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A TWO-STEP MODULUS-BASED MULTISPLITTING ITERATION METHOD FOR THE NONLINEAR COMPLEMENTARITY PROBLEM*

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Abstract In this paper, we construct a two-step modulus-based multisplitting iteration method based on multiple splittings of the system matrix for the nonlinear complementarity problem. And we prove its convergence when the system matrix is an H-matrix with positive diagonal elements. Numerical experiments show that the proposed method is efficient.

Keywords Two-step, modulus-based multisplitting method, nonlinear complementarity problem, H-matrix.

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1. Introduction

For a given matrix $A \in \mathbb{R}^{n\times n}$ and vector $q \in \mathbb{R}^n$, the nonlinear complementarity problem $NCP(A, q)$ consists of finding a vector $z \in \mathbb{R}^n$ which satisfies the conditions

$$z \geq 0,\ Az + q + \varphi(z) \geq 0,\ z^T(Az + q + \varphi(z)) = 0. \tag{1.1}$$

If $\varphi(z) = 0$, then the problem (1.1) reduces to a linear complementarity problem (LCP).

By using the matrix splitting method to LCP, Bai [1]presented the modulus-based matrix splitting iteration method. This method proved to be very efficient and attracted much attention [4, 7, 11, 12, 14, 15]. Especially, Ke, Ma and Zhang [7] established two classes of modulus-based matrix splitting iteration methods for the second-order cone linear complementarity problems. Applications to nonlinear complementarity problems(NCP) have been also considered [5, 8, 10, 13]. Xia and Li [13] presented some modulus-based matrix splitting iteration methods for a class of nonlinear complementarity problem, such as the modulus-based Gauss-Seidel iteration method (MGS) and the modulus-based SOR iteration method (MSOR). In

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papers [5] and [10], the authors presented accelerated modulus-based matrix splitting iteration methods to solve a class of nonlinear complementarity problems. Ke, Ma and Zhang [8] established a class of relaxation modulus-based matrix splitting iteration methods for circular cone nonlinear complementarity problems.

In addition, Ke and Ma [6] analyzed the convergence of the two-step modulus-based matrix splitting iteration method for LCP and they presented the convergence conditions. Bai and Zhang [2] constructed modulus-based multisplitting iteration methods for LCP based on multiple splittings of the system matrix and they presented the convergence theory. Li, Wang and Yin [9] gave the two-step modulus-based matrix splitting iteration method for a restricted class of NCP. In this paper, we construct a two-step modulus-based multisplitting iteration method based on multiple splittings of the system matrix for NCP.

This paper is organized as follows. Section 2 is the preliminaries. In Section 3, the two-step modulus-based multisplitting iteration method for NCP is introduced. The convergence of this method for H-matrices is considered in Section 4. One numerical example is given in Section 5.

2. Preliminaries

For convenience, we first briefly describe the notations.

Let $A \in \mathbb{R}^{n\times n}$ be an $n \times n$ matrix, for $A, B \in \mathbb{R}^{n\times n}$, we write $A \leq B$ if $a_{ij} \leq b_{ij}$. Calling A nonnegative if $A \geq 0$. By $|A| = (|a_{ij}|)$ we define the absolute value of $A \in \mathbb{R}^{n\times n}$. $\langle A\rangle$ denotes the comparison matrix of A. $\rho(A)$ denotes the spectral radius of A.

Lemma 2.1 ([3]). *(1) If $A \in \mathbb{R}^{n\times n}$ is an M-matrix, $B \in \mathbb{R}^{n\times n}$ is a Z-matrix, and $A \leq B$, then B is an M-matrix. (2) If $A \in \mathbb{R}^{n\times n}$ is an M-matrix, then there is a positive vector x such that $Ax > 0$.*

Lemma 2.2 ([3]). *Let $A \in \mathbb{R}^{n\times n}$ be an H-matrix, then A is nonsingular and $\left|A^{-1}\right| \leq \langle A\rangle^{-1}$.*

Lemma 2.3 ([12]). *Let $A \in \mathbb{R}^{n\times n}$ be nonnegative. If there is a positive vector x such that $Ax < x$, then $\rho(A) < 1$.*

Lemma 2.4 ([3]). *Let $A \in \mathbb{R}^{n\times n}$ be an H-matrix, then $\rho\left(|D|^{-1}|B|\right) < 1$, where $D = \operatorname{diag}(A), B = D - A$.*

3. Two-step modulus-based multisplitting method

Lemma 3.1 ([14]). *Let $A = M - N$ be a splitting of A, h be a positive constant, and Ω be a positive diagonal matrix. Then:*

(1) If z is a solution of (1.1), then $x = \frac{h}{2}\left(z - \Omega^{-1}\varphi(z)\right)$ satisfies the implicit fixed-point equation

$$(\Omega + M)x = Nx + (\Omega - A)|x| - h\left[q + \varphi\left(\frac{1}{h}(|x| + x)\right)\right]. \tag{3.1}$$

(2) If x satisfies (3.1), then $z = \frac{1}{h}(|x| + x)$ is a solution of (1.1).

To suit computational requirements of the modern high-speed multiprocessor systems, by Lemma 3.1, we establish the following two-step modulus-based multisplitting(TMM) iteration method and its several special explicit forms.

Step 1. Choose an initial vector $x^{(0)} \in \mathbb{R}^n$ and set $m := 0$;

Step 2. For $k = 1, 2, \cdots, l$, we solve the subsystem

$$\begin{aligned}(\Omega + M_k')\, x^{\left(m+\frac{1}{2},k\right)} =& N_k' x^{(m)} + (\Omega - A)\left|x^{(m)}\right| \\ & - h\left[q + \varphi\left(\frac{1}{h}\left(\left|x^{(m)}\right| + x^{(m)}\right)\right)\right],\end{aligned} \tag{3.2}$$

$$\begin{aligned}(\Omega + M_k'')\, x^{(m+1,k)} =& N_k'' x^{\left(m+\frac{1}{2},k\right)} + (\Omega - A)\left|x^{\left(m+\frac{1}{2},k\right)}\right| \\ & - h\left[q + \varphi\left(\frac{1}{h}\left(\left|x^{\left(m+\frac{1}{2},k\right)}\right| + x^{\left(m+\frac{1}{2},k\right)}\right)\right)\right];\end{aligned} \tag{3.3}$$

Step 3. $x^{(m+1)} = \sum_{k=1}^{l} E_k x^{(m+1,k)}$ and $z^{(m+1)} = \frac{1}{h}\left(\left|x^{(m+1)}\right| + x^{(m+1)}\right)$;

Step 4. If $z^{(m+1)}$ satisfies a prescribed stopping rule, then stop. Otherwise, set $m := m + 1$ and return to Step 2.

The TMM method provides a general framework of two-step modulus-based multisplitting iteration methods for solving nonlinear complementarity problems. Such iteration methods have a convenient parallel structure and can be implemented on parallel computers. In this method, taking

$$M_k' = \frac{1}{\alpha}\left(D - \beta L_k'\right),\ N_k' = \frac{1}{\alpha}\left[(1-\alpha)D + (\alpha - \beta)L_k' + \alpha U_k'\right],$$
$$M_k'' = \frac{1}{\alpha}\left(D - \beta L_k''\right),\ N_k'' = \frac{1}{\alpha}\left[(1-\alpha)D + (\alpha - \beta)L_k'' + \alpha U_k''\right],$$

we can get the two-step modulus-based multisplitting accelerated overrelaxation iteration method (TMMAOR). For $\alpha = 1$, $\beta = 0$, it becomes the two-step modulus-based multisplitting Jacobi method (TMMJ), for $\alpha = \beta = 1$, the two-step modulus-based multisplitting Gauss-Seidel method (TMMGS) and for $\alpha = \beta$, the two-step modulus-based multisplitting SOR method (TMMSOR). When $l = 1$, TMMAOR, TMMSOR, TMMGS and TMMJ becomes TMAOR, TMSOR, TMGS and TMJ, respectively.

4. Main Results

To present the following discussion, we assume that

$$\varphi(z) = \left(\varphi_1(z_1), \varphi_2(z_2), \cdots, \varphi_n(z_n)\right)^T$$

is differentiable, satisfying that $0 \leq \frac{\mathrm{d}\varphi_i(z_i)}{\mathrm{d}z_i} \leq \psi_i$, where $\psi_i \in \mathbb{R},\ i = 1, 2, \cdots, n$.

By the differential mean value theorem, there exists $\xi_i^{(m)} \in \mathbb{R}$, such that

$$\varphi_i\left(z_i^{(m)}\right) - \varphi\left(z_i^*\right) = \frac{\mathrm{d}\varphi_i\left(\xi_i^{(m)}\right)}{\mathrm{d}z_i}\left(z_i^{(m)} - z_i^*\right),\ i = 1, 2, \cdots, n.$$

Let $\psi^{(m)} = \operatorname{diag}\left(\frac{\mathrm{d}\varphi_1\left(\xi_1^{(m)}\right)}{\mathrm{d}z_1}, \frac{\mathrm{d}\varphi_2\left(\xi_2^{(m)}\right)}{\mathrm{d}z_2}, \cdots, \frac{\mathrm{d}\varphi_n\left(\xi_n^{(m)}\right)}{\mathrm{d}z_n}\right)$ and $\psi = \operatorname{diag}(\psi_1, \psi_2, \cdots, \psi_n)$, then we have

$$\varphi\left(z^{(m)}\right) - \varphi\left(z^*\right) = \psi^{(m)}\left(z^{(m)} - z^*\right) \quad \text{and} \quad \psi^{(m)} \leq \psi.$$

Theorem 4.1. *Let $A \in \mathbb{R}^{n\times n}$ be an H-matrix with positive diagonal elements and let (M_k', N_k', E_k), (M_k'', N_k'', E_k) be two multisplittings of A. Assume that $A = M_k' - N_k' = M_k'' - N_k''$ are H-splittings, $\psi_k \le \psi, h > 0$ and Ω is a positive diagonal matrix satisfying $\Omega \ge D + \psi$, then for any initial vector $x^{(0)} \in \mathbb{R}^n$ the iterative sequence $\left\{z^{(m)}\right\}_{m=0}^{\infty}$ generated by the TMM method convergences to the unique solution z^* of the $NCP(A,q)$.*

Proof. Let z^* be a solution of (1.1), then $x^* = \frac{h}{2}\left(z^* - \Omega^{-1}\varphi\left(z^*\right)\right)$ satisfies

$$(\Omega + M)x^* = Nx^* + (\Omega - A)\left|x^*\right| - h\left[q + \varphi\left(\frac{1}{h}\left(\left|x^*\right| + x^*\right)\right)\right]. \qquad (4.1)$$

To prove $\lim\limits_{m\to\infty} z^{(m)} = z^*$, we need only to prove that $\lim\limits_{m\to\infty} x^{(m)} = x^*$.

By (3.2) and (4.1), we have

$$\begin{aligned}
&\left|x^{\left(m+\frac{1}{2},k\right)} - x^*\right| \\
=&\left|(\Omega + M_k')^{-1}\left\{N_k'\left(x^{(m)} - x^*\right) + (\Omega - A)\left(\left|x^{(m)}\right| - \left|x^*\right|\right)\right.\right. \\
&\left.\left. -h\left[\varphi\left(\frac{1}{h}\left(\left|x^{(m)}\right| + x^{(m)}\right)\right) - \varphi\left(\frac{1}{h}\left(\left|x^*\right| + x^*\right)\right)\right]\right\}\right| \\
=&\left|(\Omega + M_k')^{-1}\left\{N_k'\left(x^{(m)} - x^*\right) + (\Omega - A)\left(\left|x^{(m)}\right| - \left|x^*\right|\right)\right.\right. \\
&\left.\left. -\psi^{(m)}\left[\left(\left|x^{(m)}\right| - \left|x^*\right|\right) + x^{(m)} - x^*\right]\right\}\right| \\
=&\left|(\Omega + M_k')^{-1}\left[\left(N_k' - \psi^{(m)}\right)\left(x^{(m)} - x^*\right) + \left(\Omega - A - \psi^{(m)}\right)\left(\left|x^{(m)}\right| - \left|x^*\right|\right)\right]\right| \\
\le&\left|(\Omega + M_k')^{-1}\right|\left(\left|N_k' - \psi^{(m)}\right| + \left|\Omega - A - \psi^{(m)}\right|\right)\left|x^{(m)} - x^*\right|.
\end{aligned}$$

Since $\langle M_k'\rangle - |N_k'|$ is an M-matrix, by Lemma 2.1, $\langle M_k'\rangle$ is also an M-matrix, so M_k' and $\Omega + M_k'$ are H-matrices. By Lemma 2.2, we know that $\left|(\Omega + M_k')^{-1}\right| \le (\Omega + \langle M_k'\rangle)^{-1}$, so

$$\begin{aligned}
\left|x^{\left(m+\frac{1}{2},k\right)} - x^*\right| &\le \left|(\Omega + M_k')^{-1}\right|\left(\left|N_k' - \psi^{(m)}\right| + \left|\Omega - A - \psi^{(m)}\right|\right)\left|x^{(m)} - x^*\right| \\
&\le (\Omega + \langle M_k'\rangle)^{-1}\left(\left|N_k' - \psi^{(m)}\right| + \left|\Omega - A - \psi^{(m)}\right|\right)\left|x^{(m)} - x^*\right| \\
&= l_k'\left|x^{(m)} - x^*\right|,
\end{aligned}$$

where

$$l_k' = (\Omega + \langle M_k'\rangle)^{-1}\left(\left|N_k' - \psi^{(m)}\right| + \left|\Omega - A - \psi^{(m)}\right|\right).$$

Similarly, we have

$$\left|x^{(m+1,k)} - x^*\right| \le l_k''\left|x^{\left(m+\frac{1}{2},k\right)} - x^*\right|,$$

where

$$l_k'' = (\Omega + \langle M_k''\rangle)^{-1}\left(\left|N_k'' - \psi^{\left(m+\frac{1}{2}\right)}\right| + \left|\Omega - A - \psi^{\left(m+\frac{1}{2}\right)}\right|\right).$$

So the error formula of the TMM iteration method is

$$\left|x^{(m+1)} - x^*\right| \le \sum_{k=1}^{l} E_k\left|x^{(m+1,k)} - x^*\right| \le \sum_{k=1}^{l} E_k l_k'' l_k'\left|x^{(m)} - x^*\right| = l_{\mathrm{TMM}}\left|x^{(m)} - x^*\right|,$$

where $l_{\mathrm{TMM}} = \sum_{k=1}^{l} E_k l_k'' l_k'$.

It is obvious that l_k' is nonnegative, and

$$\begin{aligned}
l_k' &= (\Omega + \langle M_k' \rangle)^{-1} \left(\left| N_k' - \psi^{(m)} \right| + \left| \Omega - A - \psi^{(m)} \right| \right) \\
&= I - (\Omega + \langle M_k' \rangle)^{-1} (\Omega + \langle M_k' \rangle) + (\Omega + \langle M_k' \rangle)^{-1} \left(\left| N_k' - \psi^{(m)} \right| + \left| \Omega - A - \psi^{(m)} \right| \right) \\
&= I - (\Omega + \langle M_k' \rangle)^{-1} \left(\Omega + \langle M_k' \rangle - \left| N_k' - \psi^{(m)} \right| - \left| \Omega - A - \psi^{(m)} \right| \right) \\
&= I - (\Omega + \langle M_k' \rangle)^{-1} \left(\langle M_k' \rangle - \left| N_k' - \psi^{(m)} \right| + \Omega - \left| \Omega - A - \psi^{(m)} \right| \right) \\
&= I - (\Omega + \langle M_k' \rangle)^{-1} \left(\langle M_k' \rangle - \left| N_k' - \psi^{(m)} \right| + \Omega - \left| \Omega - D - \psi^{(m)} \right| - |B| \right) \\
&= I - (\Omega + \langle M_k' \rangle)^{-1} \left(\langle M_k' \rangle - \left| N_k' - \psi^{(m)} \right| + D + \psi^{(m)} - |B| \right) \\
&\leq I - (\Omega + \langle M_k' \rangle)^{-1} \left(\langle M_k' \rangle - |N_k'| + D - |B| \right).
\end{aligned}$$

Since $\langle M_k' \rangle - |N_k'|$ is an M-matrix, by Lemma 2.1, there exists a positive vector $u > 0$ such that $(\langle M_k' \rangle - |N_k'|)\, u > 0$.

It is obvious that

$$a_{ii} = |a_{ii}| \geq |m_{ii}'| - |n_{ii}'|, \quad |a_{ij}| \leq \left| m_{ij}' \right| + \left| n_{ij}' \right|,$$

thus, the ith component of $(D - |B|)u$ satisfies

$$a_{ii} u_i - \sum_{j \neq i} |a_{ij}|\, u_j \geq (|m_{ii}'| - |n_{ii}'|)\, u_i - \sum_{j \neq i} \left(\left| m_{ij}' \right| + \left| n_{ij}' \right| \right) u_j > 0,$$

so $(D - |B|)u > 0$, $l_k' u \leq u - (\Omega + \langle M_k' \rangle)^{-1} (\langle M_k' \rangle - |N_k'| + D - |B|)\, u < u$.

Similarly, $l_k'' u < u$. So $l_k'' l_k' u < l_k'' u < u$, and $E_k l_k'' l_k' u < E_k u, \sum_{k=1}^{l} E_k l_k'' l_k' u < \sum_{k=1}^{l} E_k u$, i.e., $l_{\mathrm{TMM}} u < u$.

Since l_{TMM} is nonnegative, by Lemma 2.3, we have $\rho\,(l_{\mathrm{TMM}}) < 1$.

The proof is completed. □

From Theorem 4.1, we can obtain the following theorem easily.

Theorem 4.2. *Let $A \in \mathbb{R}^{n \times n}$ be an H-matrix with positive diagonal elements, $D = \mathrm{diag}(A)$, $B = D - A$, and let $(D - L_k', U_k', E_k)$, $(D - L_k'', U_k'', E_k)$ be two multisplittings of A, where L_k is a strictly lower-triangular matrix and $U_k' = D - L_k' - A$, $U_k'' = D - L_k'' - A$. Assume that $\psi_k \leq \psi$, $h > 0$ and Ω is a positive diagonal matrix satisfying $\Omega \geq D + \psi$, then for any initial vector $x^{(0)} \in \mathbb{R}^n$, the iterative sequence $\left\{ z^{(m)} \right\}_{m=0}^{\infty}$ generated by the TMMAOR method convergences to the unique solution z^* of the $NCP(A, q)$, provided that $0 < \beta \leq \alpha \leq 1$.*

5. Numerical example

One numerical example is given in this section to illustrate the efficiency of the proposed method and to verify the convergence theory established above. In all the following numerical experiments, the initial vector is chosen to be zero and $h = 1$. And set $A = D - L - U$, where $D, -L, -U$ are the diagonal, the strictly lower-triangular and the strictly upper-triangular matrices of A, respectively. Let $M_1' = M_1'' = \frac{1}{\alpha} D - L$, $N_1' = N_1'' = \frac{1}{\alpha}[(1 - \alpha) D + \alpha U]$, and $M_2' = M_2'' = \frac{1}{\alpha} D - U$,

$N_2' = N_2'' = \frac{1}{\alpha}[(1-\alpha)D+\alpha L]$, then $A = M_1' - N_1' = M_2' - N_2' = M_1'' - N_1'' = M_2'' - N_2''$ are two H-compatible splittings of A.

Since the complementarity condition $z^T(Az + q + \varphi(z)) = 0$ is equivalent to $\left\|\min\left(Az^{(k)} + q + \varphi\left(z^{(k)}\right), z^{(k)}\right)\right\|_2 = 0$, iterations are terminated when the norm of the residual vector (denoted by 'RES')

$$\mathrm{RES}\left(z^{(k)}\right) := \left\|\min\left(Az^{(k)} + q + \varphi\left(z^{(k)}\right), z^{(k)}\right)\right\|_2$$

satisfies $\mathrm{RES} \leq 10^{-5}$, or k reaches the maximal number of iteration steps, which is 1000 in our paper. All the computations are performed in MATLAB® with double machine precision where the CPU is 2.40 GHz and the memory is 4.00 GB.

Example 5.1 ([9]). Let m be a given positive integer, $n = m^2$. Choose A in (1.1) to be a block upper tridiagonal matrix as follows:

$$A = \begin{pmatrix} S & -I & -I & & \\ & S & -I & \ddots & \\ & & S & \ddots & -I \\ & & & \ddots & -I \\ & & & & S \end{pmatrix} \in \mathbb{R}^{n\times n}$$

where $S = \mathrm{tridiag}(-1, 4, -1) \in \mathbb{R}^{m\times m}$ is a tridiagonal matrix. Let $q = (1, -1, \cdots, 1, (-1)^{n-1})^T \in \mathbb{R}^n$ and

$$\varphi(z) = \left(\sqrt{z_1^2 + 0.25}, \sqrt{z_2^2 + 0.25}, \cdots, \sqrt{z_n^2 + 0.25}\right)^T \in \mathbb{R}^n.$$

The matrix A in Example 5.1 is an H_+-matrix. In actual implementation, the parameter matrix Ω is chosen to be $D + I$ in Example 5.1 for both the two-step modulus-based multisplitting successive overrelaxation method and the two-step modulus-based successive overrelaxation method, where D is the diagonal matrix of A, I is the identity matrix. For TMMSOR, we choose $E_1 = \mathrm{diag}(1, 0, 1, 0, \cdots, n \bmod 2) \in \mathbb{R}^{n\times n}$ and $E_2 = I - E_1$.

Table 1. The optimal parameters α^* for TMSOR and TMMSOR in Example 5.1.

m		α	0.8	0.9	1.0	**1.1***	1.2	1.3	1.4
256	TMSOR	IT	10	9	8	**8***	8	8	9
		CPU	0.188	0.172	0.156	**0.141***	0.156	0.156	0.172
	TMMSOR	IT	7	6	6	**6***	7	8	8
		CPU	0.112	0.085	0.076	**0.074***	0.108	0.123	0.120

In Table 1, the number of iteration steps (denoted by 'IT') and the elapsed CPU time in seconds (denoted by 'CPU') are listed for the two-step modulus-based multisplitting successive overrelaxation iteration method and the two-step modulus-based successive overrelaxation iteration method when parameter α varies from 0.8 to 1.4 with $m = 256$. The optimal parameters α^* is chosen firstly to minimize the

Table 2. Numerical results for Example 5.1.

m		MGS	MSOR	TMSOR	TMMSOR
256	IT	20	19	8	6
	CPU	0.203	0.187	0.141	0.074
	RES	9.26e-06	8.51e-06	3.30e-06	1.62e-06
512	IT	20	19	8	6
	CPU	0.890	0.860	0.688	0.594
	RES	9.26e-06	8.51e-06	4.78e-06	3.17e-06
1024	IT	21	20	8	6
	CPU	3.798	3.625	2.782	2.644
	RES	7.76e-06	6.36e-06	6.84e-06	6.27e-06
2048	IT	22	21	8	7
	CPU	19.096	18.096	14.064	13.942
	RES	6.49e-06	4.78e-06	9.76e-06	1.65e-06

number of iteration steps. When the number of iteration steps are the same, then we choose α^* to minimize the elapsed CPU time.

From Table 1, it is seen that for Example 5.1, the optimal parameter $\alpha^* = 1.1$ for both the two-step modulus-based multisplitting successive overrelaxation iteration method and the two-step modulus-based successive overrelaxation iteration method when $m = 256$. In the following, we choose $\alpha^* = 1.1$ for both the two-step modulus-based multisplitting successive overrelaxation iteration method and the two-step modulus-based successive overrelaxation iteration method.

In Table 2, the number of iteration steps, the elapsed CPU time in seconds and the residual for four methods are listed respectively when m is varying.

From Table 2, it is observed that with the same dimension, the number of iteration steps for two-step modulus-based multisplitting method is less than that for modulus-based matrix splitting method and two-step modulus-based matrix splitting method, and the two-step modulus-based multisplitting method costs less CPU time. Meanwhile, the CPU time increases when the problem size $n = m^2$ increases for all methods, while the number of the iteration steps changes few.

6. Conclusions

In this paper, the two-step modulus-based multisplitting iteration method for a class of nonlinear complementarity problems was proposed and its convergence theories were studied when the system matrix is an H-matrix with positive diagonal elements. Numerical experiments showed the new method is more effective than modulus-based matrix splitting method and two-step modulus-based matrix splitting method.

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