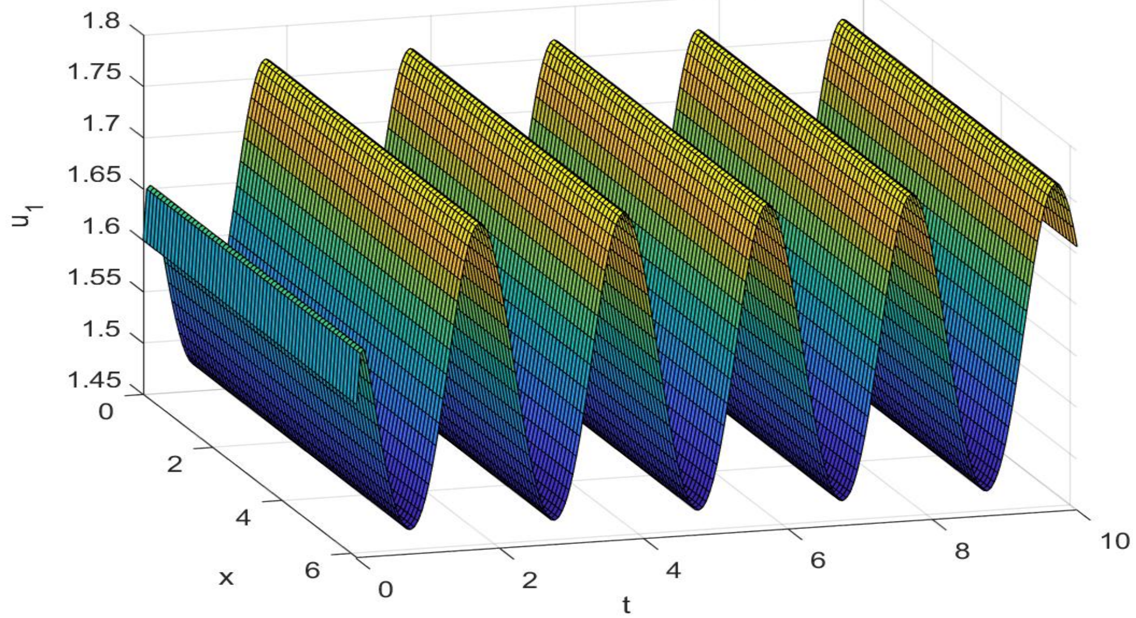

Figure 5.1 Evolution process of the density for the species $u_1(x,t)$ of systems (5.1)-(5.2)

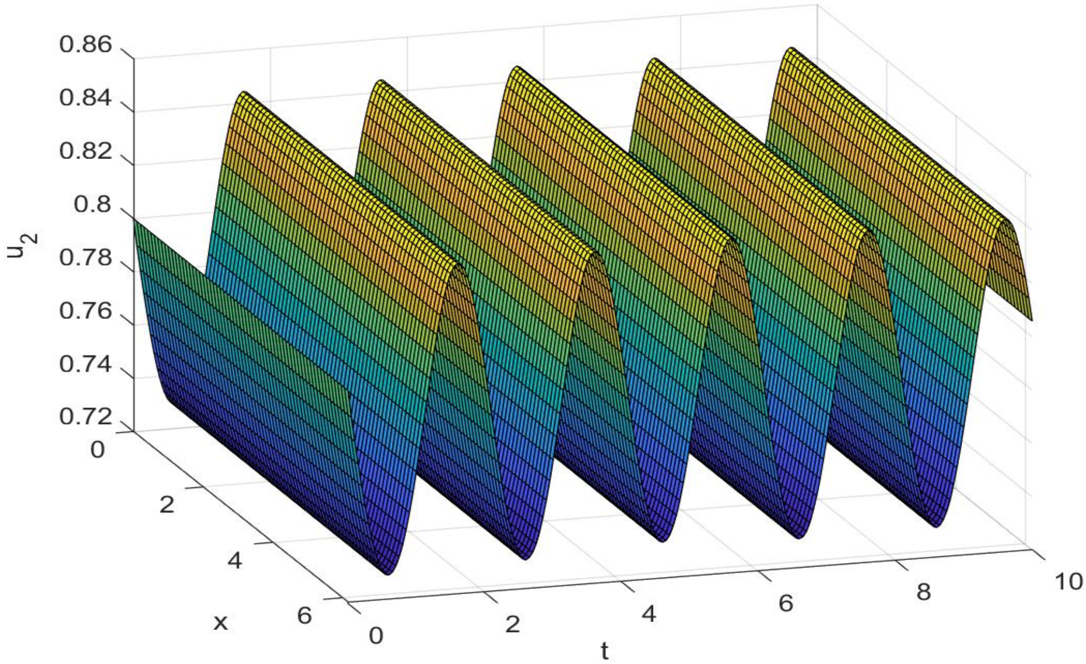

Figure 5.2 Evolution process of the density for the species $u_2(x,t)$ of systems (5.1)-(5.2)

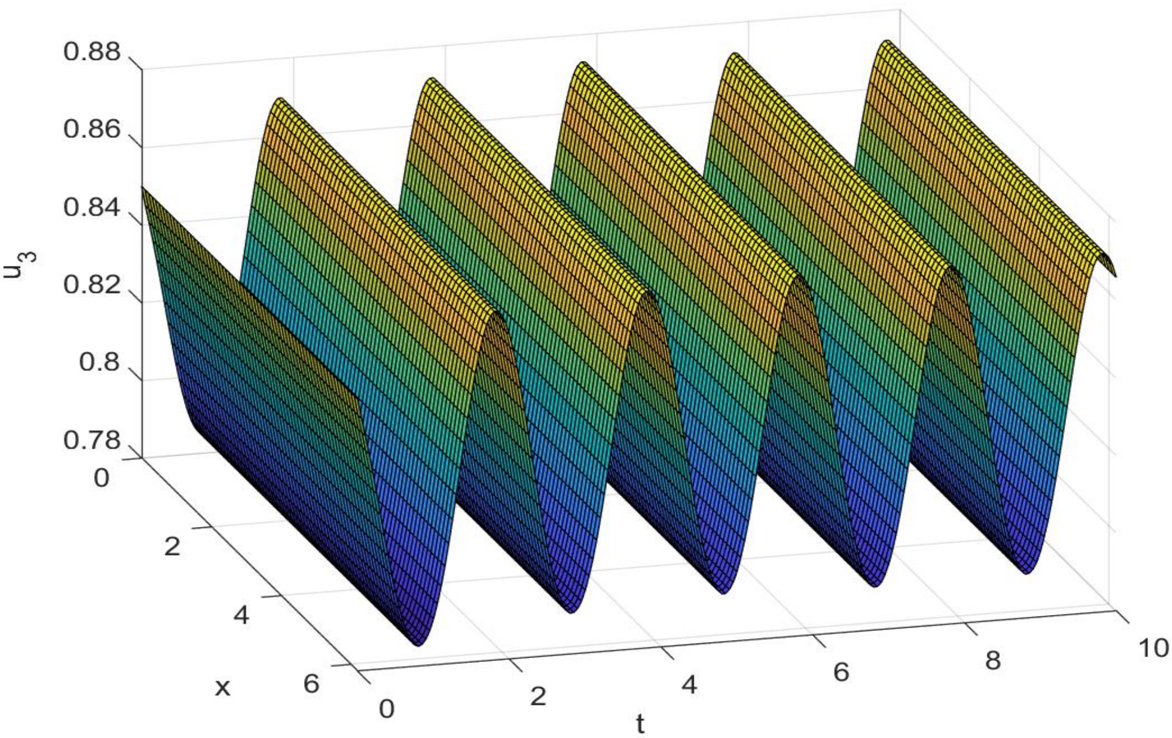

Figure 5.3 Evolution process of the density for the species $u_3(x,t)$ of systems (5.1)-(5.2)

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