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COMPUTATION ON THE TOPOLOGICAL INDICES OF HYALURONIC ACID*

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Abstract In this paper, we investigate the topological indices of Hyaluronic Acid. By constructing the graph of molecular structure and using the edge partitioning technique, we determine the general Randić index, first and second Zagreb polynomial indices, general sum-connectivity index, ordinary geometric-arithmetic index and general harmonic index of Hyaluronic Acid.

Keywords Topological index, Hyaluronic Acid, coronary heart disease, drug delivery systems.

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1. Introduction

Chemical graph theory is an important branch of mathematical chemistry, which is of significant application in solving molecular problems. In chemical graph theory, a graph stands for a molecule. It takes atoms as vertices and molecular bonds as edges of a graph. Let $G = (V(G), E(G))$ be a molecular graph, where $V(G)$ and $E(G)$ denote the graph's vertex set and edge set, respectively [5,8]. For a vertex $v \in V(G)$, we use $d(v)$ denote the degree of v in G. Chemical graph theory is expected to be associated with observable physical measurements in the experiment, and to some extent, theoretical predictions can be used to obtain chemical insights, even for molecules that do not exist yet. For laboratories in countries and regions with low budget, chemical graph theory is of great help in the determining of melting point and boiling point of compounds [5–7, 12].

In 1975, Randić [10] introduced a topological index, named as the Randić index, for measuring the degree of branching of the saturated hydrocarbon carbon atom's backbone. In 1998, Bollobás and Erdős [2] extended this index by defining as following:

$$R_k(G) = \sum_{uv \in E(G)} (d(u)d(v))^k. \tag{1.1}$$

Usually, $R_k(G)$ is called the *general Randić index*, whereas $k = -\frac{1}{2}$ corresponds to the Randić index.

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For two other special cases of the general Randić index, B. Manoochehrian, H. Yousefi-Azari and A. R. Ashrafi [9] discussed the first Zagreb index and the second Zagreb index, which can be applied to the calculation of π-electron energy of benzenoid hydrocarbons:

$$M_1(G) = \sum_{u \in V(G)} d(u)^2, \tag{1.2}$$

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v). \tag{1.3}$$

In 2007, B. Manoochehrian, etc. [9] defined the first and second Zagreb polynomial indices of the molecular graph:

$$M_1(G,x) = \sum_{uv \in E(G)} x^{d(u)+d(v)}, \tag{1.4}$$

and

$$M_2(G,x) = \sum_{uv \in E(G)} x^{d(u)d(v)}. \tag{1.5}$$

In 2009, the sum-connectivity index of G, a novel molecular descriptor derived by Zhou and Trinajstić [13], was defined, and then was generalized after a year as

$$\chi_k(G) = \sum_{uv \in E(G)} (d(u)+d(v))^k. \tag{1.6}$$

In 2011, the ordinary geometric-arithmetic index was defined as [3]:

$$OGA_k(G) = \sum_{uv \in E(G)} \left(\frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)}\right)^k. \tag{1.7}$$

The general harmonic index was investigated by Yan et al. [14] in 2015:

$$H_k(G) = \sum_{uv \in E(G)} \left(\frac{2}{d(u)+d(v)}\right)^k. \tag{1.8}$$

In the second section, we calculate indices above and show main results and proofs.

Hyaluronic Acid (HA) is both high molecular weight polymer of up to 20000 kDa and straight chain, glycosaminoglycan polymer of the extracellular matrix (ECM) composed of repeating units of the disaccharide [-D-glucuronic acid-$\beta 1$, 3-N-acetyl-D-glucosamine-$\beta 1$, 4-]n. The glycosidic linkages G to N and N to G are $1e$, $3e$ and $1e$, $4e$ separately [11]. Despite their simple structures, HA fragments have wide-ranging biological functions. There are HA polymers that are space-filling, anti-angiogenic, immunosuppressive, and that impede differentiation, possibly by suppressing cell interactions or ligand access to cell surface receptors [4].

Many pharmaceutical enterprises have made great efforts to fight against cardiovascular disease. For example, they have developed many drugs to treat coronary heart disease (CHD), such as captopril, atenolol, metoprolol, olmesartan and losartan. In recent years, some scholars discovered the promising potential of HA. The experiment designed by So Jeong Yoon et al. [15] showed that HA-based hydrogel will be of great help for treating myocardial infarction. In addition, Silvia Arpicco et al. [1] found that HA was also important in the tumorigenesis process, because HA receptors were overexpressed on tumor cells. This could be exploited in drug delivery, so as to increase the efficiency of anticancer drug.

Numerous studies had shown that there exists relationship between the chemical characteristics of drugs (such as boiling/melting points) and their molecular structures. Based on HA's molecule structure, we calculate some topological indices.

2. Main results and proofs

Let G_n denote a chemical graph formed by the molecular structure of Hyaluronic Acid, as shown in Fig. 1 and Fig. 2. We use δ and Δ to denote the minimum degree and maximum degree of G_n, respectively.

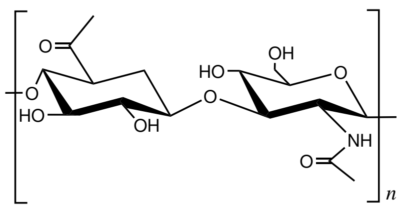

Figure 1. The molecular structure of Hyaluronic Acid.

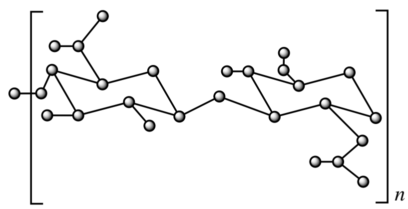

Figure 2. The chemical graph G_n of Hyaluronic Acid.

It is easy to observe that $|V(G_n)| = 26n+1$, $|E(G_n)| = 28n$, $\Delta = 3$, and $\delta \geq 1$. We define

$$E_i = \{uv \in E(G_n) \mid d(u)+d(v) = i\} \text{ for each } i \text{ with } 2\delta \leq i \leq 2\Delta,$$
$$E_j^* = \{uv \in E(G_n) \mid d(u)d(v) = j\} \text{ for each } j \text{ with } \delta^2 \leq j \leq \Delta^2.$$

Note that both E_i and E_j^* are nonempty. As discussed in [16], $E(G_n)$ can be partitioned into the union of the following subsets:

$$\begin{aligned}
E_3 &= \{uv \in E(G_n) \mid d(u)=1, d(v)=2\},\\
E_{4.1} &= \{uv \in E(G_n) \mid d(u)=1, d(v)=3\},\\
E_{4.2} &= \{uv \in E(G_n) \mid d(u)=2, d(v)=2\},\\
E_5 &= \{uv \in E(G_n) \mid d(u)=2, d(v)=3\},\\
E_6 &= \{uv \in E(G_n) \mid d(u)=3, d(v)=3\},\\
E_2^* &= \{uv \in E(G_n) \mid d(u)=1, d(v)=2\},\\
E_3^* &= \{uv \in E(G_n) \mid d(u)=1, d(v)=3\},\\
E_4^* &= \{uv \in E(G_n) \mid d(u)=2, d(v)=2\},\\
E_6^* &= \{uv \in E(G_n) \mid d(u)=2, d(v)=3\},\\
E_9^* &= \{uv \in E(G_n) \mid d(u)=3, d(v)=3\},
\end{aligned}$$

Theorem 2.1. *Let G_n be the chemical graph of Hyaluronic Acid. Then*

$$R_k(G_n) = (n+1)2^k + (7n)3^k + 4^k + (11n-1)6^k + (9n-1)9^k.$$

Proof. By the structure of G_n, it is easy to deduce that $|E_2^*| = n+1$, $|E_3^*| = 7n$, $|E_{4.2}^*| = 1$, $|E_6^*| = 11n-1$, and $|E_9^*| = 9n-1$. Set $\tau(uv) = d(u)d(v)$. Then, by the definition of the general Randić index, we get

$$\begin{aligned} R_k(G_n) &= \sum_{uv\in E(G_n)} (d(u)d(v))^k \\ &= \sum_{uv\in E_2^*} \tau(uv)^k + \sum_{uv\in E_3^*} \tau(uv)^k + \sum_{uv\in E_{4.2}^*} \tau(uv)^k + \sum_{uv\in E_6^*} \tau(uv)^k + \sum_{uv\in E_9^*} \tau(uv)^k \\ &= (n+1)2^k + (7n)3^k + 4^k + (11n-1)6^k + (9n-1)9^k. \end{aligned}$$

□

Theorem 2.2. *Let G_n be the chemical graph of Hyaluronic Acid. Then*

(1) $M_1(G_n, x) = (n+1)x^3 + (7n+1)x^4 + (11n-1)x^5 + (9n-1)x^6$.

(2) $M_2(G_n, x) = (n+1)x^2 + (7n)x^3 + x^4 + (11n-1)x^6 + (9n-1)x^9$.

Proof. Noticing that $|E_3| = n+1$, $|E_{4.1}| = 7n$, $|E_{4.2}| = 1$, $|E_5| = 11n-1$, and $|E_6| = 9n-1$, and according to the definition of the first and second Zagreb polynomial indices, we derive the following:

$$\begin{aligned} M_1(G_n, x) &= \sum_{uv\in E(G_n)} x^{d(u)+d(v)} \\ &= \sum_{uv\in E_3} x^3 + \sum_{uv\in E_4} x^4 + \sum_{uv\in E_5} x^5 + \sum_{uv\in E_6} x^6 \\ &= (n+1)x^3 + (7n+1)x^4 + (11n-1)x^5 + (9n-1)x^6. \end{aligned}$$

$$\begin{aligned} M_2(G_n, x) &= \sum_{uv\in E(G_n)} x^{d(u)d(v)} \\ &= \sum_{uv\in E_2^*} x^2 + \sum_{uv\in E_3^*} x^3 + \sum_{uv\in E_4^*} x^4 + \sum_{uv\in E_6^*} x^6 + \sum_{uv\in E_9^*} x^9 \\ &= (n+1)x^2 + (7n)x^3 + x^4 + (11n-1)x^6 + (9n-1)x^9. \end{aligned}$$

□

Theorem 2.3. *Let G_n be the chemical graph of Hyaluronic Acid. Then*

$$\chi_k(G_n) = (n+1)3^k + (7n+1)4^k + (11n-1)5^k + (9n-1)6^k.$$

Proof. By the definition of the first Zagreb polynomial index of G_n, we get easily the following:

$$\begin{aligned} \chi_k(G_n) &= \sum_{uv\in E(G_n)} (d(u)+d(u))^k \\ &= \sum_{uv\in E_3} 3^k + \sum_{uv\in E_4} 4^k + \sum_{uv\in E_5} 5^k + \sum_{uv\in E_6} 6^k \\ &= (n+1)3^k + (7n+1)4^k + (11n-1)5^k + (9n-1)6^k. \end{aligned}$$

□

Theorem 2.4. *Let G_n be the chemical graph of Hyaluronic Acid. Then*

$$OGA_k(G_n) = (n+1)(\frac{2\sqrt{2}}{3})^k + 7n(\frac{\sqrt{3}}{2})^k + (11n-1)(\frac{2\sqrt{6}}{5})^k + 9n.$$

Proof. By the definition of the ordinary geometric-arithmetic index, we obtain the following:

$$\begin{aligned} OGA_k(G_n) &= \sum_{uv\in E(G_n)} (\frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)})^k \\ &= \sum_{uv\in E_2^*} (\frac{2\sqrt{2}}{3})^k + \sum_{uv\in E_3^*} (\frac{\sqrt{3}}{2})^k + \sum_{uv\in E_4^*} 1^k + \sum_{uv\in E_6^*} (\frac{2\sqrt{6}}{5})^k + \sum_{uv\in E_9^*} 1^k \\ &= (n+1)(\frac{2\sqrt{2}}{3})^k + 7n(\frac{\sqrt{3}}{2})^k + 1^k + (11n-1)(\frac{2\sqrt{6}}{5})^k + (9n-1) \\ &= (n+1)(\frac{2\sqrt{2}}{3})^k + 7n(\frac{\sqrt{3}}{2})^k + (11n-1)(\frac{2\sqrt{6}}{5})^k + 9n. \end{aligned}$$

□

Theorem 2.5. *Let G_n be the chemical graph of Hyaluronic Acid. Then*

$$H_k(G_n) = (n+1)(\frac{2}{3})^k + (7n+1)(\frac{1}{2})^k + (11n-1)(\frac{2}{5})^k + (9n-1)(\frac{1}{3})^k.$$

Proof. The definition of the general harmonic index gives the following:

$$\begin{aligned} H_k(G_n) &= \sum_{uv\in E(G_n)} (\frac{2}{d(u)+d(v)})^k \\ &= \sum_{uv\in E_3} (\frac{2}{3})^k + \sum_{uv\in E_4} (\frac{1}{2})^k + \sum_{uv\in E_5} (\frac{2}{5})^k + \sum_{uv\in E_6} (\frac{1}{3})^k \\ &= (n+1)(\frac{2}{3})^k + (7n+1)(\frac{1}{2})^k + (11n-1)(\frac{2}{5})^k + (9n-1)(\frac{1}{3})^k. \end{aligned}$$

□

Hyaluronic acid (HA) is a naturally occurring glycosaminoglycan that is available in formulations of various molecular weight and concentration. By the calculation above,we hope it can provide a reference for further studying of HA.

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