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CAUCHY PROBLEM FOR THE GENERALIZED DAVEY-STEWARTSON SYSTEMS IN BESOV SPACES AND SOME COUNTEREXAMPLES

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Abstract In this paper, the Cauchy problem of the generalized ellipse-ellipse type Davey-Stewartson systems is discussed. When the dimension of space is greater than or equal to two, we get a unique global solution in Besov spaces by contraction mapping argument. Moreover, by using the F-expansion method, the exact periodic wave solutions for the generalized ellipse-ellipse type Davey-Stewartson systems are discussed, some counter examples are given.

Keywords Davey-Stewartson systems, F-expansion method, multi-order exact solutions, Lam function, Cauchy problem.

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1. Introduction

A large amount of work are devoted to the study of Davey-Stewartson systems. The classical Davey-Stewartson systems

$$\begin{cases} iu_t + D_1 u = r\left|u\right|^2 u + \mu uv, \\ D_2 v = D_3(|u|^2), \end{cases} \tag{1.1}$$

are originally derived by Davey and Stewartson in [4] to describe quasi-monochromatic wave pockets on the surface of a shallow liquid. Here D_1, D_2 and D_3 are partial differential operators of the second order in x, y $(x, y) \in R^2$. $D_1 = \delta\partial_x^2 + \partial_y^2$ is either elliptic or hyperbolic, and $D_2 = \partial_x^2 + m\partial_y^2 (m > 0)$ is elliptic. Later, the case that D_2 is hyperbolic, i.e. $m < 0$, is derived in [5] by taking account of the effect of surface tension (or capillary). Generally, $D_3 = \partial_x^2 u$ is a complex-valued function of $(t; x, y) \in R_+ \times R^2$, and v is a real-valued function of $(t; x, y) \in R_+ \times R^2$. And u and v are related to the amplitude and the mean velocity potential of the water wave, respectively.

As in [7], the Davey-Stewartson equations are usually classified as elliptic-elliptic, elliptic-hyperbolic, hyperbolic-elliptic and hyperbolic-hyperbolic types according to the respective types of D_1 and D_2. In recent years, Davey-Stewartson equations(1.1) have drawn much attention from many physicists and mathematicians due

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to the abundant physical and mathematical properties. In [3], by using the generalized Kudryashov method, Demiray and Bulut found the dark soliton solutions of DSE systems. Later, by employing the Bäcklund transformation, the Hamiltonian approach and the G-expansion method to the DSEs, new traveling solitary and kink wave solutions are obtained, see [23]. The well-posedness, decay of solutions, soliton solutions, solitary and standing waves, etc., have been quite extensively studied by many authors (see [7, 9, 14, 15, 17, 25, 26]).

In [10], Wang and Guo study the generalized Davey-Stewartson systems

$$\begin{cases} iu_t + Au = \lambda_1 |u|^{p_1} u + \lambda_2 |u|^{p_2} u + \mu u v_{x_1}, \\ Bv = (|u|^2)_{x_1}, \end{cases} \tag{1.2}$$

where $A := \sum_{1 \le i,j \le n} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j}$, $B := \sum_{1 \le i,j \le n} b_{ij} \frac{\partial^2}{\partial x_i \partial x_j}$, (a_{ij}) and (b_{ij}) are real invertible matrix. They discuss the initial value problem of the Davey-Stewartson systems for the elliptic-elliptic and hyperbolic-elliptic cases. The local and global existence, as well as uniqueness of solutions in H^s are shown. Moreover, they prove that the scattering operator carries a band in H^s into H^s. By using the Hirota's bilinear method, the homoclinic orbits of the Davey-Stewartson Equations are obtained through the dependent variable transformation, see [30].

In [27], Wang shows that there exists $s_c > 0$ such that the cubic (quartic) non-elliptic derivative Schrödinger equations with small data in modulation spaces $M^s_{2,1}(\mathrm{R}^{\mathrm{n}})$ for $n \ge 3 (n = 2)$ are globally well-posed if $s \ge s_c$, and ill-posed if $s < s_c$.

It should be pointed that there is little result about the existence of solutions if the second equation of (1.2) is replaced by the following equations

$$Bv = (|u|^q u)_{x_1}, \quad q \ne 2.$$

Furthermore, if $\lambda_1 |u|^{p_1} u + \lambda_2 |u|^{p_2} u$ in the first equation of (1.2) is replaced by a more general nonlinear function $f(u)$, the existence of solutions is also little known. And some physical models, such as Landau-Lifshitz equation, Navier Stokes equation, can be transformed into the equation with more general nonlinear function $f(u)$. So it is more meaningful and more difficult to study the Davey-Stewartson systems with general nonlinear term $f(u)$. System (1.1) and (1.2) are two important cases of general nonlinear Davey-Stewartson equation. Although our study (2.1) is a special case of general nonlinear Davey-Stewartequation, it includes (1.1) and (1.2).

The purpose of this paper is to investigate the Cauchy problem of the two cases mentioned above, i.e., the generalized ellipse-ellipse and hyperbolic-ellipse type. When the dimension of space is greater than or equal to two, we get a unique global solution in Besov spaces by contraction mapping argument (see Section 2).

Naturally, the reader will curious about whether these results are valid for bounded domain? To answer those interesting questions, we construct some exact periodic wave solutions for the generalized ellipse-ellipse type Davey-Stewartson systems by using the F-expansion method, and some counter examples are given (see Section 3). Furthermore, we construct some exact periodic wave solutions to show the existence of solutions for generalized ellipse-hyperbolic and hyperbolic-hyperbolic type Davey-Stewartson systems which is still open (see Section 4).

This paper is based on many kinds of methods which are raised, such as homogeneous balance method [28], hyperbolic function expansion method [6], nonlinear transformation method [11,13], trial function method [19], sine-cosine method [29], Jacobi elliptic function expansion method, etc. The solution gotten by these methods are mainly solitary wave solutions, shock solutions, see [6,11,13,19,28,29] and elliptic function, see [20–22]. To search the stability of the solutions and inspired by [18], we add perturbation in our research and discuss the evolution of the perturbation. Essentially, it is to expand the solution of nonlinear evolution equation to ε- power series and try to get the multi-order exact solutions of it. The symmetry group properties of the variable coefficient Davey-Stewartson (vcDS) systems are studied in [8]. The dromion of the Davey-Stewartson-1 equation is studied under perturbation on the large time [12]. Through the Hirota bilinear method, Ma formulate a combined fourth-order nonlinear equation while guaranteeing the existence of lump solutions of new (2+1)-dimensional nonlinear equations [16].

2. Cauchy problem

In this section we study the Cauchy problem for the generalized Davey-Stewartson systems

$$\begin{cases} iu_t + Au = f(u) + \mu u v_{x_1}, \\ Bv = (|u|^q u)_{x_1} \\ u(0,x) = u_0(x), \end{cases} \tag{2.1}$$

where $A := \sum\limits_{1\le i,j\le n} a_{ij}\frac{\partial^2}{\partial x_i \partial x_j}$, $B := \sum\limits_{1\le i,j\le n} b_{ij}\frac{\partial^2}{\partial x_i \partial x_j}$, (a_{ij}) and (b_{ij}) are real invertible matrices. $\lambda, \mu \in C$. Let exist $C > 0$, satisfied

$$\left| \sum_{1\le i,j\le n} b_{ij}\xi_i\xi_j \right| \ge C\,|\xi|^2, \quad \forall \xi \in R^n. \tag{2.2}$$

Denote $E(\psi) = F^{-1}\left[\frac{\xi_1^2}{\sum\limits_{1\le i,j\le n} b_{ij}\xi_i\xi_j}\right] F\psi$, where F is Fourier transformation. Then (2.1) is equivalent to the following form

$$\begin{cases} iu_t + Au = f(u) + \mu E(|u|^q u)u, \\ u(0,x) = u_0(x). \end{cases} \tag{2.3}$$

Here $f(u) \in C^k (k \in \mathbb{Z})$, is a nonlinear function and for any given p,

$$|f^{(k)}(u)| \le C|u|^{p+1-k}, \qquad k = 0, 1, \cdots. \tag{2.4}$$

For brevity, in the following, we denote

$$s(p) = \frac{n}{2} - \frac{2}{p}, \quad \frac{2}{\gamma(r)} = n(\frac{1}{2} - \frac{1}{r}), \quad r(p) = \frac{2n(2+p)}{n(2+p)-4}, \tag{2.5}$$

where $\frac{4}{n} \le p < \infty, \quad 1 \le r < \infty$,

$$\alpha(n) = \begin{cases} \infty, & n = 2, \\ \frac{2n}{n-2}, & n > 2. \end{cases} \tag{2.6}$$

Now, we state the main result as follows.

Theorem 2.1. *Let $n \geq 2$, $\frac{4}{n} \leq p, q < \infty$, $\max(s(q), s(p)) \leq s < \infty$, $[s] \leq p$. If $u_0 \in H^s$ and there exist a $\delta > 0$ such that $\|u_0\|_{H^s} < \delta$, then there exists a unique solution u of the Cauchy problem(2.3) satisfying*

$$u \in C(0, \infty; H^s) \cap L^{p+2}(0, \infty; B^s_{r(p),2}) \cap L^{q+3}(0, \infty; B^s_{r(q),2}). \tag{2.7}$$

Throughout this paper, we use a variety of function spaces, Lebesgue spaces L^r, Bessel potential spaces $H^{s,r}$, Besov spaces $B^s_{r,2}$. The definition of L^r and $H^{s,r}$ is as usual, and an equivalent definition of the norm on $\dot{B}^s_{r,2}$ is that

$$\|u\|_{\dot{B}^s_{r,2}} = \left(\int_0^\infty t^{2(s-[s])} \sum_{|\alpha|=[s]} \sup \|\Delta_h D^\alpha u\|^2_{L^r} \frac{dt}{t} \right)^{\frac{1}{2}}, \tag{2.8}$$

where [s] denotes the largest integer less than or equal to s, $\Delta_h u(x) = u(x+h) - u(x) = u_h - u$. For some additional basic results on Besov spaces, one can refer to [1, 24].

In the following, C stands for a constant that may be different numbers in different places. For any $r \in [1, \infty]$, r' denotes the duality number of r, i.e. $\frac{1}{r} + \frac{1}{r'} = 1$.

2.1. Main lemmas

The main tools used here are time spaces $L^p - L^{p'}$ estimates for solutions of linear Schrödinger equations in Lebesgue space. These estimates are usually named generalized Strichartz inequalities. The method of the proof of the main result is a contraction mapping argument. Let us recall that some estimates for linear Schrödinger equations in Lebesgue-Besov spaces which are established by Cazenave and Weissler in [2].

Lemma 2.1. *For all $s \in R$, $r, q \in [2, \alpha(n))$. $S(t)$ is semi-group of operator and $i\partial/\partial t + A$ is generating operator of $S(t)$, then we have*

(i) If $u_0 \in \dot{H}^s$, then $S(t)u_0 \in L^{\gamma(r)}(0, \infty; \dot{B}^s_{r,2})$, and there exists a constant $C > 0$, such that

$$\|S(t)u_0\|_{L^{\gamma(r)}(0,\infty;\dot{B}^s_{r,2})} \leq C \|u_0\|_{\dot{H}^s}; \tag{2.9}$$

(ii) If $f \in L^{\gamma(r)'}(0, \infty; \dot{B}^s_{r',2})$, then $\int_0^t S(t-\tau)f(\tau)d\tau \in L^{\gamma(q)}(0, \infty; \dot{B}^s_{q,2})$, and there exists $C > 0$, such that

$$\left\| \int_0^t S(t-\tau)f(\tau)d\tau \right\|_{L^{\gamma(q)}(0,\infty;\dot{B}^s_{q,2})} \leq C \|f\|_{L^{\gamma(r)'}(0,\infty;\dot{B}^s_{r',2})}, \tag{2.10}$$

for all $f \in L^{\gamma(r)'}(0, \infty; \dot{B}^s_{r',2})$.

The proof of this lemma can be found in [10].

Lemma 2.2 ([10]). *Let $0 \leq s < \infty$, $0 \leq r' < \infty$, $l_k, m_k, p_k, q_k > 0$ and*

$$\frac{1}{r'} = \frac{1}{l_k} + \frac{1}{m_k} = \frac{1}{p_k} + \frac{1}{q_k}, k = 0, 1, \cdots, [s].$$

Then, there exists a constant $C>0$ only depending on r', n, s, such that

$$\|uv\|_{\dot{B}^s_{r',2}} \le C\sum\nolimits_{k=0}^{[s]} (\|u\|_{\dot{H}^{k,p_k}} \|v\|_{\dot{B}^{s-k}_{q_k,2}} + \|u\|_{\dot{B}^{s-k}_{l_k,2}} \|v\|_{\dot{H}^{k,m_k}}). \tag{2.11}$$

Lemma 2.3 ([10]). *Let $-\infty < \sigma < \infty$, $1 < r$, $\mu < \infty$. Then there exists a constant $C>0$ such that, for all $u \in \dot{B}^{\sigma}_{r,\mu}$,*

$$\|E(u)\|_{\dot{B}^{\sigma}_{r,\mu}} \le C\,\|u\|_{\dot{B}^{\sigma}_{r,\mu}}. \tag{2.12}$$

By a similar method in [9], one can have the following result.

Lemma 2.4. *Let $s(p) \le s < \dfrac{n}{2}$ and $\rho = \dfrac{2n(p+2)}{n(p+2)-2}$. If $f \in C^{[s]+1}(R,R)$ satisfying one of the following conditions:*

(i) $|f^{(k)}(u)| \le C|u|^{p+1-k}$, where $k = 0, 1, ..., [s]+1, [s] < p+1$;

(ii) $|f^{(k)}(u)| \le C|u|^{p+1-k}$, when $k < p+1$; $f^{(k)}(u) = 0$, when $k < p+1$.

Then

$$\|f(u)\|_{\dot{B}^s_{\rho',2}} \le C\|u\|^r_{\dot{B}^{s-s(p)}_{\rho}} \|u\|_{\dot{B}^{[s]}_{\rho}}. \tag{2.13}$$

Proof. The proof can be divided into the following steps.

Step 1 First, consider the case $[s-s(p)] \ge 1$, one has

$$\|f(u)\|_{\dot{B}^s_{\rho'}} = (\int_0^{\infty} t^{-2(s-[s])} \sup_{|h|\le t} \sum_{|\alpha|=[s]} \|\triangle_h D^{\alpha} u\|^2_{L^{\rho'}} \frac{dt}{t})^{\frac{1}{2}}, \tag{2.14}$$

and recalling that $|f^{(q)}(u)| \le C|u|^{p+1-q}$ to obtain

$$|f^{(q)}(u) - f^{(q)}(v)| \le (|u|^{p-q} + |v|^{p-q})|u-v|.$$

Notice that $[s] \ge 1$ and (2.11) to get

$$\begin{aligned} &\sum_{|\alpha|=[s]} \|\triangle_h D^{\alpha} f(u)\|_{L_{\rho'}} \\ &\le C\sum_{q=1}^{[s]} \sum_{\Lambda^q_{[s]}} \|(|u_h|^{p-q} + |u|^{p-q})|u_h - u| \prod_{i=1}^{q} D^{\alpha_i} u\|_{L^{\rho'}} \\ &\quad + C\sum_{q=1}^{[s]} \sum_{\Lambda^q_{[s]}} \sum_{i=1}^{q} \||u_h|^{p-q+1} \prod_{j=1}^{i-1} D^{\alpha_j} u_h \prod_{j=i+1}^{q} D^{\alpha_j} u D^{\alpha_i}(u_h - u)\|_{L^{\rho'}}. \end{aligned}$$

Let

$$\Gamma_1 := \|(|u_h|^{p-q} + |u|^{p-q})|u_h - u| \prod_{i=1}^{q} D^{\alpha_i} u\|_{L^{\rho'}}, \tag{2.15}$$

$$\Gamma_2 := \sum_{i=1}^{q} \||u_h|^{p-q+1} \prod_{j=1}^{i-1} D^{\alpha_j} u_h \prod_{j=i+1}^{q} D^{\alpha_j} u D^{\alpha_i}(u_h - u)\|_{L^{\rho'}}, \tag{2.16}$$

thus

$$\sum_{|\alpha|=[s]} \|\triangle_h D^\alpha f(u)\|_{L_{\rho'}} \le C \sum_{q=1}^{[s]} \sum_{\Lambda^q_{[s]}} (\Gamma_1 + \Gamma_2). \tag{2.17}$$

Next, we estimate Γ_1 and Γ_2, respectively. Without lose of generality, it can be considered $\Lambda^q_{[s]}$ $|\alpha_q| \ge |\alpha_{q-1}| \ge \cdots \ge |\alpha_2| \ge |\alpha_1|$. Firstly, when $q = 1$, let

$$\begin{aligned} a_0 &= (p-1)(\frac{1}{\rho} - \frac{s-s(p)}{n}), \\ a_1 &= \frac{1}{\rho} - \frac{s-s(p)}{n}, \\ a_2 &= \frac{1}{\rho}. \end{aligned}$$

It is easy to see $a_0, a_1, a_2 > 0$, and

$$a_0 + a_1 + a_2 = p(\frac{1}{\rho} - \frac{s-s(p)}{n}) + \frac{1}{\rho} = \frac{1}{\rho'}. \tag{2.18}$$

By using $\dot{B}^{s-s(p)}_\rho \hookrightarrow \dot{H}^{s-s(p)}_\rho$, (2.15), and Hölder inequality, one gets

$$\begin{aligned} \Gamma_1 &\le C\|u\|^{p-1}_{\dot{H}^{s-s(p)}_\rho} \|u_h - u\|_{L^{\frac{1}{a'_0}}} \|u\|_{L^\rho} \\ &\le C\|u\|^{p-1}_{\dot{B}^{s-s(p)}_\rho} \|u_h - u\|_{L^{\frac{1}{a'_0}}} \|u\|_{L^\rho}. \end{aligned} \tag{2.19}$$

Since $[s - s(p)] \le [s]$, then

$$\begin{aligned} \Gamma_1 &\le C\|u\|^{p-1}_{\dot{B}^{s-s(p)}_\rho} \|u\|_{\dot{B}^{[s]}_\rho} \|u\|_{B^{[s]}_\rho} \\ &\le C\|u\|^{p-1}_{\dot{B}^{s-s(p)}_\rho} \|u\|_{\dot{B}^{s-s(p)}_\rho} \|u\|_{B^{[s]}_\rho} \\ &\le C\|u\|^{p}_{\dot{B}^{s-s(p)}_\rho} \|u\|_{B^{[s]}_\rho}. \end{aligned} \tag{2.20}$$

Second, consider the case $q \ge 2$, by (2.15), and let

$$\begin{aligned} a_0 &= (p-q)(\frac{1}{\rho} - \frac{s-s(p)}{n}), \\ a'_0 &= \frac{1}{\rho} - \frac{s-s(p)}{n}, \\ a_i &= \frac{1}{\rho} - \frac{s-s(p)-|a_i|}{n}, \quad i = 1, 2, \cdots, q-1, \\ a_q &= \frac{1}{\rho} - \frac{[s]-|a_q|}{n}. \end{aligned}$$

Clearly $a_0, a'_0, a_i > 0$, $(i = 1, 2, \cdots, q)$ and

$$a_0 + a'_0 + \sum_{i=1}^{q} a_i = p(\frac{1}{\rho} - \frac{s-s(p)}{n}) + \frac{1}{\rho} = \frac{1}{\rho'}. \tag{2.21}$$

By Hölder inequality, (2.15) yields

$$\Gamma_1 \le C\|u\|^{p-q}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{B}^{[s]}_\rho}\prod_{i=1}^{q-1}\|D^{\alpha_i}u\|_{\dot{H}^{s-s(p)}_\rho}\|u\|_{\dot{H}^{[s]}_\rho}. \tag{2.22}$$

Since $\dot{B}^{s-s(p)}_\rho \hookrightarrow \dot{H}^{s-s(p)}_\rho$, then (2.22) implies

$$\Gamma_1 \le C\|u\|^{p}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{B}^{[s]}_\rho}. \tag{2.23}$$

Let's consider $\Gamma_2 := \sum_{i=1}^{q}\|\prod_{j=1}^{i-1}D^{\alpha_j}u_h\prod_{i+1}^{q}D^{\alpha_j}uD^{\alpha_i}(u_h-u)|u_h|^{p-q+1}\|_{L^{\rho'}}$. There are also two scenarios to consider. First of all, when $q=1$,

$$\Gamma_2 = \|D^{\alpha_1}(u_h-u)|u_h|^p\|_{L^{\rho'}}. \tag{2.24}$$

Let

$$a_0 = p(\frac{1}{\rho}-\frac{s-s(p)}{n}),$$
$$a_1 = \frac{1}{\rho}-\frac{|\alpha_1|-|\alpha_1|)}{n},$$

then $a_0, a_1 > 0$, and $a_0+a_1 = \frac{1}{\rho'}$. and by Hölder inequality, one gets

$$\Gamma_2 \le C\|u\|^{p}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{H}^{|\alpha_1|}_\rho}. \tag{2.25}$$

Notice that $q=1$, $|\alpha_1| = [s]$, thus

$$\Gamma_2 \le C\|u\|^{p}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{B}^{[s]}_\rho}. \tag{2.26}$$

Second, let's consider the case $q \ge 2$,

$$a_0 = (p-q+1)(\frac{1}{\rho}-\frac{s-s(p)}{n}),$$
$$a_i = \frac{1}{\rho}-\frac{s-s(p)-|a_i|}{n}, \quad i=1,2,\cdots,q-1,$$
$$a_q = \frac{1}{\rho}-\frac{[s]-|a_q|}{n},$$

thus, $a_0, a_i > 0$, $(i=1,2,\cdots,q)$

$$a_0+\sum_{i=1}^{q}a_i = p(\frac{1}{\rho}-\frac{s-s(p)}{n})+\frac{1}{\rho} = \frac{1}{\rho'}. \tag{2.27}$$

It follows from Hölder inequality,

$$\begin{aligned}\Gamma_2 &\le C\|u\|^{p-q+1}_{\dot{B}^{s-s(p)}_\rho}\|u\|^{q-1}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{B}^{[s]}_\rho}\\ &\le C\|u\|^{p}_{\dot{B}^{s-s(p)}_\rho}\|u\|_{\dot{B}^{[s]}_\rho},\end{aligned} \tag{2.28}$$

which combination (2.20), (2.23), (2.26) and (2.28) yields

$$\|f(u)\|_{\dot{B}_{\rho}^{s-s(p)}} \le C\|u\|^{p}_{\dot{B}_{\rho}^{s-s(p)}}\|u\|_{\dot{B}_{\rho}^{[s]}}. \tag{2.29}$$

Step 2 Let's consider the case $[s-s(p)]=0$,

$$\|f(u)\|_{\dot{B}_{\rho'}^{s-s(p)}} = (\int_0^\infty t^{-2(s-s(p))}\sup_{|h|\le t}\|\triangle_h f(u)\|^2_{L^{\rho'}}\frac{dt}{t})^{\frac{1}{2}}. \tag{2.30}$$

Since $\dot{B}_{\rho'}^{s-s(p)} \hookrightarrow \dot{H}^{s}_{\rho\prime}$ and $|f(u)|\le |u|^{p+1}$, one has

$$p(\frac{1}{\rho}-\frac{s-s(p)}{n})+\frac{1}{\rho}-\frac{s-s(p)}{n}=\frac{1}{\rho'}-\frac{s-s(p)}{n}. \tag{2.31}$$

Let

$$\frac{1}{\beta}=\frac{1}{\rho\prime}-\frac{s-s(p)}{n},$$

and utilizing Hölder inequality to get

$$\begin{aligned}\|f(u)\|_{\dot{B}_{\rho}^{s-s(p)}} &= \||u|^{p+1}\|_{\dot{B}_{\rho}^{s-s(p)}}\\ &\le C\|u\|^{p}_{\dot{B}_{\rho}^{s-s(p)}}\|u\|_{\dot{B}_{\rho}^{s-s(p)}}\\ &\le C\|u\|^{p}_{\dot{B}_{\rho}^{s-s(p)}}\|u\|_{\dot{B}_{\rho}^{[s]}}.\end{aligned} \tag{2.32}$$

From (2.29) and (2.32), it can be seen (2.13) holds which is

$$\|f(u)\|_{\dot{B}^{s}_{\rho',2}} \le C\|u\|^{r}_{\dot{B}_{\rho}^{s-s(p)}}\|u\|_{\dot{B}_{\rho}^{[s]}}.$$

Thus the proof of this lemma is completed. □

Lemma 2.5. *Let $n\ge 2$, $\frac{4}{n}\le q<\infty$, $[s]\le q$, $0\le s<\infty$, $r=r(q)$, then we have*

$$\|E(|u|^q\, u)u\|_{\dot{B}^{s}_{r',2}} \le C\,\|u\|^{q}_{\dot{B}^{s(q)}_{r,2}}\,\|u\|^{2}_{\dot{B}^{s}_{r,2}}. \tag{2.33}$$

Proof. From Lemma 2.2, it is easy to see that

$$\begin{aligned}&\|E(|u|^q\, u)u\|_{\dot{B}^{s}_{r',2}}\\ \le& C\sum_{k=0}^{[s]}\left[\|u\|_{\dot{H}^{k,p_k}}\|E(|u|^q\, u)\|_{\dot{B}^{s-k}_{q_k,2}}+\|u\|_{\dot{B}^{s-k}_{l_k,2}}\|E(|u|^q\, u)\|_{\dot{H}^{k,m_k}}\right]\\ \le& C\sum_{k=0}^{s}[I+II],\end{aligned} \tag{2.34}$$

where $\frac{1}{r'}=\frac{1}{l_k}+\frac{1}{m_k}=\frac{1}{p_k}+\frac{1}{q_k}, k=0,1,\cdots,[s]$.

In the following, we estimate I and II, firstly,

$$\dot{H}^{k,p_k}\supset \dot{B}^{s}_{r,2},$$

and

$$\|u\|_{\dot{H}^{k,p_k}} \le C\,\|u\|_{\dot{B}^{s}_{r,2}}. \tag{2.35}$$

Setting $\frac{1}{p_k} = 1 - \frac{1}{r} - \frac{k}{n}$, $\frac{1}{q_k} = \frac{k}{n}$, then

$$\|E(|u|^q u)\|_{\dot{B}^{s-k}_{q_k,2}} \leq C \|E(|u|^q u)\|_{\dot{B}^s_{r',2}}. \tag{2.36}$$

From Lemma 2.3 and Lemma 2.4, one has

$$\|E(|u|^q u)\|_{\dot{B}^s_{r',2}} \leq C \||u|^q u\|_{\dot{B}^s_{r',2}} \leq C \|u\|^q_{\dot{B}^{s(q)}_{r,2}} \|u\|_{\dot{B}^s_{r,2}}. \tag{2.37}$$

Thus,

$$I \leq C \|u\|^q_{\dot{B}^{s(q)}_{r,2}} \|u\|^2_{\dot{B}^s_{r,2}}. \tag{2.38}$$

Similarly,

$$II \leq C \|u\|^q_{\dot{B}^{s(q)}_{r,2}} \|u\|^2_{\dot{B}^s_{r,2}}. \tag{2.39}$$

Then, one gets

$$\|E(|u|^q u)u\|_{\dot{B}^s_{r',2}} \leq C \|u\|^q_{\dot{B}^{s(q)}_{r,2}} \|u\|^2_{\dot{B}^s_{r,2}}, \tag{2.40}$$

and the proof of this lemma is completed. □

2.2. The proof of Theorem 2.1

The Cauchy problem of (2.1) is essentially equivalent to the following integral equation

$$u(t) = S(t)u_0 - i\int_0^t S(t-\tau)F(u(\tau))d\tau, \tag{2.41}$$

where $F(u) = f(u) + \mu E(|u|^q u)u$.

For all $\delta > 0$, define

$$\begin{aligned} D = \{u \in L^{p+2}(0,\infty; B^s_{r(p),2}) \cap L^{q+3}(0,\infty; B^s_{r(q),2}) \\ : \|u\|_{L^{p+2}(0,\infty;B^s_{r(p),2}) \cap L^{q+3}(0,\infty;B^s_{r(q),2})} \leq \delta\}. \end{aligned} \tag{2.42}$$

And for any $u, v \in D$, the metric $d(u,v)$ is

$$d(u,v) = \|u - v\|_{L^{p+2}(0,\infty;L^{r(p)}) \cap L^{q+3}(0,\infty;L^{r(q)})}. \tag{2.43}$$

Considering the mapping

$$J : u(t) \to S(t)u_0 - i\int_0^t S(t-\tau)F(u(\tau))d\tau, \tag{2.44}$$

and we claim that $J : (D,d) \to (D,d)$ is a contraction mapping. To show this claim, in view of Lemma 2.4 and Lemma 2.5 to get

$$\|f(u)\|_{B^s_{r(p)',2}} \leq C \|u\|^{p+1}_{B^s_{r(p),2}}, \tag{2.45}$$

and

$$\|E(|u|^q u)u\|_{B^s_{r',2}} \leq C \|u\|^{q+2}_{B^s_{r(q),2}}. \tag{2.46}$$

From $\frac{1}{(p+2)'} = \frac{p+1}{p+2}$ and $\frac{1}{(q+3)'} = \frac{q+2}{q+3}$, one gets

$$\|f(u)\|_{L^{(p+2)'}(0,\infty;B^s_{r(p)',2})} \le C\,\|u\|^{p+1}_{L^{(p+2)}(0,\infty;B^s_{r(p),2})}, \tag{2.47}$$

and

$$\|E(|u|^q\,u)u\|_{L^{(q+3)'}(0,\infty;B^s_{r(q)',2})} \le C\,\|u\|^{q+2}_{L^{(q+3)}(0,\infty;B^s_{r(q),2})}. \tag{2.48}$$

So, for any $u \in D$,

$$\begin{aligned}
\|Ju\|_L &\le \|S(t)u_0\|_L + \left\| i\int_0^t S(t-\tau)F(u(\tau))d\tau \right\|_L \\
&\le C\|u_0\|_{H^s} + C\Big(\|f(u)\|_{L^{(p+2)'}(0,\infty;B^s_{r(p)',2})} + \|E(|u|^q\,u)u\|_{L^{(q+3)'}(0,\infty;B^s_{r(q)',2})}\Big) \\
&\le C\,\|u_0\|_{H^s} + C(\|u\|^{p+1}_{L^{(p+2)}(0,\infty;B^s_{r(p),2})} + \|u\|^{q+2}_{L^{(q+3)}(0,\infty;B^s_{r(q),2})}) \\
&\le C\,\|u_0\|_{H^s} + 2C(\delta^{p+1} + \delta^{q+2}) \\
&\le \delta,
\end{aligned}$$

where

$$L := L^{p+2}(0,\infty;B^s_{r(p),2}) \cap L^{q+3}(0,\infty;B^s_{r(q),2}). \tag{2.49}$$

Now, one can get $J : D \to D$. Further, for any $u, v \in D$,

$$\begin{aligned}
&d(Ju, Jv) \\
&= \|Ju - Jv\|_{L^{p+2}(0,\infty;L^{r(p)})\cap L^{q+3}(0,\infty;L^{r(q)})} \\
&= \left\| \int_0^t S(t-\tau)(F(u(\tau)) - F(v(\tau)))d\tau \right\|_{L^{p+2}(0,\infty;L^{r(p)})\cap L^{q+3}(0,\infty;L^{r(q)})} \\
&\le C\,\|f(u) - f(v)\|_{L^{(p+2)'}(0,\infty;L^{r(p)})} + C\,\|E(|u|^q\,u)u - E(|v|^q\,v)v\|_{L^{(q+3)'}(0,\infty;L^{r(q)})} \\
&\le C\,\||u-v|\,(|u|^p + |v|^p)\|_{L^{(p+2)'}(0,\infty;L^{r(p)})} \\
&\quad + C\,\||u-v|\,(E(|u|^q\,u) + E(|v|^q\,v))\|_{L^{(q+3)'}(0,\infty;L^{r(q)})} \\
&\le C\,\|u-v\|_{L^{p+2}(0,\infty;L^{r(p)})\cap L^{q+3}(0,\infty;L^{r(q)})}\,(\|u\|^p_{L^{(p+2)}(0,\infty;L^{r(p)})} + \|v\|^p_{L^{(p+2)}(0,\infty;L^{r(p)})} \\
&\quad + \|u\|^{q+1}_{L^{(q+3)}(0,\infty;L^{r(q)})} + \|v\|^{q+1}_{L^{(q+3)}(0,\infty;L^{r(q)})}) \\
&\le C\,\|u-v\|_{L^{p+2}(0,\infty;L^{r(p)})\cap L^{q+3}(0,\infty;L^{r(q)})}\,(\delta^p + \delta^{q+1}) \\
&\le \frac{1}{2}d(u,v).
\end{aligned}$$

Thus, J is a contraction mapping on (D,d), and has a unique fixed point $u \in D$. From Lemma 1, we deduce that there exists a unique solution u of the Cauchy problem (2.1) satisfying

$$u \in C(0,\infty;H^s) \cap L^{p+2}(0,\infty;B^s_{r(p),2}) \cap L^{q+3}(0,\infty;B^s_{r(q),2}).$$

This finishes the proof of the Theorem.

3. Explicit periodic wave solutions and some counter examples

The F-expansion method is the generalization of Jacobi elliptic function expansion method. In this section we mainly consider the general Davey-Stewartson systems

$$\begin{cases} iu_t + \Delta u + r\left|u\right|^2 u - \mu u v_{x_1} = 0, \\ \Delta v + b(|u|^2)_{x_1} = 0, \end{cases} \tag{3.1}$$

where u is a complex-valued function, r, μ, b are real constants.

Let

$$u = \exp(i\eta)w(x,t), \quad \eta = \Sigma_{i=1}^{n}\alpha_i x_i + \lambda t + \eta_0, \tag{3.2}$$

where $w(x,z)$ is real, $\alpha_i(i = 1, 2, \cdots, n)$, λ are undetermined coefficients, η_0 is an arbitrary $n-$dimensional constant vector.

From (3.2), one gets

$$u_t = i\lambda \exp(i\eta)w + \exp(i\eta)w_t, \tag{3.3}$$

and

$$u_{x_i x_i} = -\alpha_i^2 \exp(i\eta)w + 2\alpha w_{x_i} \exp(i\eta)i + \exp(i\eta)w_{x_i x_i}. \tag{3.4}$$

Combining (3.2) and (3.3)-(3.4), it follows that

$$\begin{cases} w_t + 2\Sigma_{i=1}^{n}\alpha_i w_{x_i} = 0, \\ \Delta w + rw^3 - \mu w v_{x_1} - (\lambda + \Sigma_{i=1}^{n}\alpha_i^2)w = 0, \\ \Delta v + b(w^2)_{x_1} = 0. \end{cases} \tag{3.5}$$

Supposing the problem (3.5) has wave solution as follows

$$w = w(\xi) = w(\Sigma_{i=1}^{n}k_i x_i + nt + \xi_0), \tag{3.6}$$

and

$$v = v(\xi) = v(\Sigma_{i=1}^{n}k_i x_i + nt + \xi_0), \tag{3.7}$$

where $k_i(i = 1, 2, \cdots, n)$, are undetermined constants, ξ_0 is an arbitrary constant.

Combining (3.5) and (3.6)-(3.7), one can gets simultaneous differential equations of $w(\xi), v(\xi)$,

$$n + 2\Sigma_{i=1}^{n}\alpha_i k_i = 0, \tag{3.8}$$

$$(\Sigma_{i=1}^{n}k_i^2)w'' + rw^3 - \mu k_1 w v' - (\lambda + \Sigma_{i=1}^{n}\alpha_i^2)w = 0, \tag{3.9}$$

$$(\Sigma_{i=1}^{n}k_i^2)v'' + 2bk_1 w w' = 0. \tag{3.10}$$

In view of the F-expansion, the homogeneous balance of $(\Sigma_{i=1}^{n}k_i^2)w''$ and $rw^3 - \mu k_1 w v'$ in (3.9), $(\Sigma_{i=1}^{n}k_i^2)v''$ and $2bk_1 w w'$ in (3.10) should be considered. So, let

$$w = a_1 F + a_0, \tag{3.11}$$

$$v = b_1 F + b_0, \tag{3.12}$$

where a_0, a_1, b_0, b_1 are undetermined constants, $F(\xi)$satisfies

$$F^{'2} = PF^4 + QF^2 + R, \tag{3.13}$$

where P, Q, R are real constants.

Combining (3.8)-(3.12), one gets the polynomials of $F(\xi)$,

$$\begin{aligned}&[a_1(\Sigma_{i=1}^n k_i^2)(2PF^3 + QF) + r(a_1F + a_0)^3 - (\lambda + \Sigma_{i=1}^n \alpha_i^2)(a_1F + a_0)]^2 \\ &-[\mu k_1 b_1(a_1F + a_0)]^2(PF^4 + QF^2 + R) = 0.\end{aligned} \tag{3.14}$$

$$[b_1(\Sigma_{i=1}^n k_i^2)(2PF^3 + QF)]^2 - [2a_1bk_1(a_1F + a_0)]^2(PF^4 + QF^2 + R) = 0. \tag{3.15}$$

Setting the coefficients of the polynomials to zeros, one can get the functions of the undetermined parameters as follows,

$$F^6: \quad 4(\Sigma_{i=1}^n k_i^2)^2P^2 + r^2a_1^4 + 4r(\Sigma_{i=1}^n k_i^2)Pa_1^2 = \mu^2k_1^2b_1^2P, \tag{3.16}$$

$$F^5: \quad 3a_1^3a_0r^2 + 6(\Sigma_{i=1}^n k_i^2)a_1a_0P = \mu^2k_1^2b_1^2Pa_0, \tag{3.17}$$

$$\begin{aligned}F^4: \quad &4a_1^2(\Sigma_{i=1}^n k_i^2)^2PQ + 9r^2a_1^2a_0^2 + 6r^2a_1^4a_0^2 + 2a_1^4r(\Sigma_{i=1}^n k_i^2)Q \\ &+12a_1^2a_0^2r(\Sigma_{i=1}^n k_i^2)P - 4a_1^2(\lambda + \Sigma_{i=1}^n \alpha_i^2)(\Sigma_{i=1}^n k_i^2)P \\ &-2ra_1^4(\lambda + \Sigma_{i=1}^n \alpha_i^2) = \mu^2k_1^2b_1^2(a_1^2Q + a_0^2P),\end{aligned} \tag{3.18}$$

$$\begin{aligned}F^3: \quad &a_1^2a_0^3r^2 + 9a_1a_0^3r^2 + 3a_1a_0r(\Sigma_{i=1}^n k_i^2) - 2Pa_0(\Sigma_{i=1}^n k_i^2)(\lambda + \Sigma_{i=1}^n \alpha_i^2) \\ &+2Pa_1a_0^3r(\Sigma_{i=1}^n k_i^2) - r(\lambda + \Sigma_{i=1}^n \alpha_i^2)(3a_1a_0 + a_1^2a_0) = \mu^2k_1^2b_1^2Qa_0,\end{aligned} \tag{3.19}$$

$$\begin{aligned}F^2: \quad &a_1^2(\Sigma_{i=1}^n k_i^2)^2Q^2 + 9r^2a_1^2a_0^4 + 6r^2a_1a_0^4 + a_1^2(\lambda + \Sigma_{i=1}^n \alpha_i^2)^2 \\ &-2a_1^2Q(\lambda + \Sigma_{i=1}^n \alpha_i^2)(\Sigma_{i=1}^n k_i^2) - 6r(\lambda + \Sigma_{i=1}^n \alpha_i^2)(a_1^2a_0^2 + a_1a_0^2) \\ &+6a_1^2a_0^2Qr(\Sigma_{i=1}^n k_i^2) = \mu^2k_1^2b_1^2(a_1^2R + a_0^2Q),\end{aligned} \tag{3.20}$$

$$\begin{aligned}F^1: \quad &3r^2a_1a_0^4 + (\lambda + \Sigma_{i=1}^n \alpha_i^2)^2a_0^3 - Q(\lambda + \Sigma_{i=1}^n \alpha_i^2)(\Sigma_{i=1}^n k_i^2)a_0 \\ &+rQ(\Sigma_{i=1}^n k_i^2)a_0 - 4r(\lambda + \Sigma_{i=1}^n \alpha_i^2)a_0^3 = \mu^2k_1^2b_1^2Ra_0,\end{aligned} \tag{3.21}$$

$$F^0: \quad r^2a_0^6 + (\lambda + \Sigma_{i=1}^n \alpha_i^2)^2a_0^2 - 2r(\lambda + \Sigma_{i=1}^n \alpha_i^2)a_0^4 = \mu^2k_1^2b_1^2Ra_0^2, \tag{3.22}$$

$$F^6: \quad P^2(\Sigma_{i=1}^n k_i^2)^2b_1^2 = b^2k_1^2Pa_1^4, \tag{3.23}$$

$$F^5: \quad b^2k_1^2Pa_1^3a_0 = 0, \tag{3.24}$$

$$F^4: \quad PQ(\Sigma_{i=1}^n k_i^2)^2b_1^2 = b^2k_1^2a_1^2(a_1^2Q + a_0^2P), \tag{3.25}$$

$$F^3: \quad b^2k_1^2Qa_1^3a_0 = 0, \tag{3.26}$$

$$F^2: \quad Q^2(\Sigma_{i=1}^n k_i^2)^2b_1^2 = 4b^2k_1^2a_1^2(a_1^2R + a_0^2Q), \tag{3.27}$$

$$F^1: \quad b^2k_1^2Ra_1^3a_0 = 0, \tag{3.28}$$

$$F^0: \quad b^2k_1^2Ra_0^2 = 0. \tag{3.29}$$

Solving the algebraic equations (3.16)-(3.29) to get

$$\begin{cases} a_0 = 0, a_1 = \pm(\Sigma_{i=1}^n k_i^2)\sqrt{\frac{-2P}{r(\Sigma_{i=1}^n k_i^2)+\mu b k_1^2}}, \\ b_0 = const, b_1 = \pm\frac{2bk_1(\Sigma_{i=1}^n k_i^2)}{r(\Sigma_{i=1}^n k_i^2)+\mu b k_i^2}\sqrt{P}, \\ \lambda = (\Sigma_{i=1}^n k_i^2)Q - (\Sigma_{i=1}^n \alpha_i^2), \\ Q^2 = 4PR, \end{cases} \tag{3.30}$$

where $k_i, \alpha_i (i = 1, 2, \cdots, n)$ are constants, and $r(\Sigma_{i=1}^n k_i^2) + \mu b k_1^2 < 0$.

Since $Q^2 = 4PR$, in view of (3.13), one has

$$F = \frac{R}{P}(\tan[\sqrt[4]{PR}(\xi + c)])^4, \tag{3.31}$$

where c is a constant.

Combining (3.30)-(3.31) and (3.11)-(3.12), and in view of (3.3), one can get solutions of (3.1), which are as follows

$$u = \pm(\Sigma_{i=1}^n k_i^2)\sqrt{\frac{-2P}{r(\Sigma_{i=1}^n k_i^2) + \mu b k_1^2}} \exp(i\eta)\frac{R}{P}(\tan[\sqrt[4]{PR}(\xi + c)])^4, \tag{3.32}$$

$$v = b_0 \pm \frac{2bk_1(\Sigma_{i=1}^n k_i^2)}{r(\Sigma_{i=1}^n k_i^2) + \mu b k_1^2}\frac{R}{\sqrt{P}}(\tan[\sqrt[4]{PR}(\xi + c)])^4, \tag{3.33}$$

where

$$\begin{aligned} \eta &= \Sigma_{i=1}^n \alpha_i x_i + [(\Sigma_{i=1}^n k_i^2)Q - (\Sigma_{i=1}^n \alpha_i^2)]t + \eta_0, \\ \xi &= \Sigma_{i=1}^n k_i x_i - 2(\Sigma_{i=1}^n \alpha_i k_i)t + \xi_0, \end{aligned} \tag{3.34}$$

k_i, α_i are constants, $r\Sigma_{i=1}^n k_i^2 + \mu b k_1^2 < 0$, η_0 is an arbitrary constant.

Remark 3.1. (3.32) and (3.33) show that if R^n is replaced by bounded domain, then there are some counter examples for nonhomogeneous initial values problems to elliptic-elliptic Davey-Stewartson systems.

4. Multi-order exact solutions

4.1. Lam equation and Lam function

In this chapter, we aim to construct Multi-order exact soltions for DSI (Davey-Stewartson systems of elliptic-hyperbolic types). Firstly, we recall the Lam equation and Lam function. Usually, the Lam equation of $y(x)$ can be written as

$$\frac{d^2y}{dx^2} + [\lambda - n(n+1)m^2 sn^2 x]y = 0, \tag{4.1}$$

where λ is eigenvalue, n is positive integer, snx is Jacobi elliptic sine function, m is the modulus and $0 < m < 1$, $x \in R^1$ in this subsection.

Making a change of independent variable

$$z = sn^2 x, \tag{4.2}$$

then, (4.1) is rewritten as

$$\frac{d^2y}{dz^2}+\frac{1}{2}[\frac{1}{z}+\frac{1}{z-1}+\frac{1}{z-h}]\frac{dy}{dz}-\frac{\mu n(n+1)z}{4z(z-1)(z-h)}y=0, \tag{4.3}$$

where $h=m^{-2}>1$, $\mu=-h\lambda$. The equation (4.3) is a Fuch-type equation which has four singular points, i.e. $z=0,1,h,\infty$. The solution of (4.3) is called Lam function.

Especially,

(i) When $n=2,\lambda=1+m^2,\mu=-(1+m^2)$, the Lam equation is

$$\frac{d^2y}{dx^2}+[(1+m^2)-6m^2sn^2x]y=0. \tag{4.4}$$

The corresponding Lam function is defined by $L_2^s(x)\equiv(1-z)^{1/2}(1-h^{-1}z)^{1/2}=cnxdnx$, where cnx,dnx are Jacobi elliptic cosine functions and the third-class Jacobi elliptic functions respectively.

(ii) When $n=2,\ \lambda=(1+4m^2)$, the Lam equation is

$$\frac{d^2y}{dx^2}+[(1+4m^2)-6m^2sn^2x]y=0. \tag{4.5}$$

The corresponding Lam function is defined by

$$L_2^c(x)\equiv snxdnx. \tag{4.6}$$

(iii) When $n=2,\ \lambda=4+m^2$, the Lam equation is

$$\frac{d^2y}{dx^2}+[(4+m^2)-6m^2sn^2x]y=0. \tag{4.7}$$

The corresponding Lam function is defined by

$$L_2^d(x)\equiv snxcnx. \tag{4.8}$$

(iv) When $n=3,\ \lambda=4(1+m^2),\ [\mu=-4(1+m^{-2})]$, the Lam equation is

$$\frac{d^2y}{dx^2}+[4(1+m^2)-12m^2sn^2x]y=0. \tag{4.9}$$

The corresponding lam function is defined by

$$L_3(x)\equiv z^{1/2}(1-z)^{1/2}(1-h^{-1}z)^{1/2}=snxcnxdnx. \tag{4.10}$$

4.2. Multi-order exact solutions of DSI

In this section, we shall consider the following elliptic-hyperbolic types systems (DSI)

$$\begin{cases} iu_t+\Delta u+r\left|u\right|^2u-2uv=0 \\ \Sigma_{j=1}^{l}v_{x_jx_j}-\Sigma_{j=l+1}^{n}v_{x_jx_j}-\Sigma_{j=1}^{k}r_j(|u|^2)_{x_jx_j}=0,\quad 1\le l,\ k<n. \end{cases} \tag{4.11}$$

Setting

$$u = \exp(i\eta)w(x,t), \quad x \in R^n, \quad \eta = \Sigma_{i=1}^n \alpha_i x_i + \lambda t + \eta_0, \tag{4.12}$$
$$w = w(\xi) = w(\Sigma_{i=1}^n k_i x_i + nt + \xi_0), \tag{4.13}$$
$$v = v(\xi) = v(\Sigma_{i=1}^n k_i x_i + nt + \xi_0). \tag{4.14}$$

Thus, (4.11) can be rewrite as

$$n + 2\Sigma_{j=1}^n \alpha_j k_j = 0, \tag{4.15}$$
$$(\Sigma_{j=1}^n {k_j}^2)w'' + rw^3 - 2wv - (\lambda + \Sigma_{j=1}^n {\alpha_j}^2)w = 0. \tag{4.16}$$
$$(\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)v'' - 2\Sigma_{j=1}^k r_j {k_j}^2 (w'^2 + ww") = 0, \tag{4.17}$$

Let

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \cdots, \tag{4.18}$$
$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots, \tag{4.19}$$

where $0 < \varepsilon << 1$, $w_0, w_1, w_2 \cdots v_0, v_1, v_2 \cdots$ are the exact solutions of the zeroth-order equation, the first-order equation and the second-order equation and so on, respectively.

Combining (4.16)-(4.19), one gets equation of each order. The equation of ε^0-order is

$$\begin{cases} (\Sigma_{j=1}^n {k_j}^2)w_0'' + rw_0^3 - 2w_0 v_0 - (\lambda + \Sigma_{j=1}^n {\alpha_j}^2)w_0 = 0, \\ (\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)v_0'' - 2\Sigma_{j=1}^k r_j {k_j}^2 (w_0'^2 + w_0 w_0'') = 0. \end{cases} \tag{4.20}$$

The equation of ε^1-order is

$$\begin{cases} (\Sigma_{j=1}^n {k_j}^2)w_1'' + 3rw_0^2 w_1 - 2(w_0 v_1 + w_1 v_0) - (\lambda + \Sigma_{j=1}^n {\alpha_j}^2)w_0 = 0, \\ (\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)v_1'' - 2\Sigma_{j=1}^k r_j {k_j}^2 (2w_0' w_1' + w_0 w_1'' + w_0'' w_1) = 0. \end{cases} \tag{4.21}$$

The equation of ε^2-order is

$$\begin{cases} (\Sigma_{j=1}^n {k_j}^2)w_2'' + 3r(w_0^2 w_2 + w_0 w_1^2) - 2(w_0 v_2 + w_1 v_1 + w_2 v_0) = (\lambda + \Sigma_{j=1}^n {\alpha_j}^2)w_2, \\ (\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)v_2'' - 2\Sigma_{j=1}^k r_j {k_j}^2 (2w_0' w_2' + w_1''^2 + w_0'' w_2 + w_0 w_2'' + w_1' w_1') = 0. \end{cases} \tag{4.22}$$

For (4.20), one can apply the Jacobi elliptic function expansion method. Firstly, setting

$$w_0 = a_0 + a_1 sn\xi, \quad v_0 = b_0 + b_1 sn\xi + b_2 sn^2\xi. \tag{4.23}$$

Combining (4.20) and (4.23), it can easily be obtained

$$\begin{cases} a_0 = 0, a_1 = \pm\sqrt{\frac{2m^2(\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)}{r}}, \\ b_0 = const, b_1 = 0, b_2 = 2\Sigma_{j=1}^l {k_j}^2 m^2, \\ \lambda = -(\Sigma_{j=1}^n {k_j}^2)(1 + m^2) - \Sigma_{j=1}^n {\alpha_j}^2 - 2c. \end{cases} \tag{4.24}$$

Thus, the zeroth-order solution of (4.18) can be get

$$\begin{aligned} w_0 &= \pm m\sqrt{\frac{2(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r}}sn\xi, \\ v_0 &= c + 2\Sigma_{j=1}^{l}{k_j}^2 m^2 sn^2\xi, \\ \xi &= \Sigma_{j=1}^{n}k_j x_j - (2\Sigma_{j=1}^{n}\alpha_j k_j)t + \xi_0, \end{aligned} \tag{4.25}$$

where k_j, α_j are constants, ξ_0 is arbitrary constant, and $\frac{(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r} > 0$.

Notice that one can deduce that $v_0 = \frac{\Sigma_{j=1}^{l}{k_j}^2 r w_0^2}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2} + c$ from (4.25) and $v_1 = \frac{2\Sigma_{j=1}^{l}{k_j}^2 r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2} w_0 w_1$ from the second equation of (4.21).

Then one gets the transformation of (4.21)

$$\begin{aligned} &(\Sigma_{j=1}^{n}{k_j}^2)w_1'' + 6\left(r - \frac{2\Sigma_{j=1}^{l}{k_j}^2 r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}\right)\frac{m^2(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r} sn^2\xi w_1 \\ &+ (\Sigma_{j=1}^{n}{k_j}^2)(1+m^2)w_1 = 0. \end{aligned}$$

Simplifying this equation to have

$$w_1'' + [(1+m^2) - 6m^2 sn^2\xi]w_1 = 0. \tag{4.26}$$

From (4.26), the first-order term of (4.18) is

$$\begin{aligned} w_1(\xi) &= AL_2^s = Acn\xi dn\xi, \\ v_1(\xi) &= \pm 2A\Sigma_{j=1}^{l}{k_j}^2 m\sqrt{\frac{2r}{(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}} sn\xi cn\xi dn\xi. \end{aligned} \tag{4.27}$$

For the second-order equation of (4.22), combining (4.27), (4.25) and $v_2 = \frac{\Sigma_{j=1}^{l}{k_j}^2 r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}(2w_0w_2 + w_1^2)$ from the second equation of (4.22), one has

$$w_2'' + [(1+m^2) - 6m^2 sn^2\xi]w_2 = \pm 3\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}} mA^2 sn\xi cn^2\xi dn^2\xi,$$

i.e.

$$\begin{aligned} &w_2'' + [(1+m^2) - 6m^2 sn^2\xi]w_2 \\ &= \pm 3\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}} mA^2[sn\xi - (1+m^2)sn^3\xi + m^2 sn^5\xi], \end{aligned} \tag{4.28}$$

by using $cn^2\xi = 1 - sn^2\xi, dn^2\xi = 1 - m^2 sn^2\xi$.

Noticing that (4.28) is an inhomogeneous Lam equation and the key step is to find a particular solution of the inhomogeneous term of (4.28).

Letting

$$w_2 = c_1 sn\xi + c_3 sn^3\xi. \tag{4.29}$$

Considering the (4.28), one gets

$$c_1 = \mp\frac{1+m^2}{4m}\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}} A^2, \tag{4.30}$$

$$c_3 = \pm\frac{1}{2}\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}}mA^2. \tag{4.31}$$

Then the second-order solution of (4.18) is

$$\begin{aligned} w_2(\xi) = &\mp \frac{1+m^2}{4m}\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}}A^2 sn\xi \\ &\pm \frac{1}{2}\sqrt{\frac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}}mA^2 sn^3\xi, \end{aligned} \tag{4.32}$$

$$v_2(\xi) = A^2\frac{\Sigma_{j=1}^{l}{k_j}^2 r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}[cn^2\xi dn^2\xi \mp (1+m^2)sn^2\xi \pm 2m^2 sn^4\xi)]. \tag{4.33}$$

Thus one has the multi-order solution of DSI

$$\begin{aligned} u_0(\xi) &= \pm m\sqrt{\frac{2(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r}}sn\xi\exp(i\eta), \\ v_0(\xi) &= c + 2\Sigma_{j=1}^{l}{k_j}^2 m^2 sn^2\xi, \end{aligned} \tag{4.34}$$

$$\begin{aligned} u_1(\xi) &= AL_2^s = Acn\xi dn\xi\exp(i\eta) \\ v_1 &= \pm 2A\Sigma_{j=1}^{l}{k_j}^2 m\sqrt{\frac{2r}{(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}}sn\xi cn\xi dn\xi, \end{aligned} \tag{4.35}$$

$$\begin{aligned} u_2(\xi) &= \mp\tfrac{1+m^2}{4m}\sqrt{\tfrac{2r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}}A^2 sn\xi(1 - \tfrac{2m^2}{1+m^2}sn^2\xi)\exp(i\eta) \\ v_2(\xi) &= A^2\tfrac{\Sigma_{j=1}^{l}{k_j}^2 r}{\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2}[cn^2\xi dn^2\xi \mp (1+m^2)sn^2\xi \pm 2m^2 sn^4\xi)], \end{aligned} \tag{4.36}$$

where $\xi = \Sigma_{j=1}^{n}k_j x_j - (2\Sigma_{j=1}^{n}\alpha_j k_j)t + \xi_0$, $\eta = \Sigma_{j=1}^{n}\alpha_j k_j - [(\Sigma_{j=1}^{n}{k_j}^2)(1+m^2) + \Sigma_{j=1}^{n}{\alpha_j}^2 + 2c]t + \eta_0$, k_j, α_j are constants, ξ_0, η_0 are arbitrary constants, and $\frac{(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r} > 0$.

4.3. Degenerate solution

When the $m \to 1$, $sn\xi \to \tanh\xi$, the zeroth-order solution of DSI degenerates into

$$\begin{aligned} u_0 &= \pm\sqrt{\frac{2(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r}}\tanh\xi\exp(i\eta), \\ v_0 &= c + 2\Sigma_{j=1}^{l}{k_j}^2\tanh^2\xi, \end{aligned} \tag{4.37}$$

where

$$\begin{aligned} \xi &= \Sigma_{j=1}^{n}k_j x_j - (2\Sigma_{j=1}^{n}\alpha_j k_j)t + \xi_0, \\ \eta &= \Sigma_{j=1}^{n}\alpha_j k_j - [2(\Sigma_{j=1}^{n}{k_j}^2) + \Sigma_{j=1}^{n}{\alpha_j}^2 + 2c]t + \eta_0, \end{aligned} \tag{4.38}$$

and k_j, α_j are constants, ξ_0, η_0 are arbitrary constants, and $\frac{(\Sigma_{j=1}^{l}{k_j}^2 - \Sigma_{j=l+1}^{n}{k_j}^2)}{r} > 0$.

This is solitary wave solution that we frequently see, and we call it shock wave solution.

Similarly by $cn\xi \to \sec h, dn\xi \to \sec h\xi$, when $m \to 1$, the first-order solution of DSI is to degenerate into

$$u_1(\xi) = A \sec h^2\xi \exp(i\eta), \tag{4.39}$$

$$v_1(\xi) = \pm A\Sigma_{j=1}^{l} {k_j}^2 \sqrt{\frac{2r}{(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}} \tanh\xi \sec h^2\xi. \tag{4.40}$$

This is a bell shaped solitary wave solution, pulse shock wave solution.

The second-order solution of DSI is to degenerate into

$$u_2(\xi) = \mp\frac{1}{2}\sqrt{\frac{2r}{\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2}} A^2 \tanh\xi(1 - \tanh^2\xi) \exp(i\eta), \tag{4.41}$$

$$v_2(\xi) = A^2 \frac{\Sigma_{j=1}^{l} {k_j}^2 r}{\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2} [\sec h^4\xi \mp 2(\tanh^2\xi - \tanh^4\xi)]. \tag{4.42}$$

It is a new solitary wave solution.

4.4. The more exact solution of DSI

One can get more solutions of Davey-Stewartson equations:

(i) If $w_0 = a_0 + a_1 cn\xi$, $v_0 = b_0 + b_1 cn\xi + b_2 cn^2\xi$ in (4.23), one can get the zeroth-order solution of DSI which is

$$u_0 = \pm m\sqrt{\frac{-2(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}{r}} cn\xi \exp(i\eta), \quad \frac{-2(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}{r} > 0,$$

$$v_0 = c - 2\Sigma_{j=1}^{l} {k_j}^2 m^2 cn^2\xi,$$

$$\eta = \Sigma_{j=1}^{n} \alpha_j k_j + [(\Sigma_{j=1}^{n} {k_j}^2)(2m^2 - 1) - \Sigma_{j=1}^{n} {\alpha_j}^2 - 2c]t + \eta_0. \tag{4.43}$$

The first-order solution is

$$u_1(\xi) = AL_2^c \exp(i\eta) = A sn\xi dn\xi \exp(i\eta),$$

$$v_1(\xi) = \pm 2A\Sigma_{j=1}^{l} {k_j}^2 m \sqrt{\frac{-2r}{(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}} sn\xi cn\xi dn\xi. \tag{4.44}$$

The second-order solution is

$$u_2(\xi) = \mp\sqrt{\frac{-2r}{\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2}} \frac{A^2(2m^2 - 1)}{4m} cn\xi(1 - \frac{2m^2}{2m^2 - 1} cn^2\xi) \exp(i\eta),$$

$$v_2(\xi) = A^2 \frac{\Sigma_{j=1}^{l} {k_j}^2 r}{\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2} [sn^2\xi dn^2\xi \mp (2m^2 - 1)cn^2\xi \pm 2m^2 cn^4\xi)]. \tag{4.45}$$

(ii) If $w_0 = a_0 + a_1 dn\xi$,$v_0 = b_0 + b_1 dn\xi + b_2 dn^2\xi$ in (4.23), one can get the zeroth-order solution of DSI which is

$$u_0 = \pm\sqrt{\frac{-2(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}{r}} dn\xi \exp(i\eta), \quad \frac{-2(\Sigma_{j=1}^{l} {k_j}^2 - \Sigma_{j=l+1}^{n} {k_j}^2)}{r} > 0,$$

$$v_0 = c - 2ldn^2\xi,$$
$$\eta = \Sigma_{j=1}^n \alpha_j k_j + [(\Sigma_{j=1}^n {k_j}^2)(2 - m^2) - \Sigma_{j=1}^n {\alpha_j}^2 - 2c]t + \eta_0. \tag{4.46}$$

The first-order solution is

$$u_1(\xi) = AL_2^d \exp(i\eta) = Asn\xi cn\xi \exp(i\eta),$$
$$v_1(\xi) = \pm 2A\Sigma_{j=1}^l {k_j}^2 \sqrt{\frac{-2r}{(\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)}} sn\xi cn\xi dn\xi. \tag{4.47}$$

The second-order solution is

$$u_2(\xi) = \mp \sqrt{\frac{-2r}{\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2}} \frac{A^2(2-m^2)}{4m^4} dn\xi(1 - \frac{2}{2-m^2} dn^2\xi) \exp(i\eta),$$
$$v_2(\xi) = A^2 \frac{\Sigma_{j=1}^l {k_j}^2 r}{\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2} [sn^2\xi cn^2\xi \mp \frac{2-m^2}{m^4} dn^2\xi(1 - \frac{2}{2-m^2} dn^2\xi)]. \tag{4.48}$$

These are periodic wave solutions of DSI expressed by Jacobi elliptic functions.

When $m \to 1$, one can get the degenerate solutions,

$$u_0(\xi) = \pm \sqrt{\frac{-2(\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)}{r}} \sec h\xi \exp(i\eta),$$
$$v_0(\xi) = c - 2\Sigma_{j=1}^l {k_j}^2 \sec h^2\xi, \tag{4.49}$$
$$\eta = \Sigma_{j=1}^n \alpha_j k_j + [(\Sigma_{j=1}^n {k_j}^2) - \Sigma_{j=1}^n {\alpha_j}^2 - 2c]t + \eta_0,$$
$$u_1(\xi) = A \tanh\xi \sec h\xi \exp(i\eta),$$
$$v_1(\xi) = \pm 2A\Sigma_{j=1}^l {k_j}^2 \sqrt{\frac{-2r}{(\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2)}} \tanh\xi \sec h^2\xi, \tag{4.50}$$
$$u_2(\xi) = \mp \sqrt{\frac{-2r}{\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2}} \frac{A^2}{4} \sec h\xi(1 - 2\sec h^2\xi) \exp(i\eta),$$
$$v_2(\xi) = A^2 \frac{\Sigma_{j=1}^l {k_j}^2 r}{\Sigma_{j=1}^l {k_j}^2 - \Sigma_{j=l+1}^n {k_j}^2} [\tanh^2\xi \sec h^2\xi \mp \sec h^2\xi \pm 2\sec h^4\xi)]. \tag{4.51}$$

Remark 4.1. It is open on the existence of global smooth solutions for Cauchy problems to elliptic-hyperbolic types Davey-Stewartson systems. (4.27) indicates that there are some examples of global smooth solutions.

5. Conclusion

In this paper, we prove that the Cauchy problem of generalized Davey-Stewartson systems has a unique solution in $C(0,\infty;H^s) \cap L^{p+2}(0,\infty;B^s_{r(p),2}) \cap L^{q+3}(0,\infty;B^s_{r(q),2})$. What's more interesting, we construct some explicit period wave solution of the generalized Davey-Stewartson by F-expansion method, as well as some multi-order exact solutions.

From the discussion above, it can be seen that

(i) One can get many zeroth-order solutions of nonlinear evolution equations by using F-expansion or Jacobi elliptic function expansion, which only related to the correlation chart of P, Q, R and the solution of $PF^4 + QF^2 + R$.

(ii) The form of the first-order equation is the same as that of the Lam equation. So one can get the first-order solution by solving the Lam equation. The form of the second-order equation is the same as the inhomogeneous Lam equation, and one can obtain the second-order solution by the particular solution of the inhomogeneous term.

(iii) One can obtain the degenerate solution by discussing the limit cases of the multi-order exact solutions. The method is valid to get the multi-order exact solutions of some other nonlinear evolution equations. At the same time, one can get many kinds of solitary wave solutions.

(iv) By the contraction mapping theorem, one can deduce that there exists a unique solution of the Cauchy problem of generalized Davey-Stewartson systems.

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