Global asymptotical stability for a fishery model with seasonal harvesting ☆

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Abstract

A new fishery model is proposed by using the strategy of seasonal harvesting. Sufficient and necessary conditions are established to ensure the existence of a unique equilibrium or a periodic solution by the approach of Poincaré maps. It is shown that the equilibrium or the periodic solution is globally asymptotically stable. Numerical examples are provided to demonstrate the model dynamics and some biological implications are given as well.

Keywords: Seasonal harvesting, Fishery model, Poincaré map, Periodic solution, Global stability

1. Introduction

In recent years, various population models have been proposed for single or multiple species, and their dynamics have been extensively studied [1–11]. Moreover, much attention has been paid on non-smooth population models [12–24], which arise from discontinuous control strategies such as economic threshold [14], seasonal harvesting [3, 4], periodically releasing [21–24] and so on. These non-smooth models often bring difficulties and challenges such as sliding motions [20, 25], limit cycles [26–28], bifurcation [29–32] and global dynamics [20–22].

To study the dynamics of population, the simplest model introduced by Schaefer

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[2] is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right) - Ex, \tag{1.1}$$

where x represents the fish population, $r > 0$ is the intrinsic growth rate, $K > 0$ is the carrying capacity and E is the fishing effort. In general, $E = E(t)$ is variable in time t, see [1, 2] for example. In order to efficiently exploit stocks and prevent the collapse of key fisheries, the authors [15] proposed a two-dimensional fishery model with discontinuous harvesting as follows

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} &= rx\left(1 - \frac{x}{K}\right) - \psi(x)Ex, \\ \frac{\mathrm{d}E}{\mathrm{d}t} &= k\psi(x)(px - c)E, \end{cases} \tag{1.2}$$

where an equation of fishing effort was added to the model (1.1). The parameters k, p and c are positive, and $\psi(x)$ is usually discontinuous denoting the harvesting strategy. It is shown that discontinuous harvesting strategies are superior to continuous ones in maintaining the fish population at a sustainable level [15]. This discontinuous harvesting strategies require constantly monitoring the number of fish species and determining whether they have reached the threshold, which wastes a lot of manpower in practical operation. In fact, harvesting commonly practiced in fishery does not always occur because of seasonal environmental fluctuations. The seasonal harvesting is also an effective strategy to maintain the fish population at a desirable level, which permits harvesting only during a specified harvest season, while during the rest of the year, the fishing moratorium, no harvesting or less harvesting is allowed.

The periodically switched model has recently been investigated in several studies in population dynamics. In [21, 22], the authors proposed a mosquito population suppression model ignoring the dynamics with respect to the sterile mosquitoes. Thus the dimension of the model is reduced and the mathematical analysis becomes more tractable. Comparing with the massive literature devoted to the analysis of mathematical models with continuous harvesting, discontinuous harvesting strategies have received surprisingly little attention. In a much earlier paper, the author in [33] studied the global dynamics of a logistic equation with seasonal constant-yield harvesting. Constant-yield harvesting is assumed that the population is caught at a constant rate per unit time. It's difficult to keep constant-yield harvesting when the population density is low. The recent work [34] discussed a two-species competition model with seasonal succession and different harvesting strategies, where the author assumed that the fish species is experiencing a bad season if it is in the non-growing season. Although the large fish do not lay eggs in bad seasons, the small fish will continue to grow.

Inspired by the research works presented above, we propose a fishery model by using the strategy of seasonal constant-effort harvesting in this paper. The population of the species obeys logistic type equation in the fishing moratorium. The model is a periodically switched system to describe the phenomenon of seasonal harvesting. This brings some new challenges to mathematical analysis. We will study the global dynamics of this model and establish sufficient and necessary conditions to ensure the existence of a globally asymptotically stable T-periodic solution.

The rest of the paper is organized as follows. Our model is described and some preliminary results are given in Section 2. Section 3 is devoted to the global dynamics of the model. Finally in Section 4, we give some biological implications and numerical simulations to verify our results obtained in this paper.

2. Model description and preliminary results

In this paper, by incorporating the strategy of seasonal harvesting into the model (1.1), we consider the following fishery model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right) - E(t)x, \tag{2.1}$$

where x, K and r are the same as in (1.1),

$$E(t) = \begin{cases} v, & t \in [iT, iT + \bar{T}) \\ 0, & t \in [iT + \bar{T}, (i+1)T), \end{cases} \quad i = 0, 1, 2, \cdots, \tag{2.2}$$

is a discontinuous harvesting function, $T > 0$ represents the time of capture cycle, $\bar{T} \leq T$ is the time of harvesting in the cycle and v is the harvesting rate.

Obviously, model (2.1) consists of the following two sub-equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right) - vx \tag{2.3}$$

and

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right). \tag{2.4}$$

Moreover, model (2.1) is T-periodic and discontinuous with respect to t. We employ the approach of Filippov [25] to define solutions of (2.1). Without loss of generality, we assume that the initial time of a solution is $t = 0$.

Definition 2.1 *A function $x(t) = x(t; 0, u)$ is a solution on $[0, +\infty)$ with initial value $x(0) = u$ of system (2.1), if $x(t)$ is continuous and piecewise differentiable on $[0, +\infty)$, and satisfies the equation (2.3) on $(iT, iT + \bar{T})$ and the equation (2.4) on $(iT + \bar{T}, (i+1)T)$ for $i = 0, 1, 2, \cdots$.*

As usual we only consider the regime where $x(t) \geq 0$. In fact, $x(t) < 0$ for some $t > 0$ implies this species has become extinct. In what follows, the well-posedness of system (2.1) is established.

Proposition 2.1 *Let $x(t; 0, u)$ be a solution of the system (2.1) with initial condition $x(0) = u \geq 0$, then $x(t) \geq 0$ for $t \in [0, t_0)$, where $0 < t_0 < +\infty$.*

Proof. For $t \in [0, \bar{T})$ and any given $u \geq 0$, system (2.1) becomes the initial-value problem for the following ODE on $t \in [0, \bar{T})$:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right) - vx, \quad x(0) = u. \tag{2.5}$$

It is obvious that $rx\left(1 - \frac{x}{K}\right) - vx$ is a locally Lipschitz continuous function, then system (2.1) admits a unique solution on its interval of existence. Moreover, we can get

$$x(t) = \frac{K(r-v)ue^{(r-v)t}}{r\left[e^{(r-v)t} - 1\right]u + K(r-v)}. \tag{2.6}$$

Since $x(0) = u \geq 0$, we have that $x(t) \geq 0$ for $t \in [0, \bar{T})$.

For $t \in [\bar{T}, T)$, from Definition 2.1, the solution of system (2.1) can be determined uniquely by the initial-value problem for the following ODE:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1 - \frac{x}{K}\right), \quad x(\bar{T}) = x(\bar{T}^-). \tag{2.7}$$

Similarly, it is obvious that $rx\left(1 - \frac{x}{K}\right)$ is a locally Lipschitz continuous function, then system (2.1) admits a unique solution on its interval of existence. Meanwhile, the initial value problem (2.7) has a solution

$$x(t) = \frac{Krx(\bar{T})e^{rt}}{r\left[e^{rt} - 1\right]x(\bar{T}) + Kr}. \tag{2.8}$$

It is easy to get $x(\bar{T}^-) \geq 0$ from (2.6). Then we have that $x(t) \geq 0$ for $t \in [\bar{T}, T)$.

Next, we can use the same manner of steps on each time interval $[iT, (i+1)T)$. Thus, the uniqueness and positivity of solutions for model (2.1) are allowed on $[iT, (i+1)T)$. Since i can be chosen sufficiently large, it follows that the solution $x(t) \geq 0$ with $x(0) = u \geq 0$ on $[0, t_0)$, where $t_0 \in [iT, (i+1)T)$. $\square$

Proposition 2.2 *Let $x(t; 0, u)$ be a solution of the system (2.1) with initial condition $u \geq 0$, then the solution $x(t; 0, u)$ is bounded and hence exists on $[0, +\infty)$.*

Proof. If $x(t;0,u)$ be a solution of system (2.1) with initial condition $x(0)=u\geq 0$ on the time interval $[0,t_0)$, where $t_0\in(0,+\infty)$. Since $u\geq 0$, by Proposition 2.1, we have that $x(t;0,u)\geq 0$ for $t\in[0,t_0)$. When $x(t)>K$, we have that

$$\frac{\mathrm{d}x}{\mathrm{d}t}=\begin{cases} rx\left(1-\frac{x}{K}\right)-vx<0, & t\in[iT,iT+\bar{T}),\\ rx\left(1-\frac{x}{K}\right)<0, & t\in[iT+\bar{T},iT+T),\end{cases} \tag{2.9}$$

which implies that $x(t)\leq\max\{x(0),K\}$. In conclusion, the solution $x(t)$ is bounded on $[0,t_0)$. This, together with the virtue of the continuation theorem (see[25]), implies that the solution $x(t)$ exists on the time interval $[0,+\infty)$. □

From the uniqueness of solution of the equations (2.3) and (2.4), we can obtain the uniqueness of the solution of equation (2.1). Moreover,

$$x(t+T;T,u)=x(t;0,u) \tag{2.10}$$

in view of the T-periodicity of system (2.1). Define a Poincaré map $P(\cdot)$ as

$$P(u)=x(T;0,u) \tag{2.11}$$

and a displacement function as

$$d(u)=P(u)-u \tag{2.12}$$

for $u>0$. Then for a given $u>0$, $x(t;0,u)$ is a T-periodic solution if and only if $d(u)=0$. For the sake of convenience, we further define

$$\bar{P}(u)=x(\bar{T};0,u)$$

and

$$P_n(u)=\underbrace{P\circ P\circ\cdots\circ P}_{n}(u),\quad n=1,2,\cdots. \tag{2.13}$$

Then it follows from (2.10) and the uniqueness of solutions that

$$P_n(u)=x(nT;0,u),\quad n=1,2,\cdots. \tag{2.14}$$

Integrating (2.3) from 0 to $\bar{T}$ with initial value u, we have

$$\bar{P}(u)=\frac{K(r-v)ue^{(r-v)\bar{T}}}{r\left[e^{(r-v)\bar{T}}-1\right]u+K(r-v)}. \tag{2.15}$$

Similarly,

$$P(u)=\frac{K\bar{P}(u)e^{r(T-\bar{T})}}{\left[e^{r(T-\bar{T})}-1\right]\bar{P}(u)+K} \tag{2.16}$$

by integrating (2.4) from $\bar{T}$ to T with initial value $\bar{P}(u)$. Some properties of the Poincaré map and the displacement function defined by (2.11) and (2.12) respectively are listed by the following lemmas, which will play an important role in proving our main results.

Lemma 2.1 *Let $P(\cdot)$ and $P_n(\cdot)$ be defined by (2.11) and (2.13) respectively. Then for any $u > 0$ the following statements hold:*

(i) *$\{P_n(u)\}$ is strictly increasing if $P(u) > u$ while strictly decreasing if $P(u) < u$;*

(ii) *$P'(u) = \frac{P^2(u)}{u^2} e^{v\bar{T} - rT}$ and $\lim_{u \to 0^+} P'(u) = e^{rT - v\bar{T}}$.*

Proof. Assume $P(u) > u$, namely $x(T; 0, u) > u$. Then the uniqueness of the solution $x(t; 0, u)$ implies that

$$x(nT; 0, x(T; 0, u)) > x(nT; 0, u) = P_n(u). \tag{2.17}$$

On the other hand, it follows from (2.14) that

$$P_{n+1}(u) = P_n(P(u)) = x(nT; 0, x(T; 0, u)). \tag{2.18}$$

It is obtained from (2.17) and (2.18) that $P_{n+1}(u) > P_n(u)$, which means that $\{P_n(u)\}$ is strictly increasing. Similarly, $\{P_n(u)\}$ is strictly decreasing if $P(u) < u$. Thus the assertion (i) is true.

By direct computations, we get

$$\bar{P}'(u) = \frac{\bar{P}^2(u)}{u^2} e^{-(r-v)\bar{T}} \quad \text{and} \quad P'(u) = \frac{P^2(u)\bar{P}'(u)}{\bar{P}^2(u)} e^{-r(T-\bar{T})}$$

from (2.15) and (2.16) respectively. This means that

$$P'(u) = \frac{P(u)^2}{u^2} e^{v\bar{T} - rT}. \tag{2.19}$$

Notice that $\lim_{u \to 0^+} \bar{P}(u) = \lim_{u \to 0^+} P(u) = 0$ and

$$\lim_{u \to 0^+} \frac{P(u)}{u} = \lim_{u \to 0^+} \frac{P(u)}{\bar{P}(u)} \frac{\bar{P}(u)}{u} = e^{rT - v\bar{T}}$$

again by (2.15) and (2.16). Therefore, it is seen from (2.19) that $\lim_{u \to 0^+} P'(u) = e^{rT - v\bar{T}}$. The proof is finished. $\square$

With the help of Lemma 2.1, some properties of $d(\cdot)$ are stated by the following lemma.

Lemma 2.2 *Let $d(\cdot)$ be defined for $u > 0$ by (2.12). Then $d(u) < 0$ if $\bar{T} \geq rT/v$, and $d(u)$ has a unique zero if $\bar{T} < rT/v$.*

Proof. Suppose that $\bar{T} = rT/v$. Then $r < v$ in view of $T > \bar{T}$. It is obtained from (2.15) and (2.16) respectively that $\bar{P}(u) < u$ and $P(u) < \bar{P}(u)$. Consequently, $P(u) < u$ for $u > 0$, which means $d(u) < 0$ for $u > 0$.

Next we show that $d(\cdot)$ has at most one zero if $\bar{T} \neq rT/v$. Since the proof for the case $\bar{T} < rT/v$ is analogous, we only give the proof for the case $\bar{T} > rT/v$ by the way of contradiction. Assume that $d(\cdot)$ has two adjacent zeros u_1^* and u_2^* with $0 < u_1^* < u_2^*$. Then $d(\cdot)$ has no zeros between u_1^* and u_2^*. By the assertion (ii) of Lemma 2.1, we have $P(u_i^*) = u_i^*$ and

$$d'(u_i^*) = P'(u_i^*) - 1 = e^{v\bar{T} - rT} - 1 > 0, \quad i = 1, 2.$$

Consequently, there is $\delta \in (0, u_2^* - u_1^*)$ such that $d(u) > 0$ for $u \in (u_1^*, u_1^* + \delta)$ and $d(u) < 0$ for $u \in (u_2^* - \delta, u_2^*)$. This implies that there exists another zero between u_1^* and u_2^*, which is a contradiction to the adjacency of the zeros u_1^* and u_2^*.

Notice that $P(K) < K$, i.e.,

$$d(K) < 0 \tag{2.20}$$

in view of $\frac{\mathrm{d}x}{\mathrm{d}t}\big|_{x=K} = -vK < 0$ for the subsystem (2.3) and $\frac{\mathrm{d}x}{\mathrm{d}t}\big|_{x=K} = 0$ for the subsystem (2.4). If $\bar{T} > rT/v$, we claim by contradiction that $d(\cdot)$ has no zeros and $d(u) < 0$ for $u > 0$. Indeed, if $d(\cdot)$ has a zero $u^* > 0$, we have $d'(u^*) > 0$ and there is $\bar{\delta} > 0$ and $u_1 \in (u^*, u^* + \bar{\delta})$ such that

$$d(u_1) > 0. \tag{2.21}$$

Thus it is deduced from (2.20) and (2.21) that $d(\cdot)$ has another zero $u^\star$ with $u^\star > u^*$. This is a contradiction to the uniqueness of a zero. Therefore, $d(u) < 0$ for $u > 0$ if $\bar{T} > rT/v$.

When $\bar{T} < rT/v$, it follows from $\lim\limits_{u \to 0^+} P'(u) = e^{rT - v\bar{T}} > 1$ that there is $u_2 \in (0, u^*)$ such that $P(u_2) > u_2$, i.e.,

$$d(u_2) > 0. \tag{2.22}$$

Thus (2.20) and (2.22) imply that $d(\cdot)$ has a zero between 0 and K. Moreover, it is unique since $d(\cdot)$ has at most one zero. The proof is completed. $\square$

3. Main results

Making use of the properties of the Poincaré map $P(\cdot)$ and the displacement function $d(\cdot)$ stated by the lemmas in Section 2, we discuss the global dynamics of the discontinuous fishery model (2.1) in this section. Note that $x = 0$ is a trivial solution of (2.1), which is called an equilibrium denoted by E_0.

Theorem 3.1 *The equilibrium* $E_0 = 0$ *of the system* (2.1) *is globally asymptotically stable if and only if* $\bar{T} \geq rT/v$.

Proof. We show the necessity by the way of contradiction at first. Suppose $\bar{T} < rT/v$. Then it follows from Lemma 2.1 that $\lim_{u\to 0^+} P'(u) = e^{rT - v\bar{T}} > 1$. Consequently, there exists $\delta_1 > 0$ sufficiently small such that $P(u) > u$ for $u \in (0, \delta_1)$. Furthermore, for a given $u \in (0, \delta_1)$, $\{P_n(u)\}$ is strictly increasing by Lemma 2.1. This is a contradiction with the global asymptotical stability of E_0.

Next we verify the sufficiency. Assume $\bar{T} \geq rT/v$. Then for any $u > 0$, $P(u) < u$ by Lemma 2.2, so $\{P_n(u)\}$ is strictly decreasing by Lemma 2.1. We will prove that E_0 is locally stable. In fact, it is obtained from the continuous dependence that for any $\varepsilon > 0$ there is $\delta_2 > 0$ such that

$$|x(t; 0, u)| < \varepsilon, \quad t \in [0, T] \tag{3.1}$$

as long as $u \in (0, \delta_2)$. For any $t > 0$, there are an integer m and $t_0 \in [0, T]$ such that $t = mT + t_0$. Thus

$$x(t; 0, u) = x(mT + t_0; mT, x(mT; 0, u)) = x(t_0; 0, P_m(u))$$

by (2.10) and (2.14). Note that $P_m(u) < u$ by the monotonicity of $\{P_n(u)\}$. We obtain $P_m(u) \in (0, \delta_2)$ for $u \in (0, \delta_2)$ and hence

$$|x(t; 0, u)| = |x(t_0; 0, P_m(u))| < \varepsilon$$

for any $t \geq 0$ by (3.1). This means E_0 is locally stable. Again by the monotonicity of $\{P_n(u)\}$, we get

$$\lim_{n\to\infty} P_n(u) = 0$$

for any $u > 0$. This implies that E_0 is globally attractive. Thus E_0 is globally asymptotically stable, which completes the proof. □

When $\bar{T} < rT/v$, it will be seen from the following theorem that there is a unique T-periodic solution, which is globally asymptotically stable. Moreover, the T-periodic solution does not exist if $\bar{T} \geq rT/v$.

Theorem 3.2 *For the system* (2.1)*, there is a unique globally asymptotically stable* T*-periodic solution if and only if* $\bar{T} < rT/v$.

Proof. Let us verify the necessity by the way of contradiction firstly, and assume $\bar{T} \geq rT/v$. Then it follows from Lemma 2.2 that $d(u) < u$ for $u > 0$. Hence there is no T-periodic solutions for the system (2.1).

When $\bar{T} < rT/v$, it is obtained from Lemma 2.2 that $d(\cdot)$ has a unique zero $\tilde{u}^* > 0$. So the system (2.1) has a unique T-periodic solution. Next we need to show the T-periodic solution is globally asymptotically stable. The local stability can be similarly proved by the proofs of Theorem 3.1, thus it is omitted here. Notice that $\tilde{u}^* \in (0, K)$ is the unique zero of $d(\cdot)$ and $d(K) < 0$ from (2.20). We have $d(u) > 0$ for $u \in (0, \tilde{u}^*)$ and $d(u) < 0$ for $u \in (\tilde{u}^*, +\infty)$. Again by Lemma 2.1, $\{P_n(u)\}$ is strictly increasing for $u \in (0, \tilde{u}^*)$ while decreasing for $u \in (\tilde{u}^*, +\infty)$. This means that $\lim_{n\to\infty} P_n(u) = \tilde{u}^*$ for $u \in (0, +\infty)$ and the T-periodic solution is globally asymptotically stable. □

Remark 3.1 *When $\bar{T} = 0$ or $\bar{T} = T$ and $r > v$, the differential equations* (2.1) *with a continuous righthand sides, is no harvesting or constant-effort harvesting. It is well-known that there exists a unique positive equilibrium. Under seasonal constant-effort harvesting, we verify that the dynamical behavior is analogous, but with periodic solution instead of equilibrium.*

4. Biological implication and numerical simulations

Making use of the strategy of seasonal harvesting, a new fishery model is established in this paper and its global dynamics is investigated thoroughly. We have shown that there is a unique equilibrium or a unique periodic solution. Furthermore, the equilibrium or the periodic solution is globally asymptotically stable.

Our results suggest that taking appropriate fishing effort v and harvesting time $\bar{T}$ will maintain the normal reproduction of aquatic organisms and prevent the fish stock from extermination. When $v\bar{T} < rT$, Theorem 3.2 tells us that there is a globally asymptotically stable T-periodic solution, see Fig. 1. This means the fish stock will not die out and can be maintained at some desirable level.

However, when fishing effort v is large and harvesting time $\bar{T}$ is long, Theorem 3.1 implies that $E_0 = 0$ is globally asymptotically stable as long as $v\bar{T} \geq rT$, see Fig. 2. Thus large fishing effort or long harvesting time may lead to the extermination of fish stock.

The above conclusions highlight the crucial importance of the time of capture and the harvesting intensity during the time of capture. Although the harvest intensity of the species is not large if the proportion of the time of capture is too large, the fish population will become extinct. Hence, in order to keep the persistence of the fish population, the harvesting intensity on population in the time of capture should not be too high (i.e.,$v < rT/\bar{T}$).

Conflict of interest

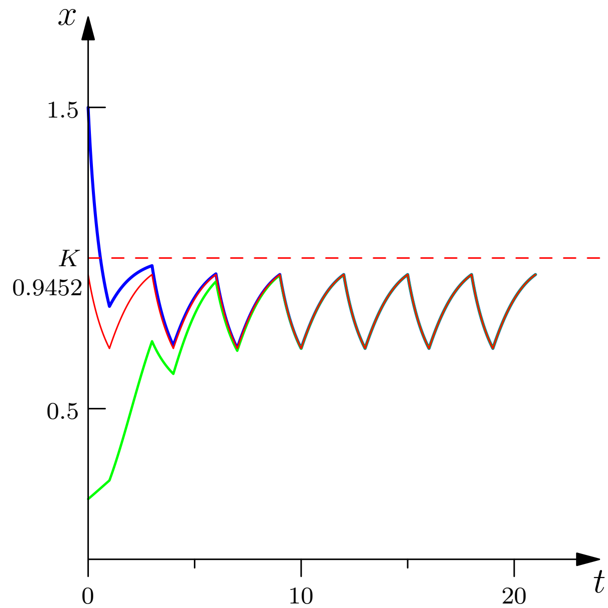

Fig. 1: Global asymptotical stability of the unique periodic solution with initial value $x(0) = 0.9452$, where $r = 1, K = 1, v = 0.5, \bar{T} = 1$ and $T = 3$.

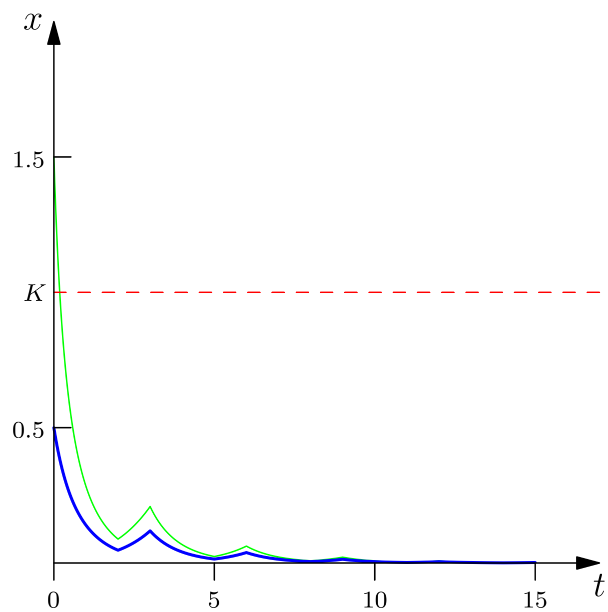

Fig. 2: Global asymptotical stability of $E_0 = 0$, where $r = 1, K = 1, v = 2, \bar{T} = 2$ and $T = 3$.

We declare that we have no conflict of interest.

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