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ATTRACTOR FOR THE NON-AUTONOMOUS LONG WAVE-SHORT WAVE RESONANCE INTERACTION EQUATION WITH DAMPING*

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Abstract In this paper, the long wave-short wave resonance interaction equation with a nonlinear term in bounded domain was studied. When $\beta \geq \frac{3}{2}$, we obtained the existence and uniqueness of the weak solution of system (1.1)-(1.4) by Galërkin's method, and further proved the existence of the compact uniform attractor for damped driven by the non-autonomous long wave-short wave resonance interaction equation.

Keywords Long wave-short wave resonance interaction equation, a priori estimates, well-posedness, attractor.

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1. Introduction

The long wave-short wave (LS) resonance equation appeared in a recent study of the interaction of surface waves with gravity and capillary modes, as well as the analysis of internal waves and Rosby waves [12]. In the plasma physics, the long wave-short wave resonance equation explains the high frequency electron plasma resonance and associated low frequency ion density perturbation [22]. A general theory on the interaction between short wave and electromagnetic wave was presented [6].

The long wave-short wave resonance equation has attracted extensive attention from many physicists and mathematicians, due to its rich physical and mathematical properties. For one-dimensional wave propagation, there were many studies on this interaction. Guo [7, 13] verified the existence of global solutions for the long wave-short wave equation and the generalized long wave-short wave equation, respectively. In [14], Guo studied the orbital stability of the solitary waves of the long wave-short wave resonance equation. In [15], Guo studied the asymptotic behavior of the solutions of long wave-short wave equations with zero order dissipation in $H^2_{\text{per}} \times H^1_{\text{per}}$. The approximate inertial manifolds of LS equation was in [16]. In [4, 5, 19, 24,

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25, 29], the well-posedness of Cauchy problem for long wave-short wave resonance equation was studied.

In recent years, the study of attractors in dynamics has attracted extensive attention [20]. We can get its global attractors in [10,19,28,29] for autonomous systems. But unlike autonomous systems, non-autonomous systems have special temporal correlation. So we have obtained the attractors of non-autonomous system, such as pullback attractors (see [2,18,23]) and uniform attractors (see [1,27]). In this paper, we prove the uniform attractor of the long wave-short wave resonance interaction equation with damping (1.1)-(1.3) by defining the relevant uniform attractor in [9].

In this paper, we consider the following long wave-short wave resonance interaction equation with damping:

$$iu_t + u_{xx} - uv + |u|^{\beta-1}u + i\gamma u = f(x,t), \tag{1.1}$$

$$v_t + \alpha v + \gamma|u|_x^2 = g(x,t). \tag{1.2}$$

Under the following initial condition

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad \forall x \in \Omega, \tag{1.3}$$

and the boundary value conditions

$$u(x-D,t) = u(x+D,t), \;\; v(x-D,t) = v(x+D,t), \;\; \forall x \in \Omega, \tag{1.4}$$

where $x \in \Omega = [-D, D] \subset \mathbb{R}$, $D > 0$ and α, β and γ are positive constants. u and v are unknown functions, u and f are complex functions, v and g are real functions. Non-autonomous terms f and g are time-dependent external forces. When $\beta = 2$ or 3, the well-posedness of the solution of the long wave-short wave resonance interaction equation has been studied by many people. When $\beta = 3$, Gao first studied the well-posedness of the solution of the non-autonomous long wave-short wave resonance interaction equation and obtained the existence of attractors in [11], and then Cui proved the existence of uniform attractors in [9]. In [21], the author has also proved the well-posedness of the solution for the long wave-short wave resonance interaction equation and studied the existence of global attractors for the equation when $\beta = 2$. In this paper, our aim here is, firstly, to get the well-posedness of solutions for problem (1.1)-(1.4) for $\beta \geq \frac{3}{2}$ and then to derive the existence of the compact uniform attractor.

The rest of this paper is organized as follows. In section 2, we introduce symbols and preliminary results, and recall some facts about the uniform attractor. In section 3, a priori estimate of the solution is obtained. In section 4, existence and uniqueness of the weak solution of the system (1.1)-(1.4) are proved. In section 5, the existence of the strong compact uniform attractor for (1.1)-(1.4) is obtained.

2. Preliminary

In this section, we introduce some notations and preliminary results used in this paper. Firstly, We have added the subscript "per" to the usual Sobolev space to represent the Sobolev space over the periodic region. Now, we denote some notations:

$$L^p_{\text{per}} =: L^p_{\text{per}}(\Omega) = \{u \in L^p(\Omega), u(x-D) = u(x+D), 1 \leq p \leq \infty\},$$
$$H^p_{\text{per}} =: H^p_{\text{per}}(\Omega) = \{u \in L^2_{per}(\Omega), D^\alpha u \in L^2_{per}(\Omega), |\alpha| \leq p, 1 \leq p < \infty\},$$

where $\Omega = [-D, D] \subset \mathbb{R}, D > 0$. Especially, when $p = 2$, the first formula becomes the space L^2_{per}, and $(\cdot,\cdot)$, $||\cdot||$ denote the inner product and norm of $L^2_{per}(\Omega)$, which are defined as follows:

$$(u, v) = \int_\Omega u(x)\overline{v}(x)dx,\ u,\ v \in L^2_{\rm per},\quad ||u||^2 = \int_\Omega u(x)\overline{u}(x)dx,\ u \in L^2_{\rm per},$$

and $\overline{u}$ denotes the conjugate complex quantity of u. Similarly, we denote the norm of $L^p_{\rm per}(\Omega)$ for all by $\|\cdot\|_p$. And $\|\cdot\|_{H^p}$ denotes the norm of $H^p_{per}(\Omega)$, which is defined by $\|u\|^2_{H^p} = \sum\limits_{|\alpha|\le p} \|D^\alpha u\|^2$ for all $1 \le p < \infty$.

For simplicity and convenience, the letter C represents a constant, which may vary in different lines. $C(\cdot,\cdot)$ represents the constant C represented by the parameters appearing in parentheses.

Next, we introduce Sobolev embedding theorem for one-dimensional domain used in the following section.

Theorem 2.1. *Let Ω be a bounded open subset of $\mathbb{R}^1$, and suppose $\partial\Omega$ is C^1. Assume $p > 2$, and $u \in H^1(\Omega)$. Then $u \in L^p(\Omega)$, with the estimate*

$$\|u\|_p \le C\|u\|^{\frac{p+2}{2p}}\|u_x\|^{\frac{p-2}{2p}}, \tag{2.1}$$

the constant C depending only on p and Ω.

Finally, the definitions(see [7–9]) and some main lemmas(see [3, 8, 17]) about uniform attractors are clarified. Let $(X, \|\cdot\|_X)$ be a Banach space, then we have the following definitions and lemma.

Definition 2.1. Suppose $f(t) : \mathbb{R} \to X$ is a function, and $T(\cdot)$ is the translation operator. The set

$$\mathcal{H}(f) = \overline{\{T(s)f(t) = f(t+s)|s \in \mathbb{R}\}}. \tag{2.2}$$

is defined the hull of f in X, denoted by $\mathcal{H}(f)$.

(i) f is said to be translation bounded in $L^2(\mathbb{R}; X)$ if $\mathcal{H}(f)$ is bounded in which

$$\|f\|^2_{L^2_b(\mathbb{R};X)} := \sup_{t\in\mathbb{R}} \int_t^{t+1} \|f(t)\|^2_X ds < \infty, \tag{2.3}$$

then $L^2_b(\mathbb{R}; X)$ consists of all the translation bounded functions in $L^2(\mathbb{R}; X)$;

(ii) The collection of all the translation compact functions in $L^2_{loc}(\mathbb{R}; X)$ is denoted by $L^2_c(\mathbb{R}; X)$.

Definition 2.2. Suppose $\sum$ is a parameter set. If for each $\sigma \in \sum$, the mapping $U_\sigma(t,\tau) : X \to X$ satisfies

(i) $U_\sigma(t, s) \circ U_\sigma(s, \tau) = U_\sigma(t, \tau),\quad \forall t \ge s \ge \tau,\ \tau \in \mathbb{R}$,

(ii) $U_\sigma(\tau, \tau) = I$ (the identity operator on X), $\tau \in \mathbb{R}$,

where $\{U_\sigma(t, \tau), t \ge \tau, \tau \in \mathbb{R}\}$, $\sigma \in \sum$ is said to be a family of processes in X.

Let B_0, $B \in \mathcal{B}(E)$ be the set of bounded subsets of E. If for any $\tau \in \mathbb{R}$, there exists $t_0 = t_0(\tau, B) \geq \tau$ such that $\cup_{\sigma\in\sum} U_\sigma(t,\tau)B \subseteq B_0$ for all $t \geq t_0$. Then B_0 is said to be uniformly absorbing set for the family of processes $\{U_{\sigma\in\sum}(t,\tau)\}$,

A set $Y \subset E$ is called uniformly attracting for the family of process $\{U_\sigma(t,\tau)\}$, $\sigma \in \sum$ if, for each fixed $\tau \in \mathbb{R}$ and every $B \in \mathcal{B}(E)$, it satisfies that

$$\lim_{t\to+\infty}\left(\sup_{\sigma\in\sum} \mathrm{dist}_E(U_\sigma(t,\tau)B, Y)\right) = 0. \tag{2.4}$$

Definition 2.3. A closed set $\mathcal{A}_\sum \subset X$ is called the uniform attractor of the family of processes $\{U_\sigma(t,\tau)\}_{\sigma\in\sum}$ if it is uniformly attracting (attracting property) and it is contained in any closed uniformly attracting set $\mathcal{A}'$ of the family of processes $\{U_\sigma(t,\tau)\}_{\sigma\in\sum} : \mathcal{A}_\sum \subseteq \mathcal{A}'$ (minimality property).

Definition 2.4. $\{U_\sigma(t,\tau)\}_{\sigma\in\sum}$, a family of processes in X, is said to be $(X\times\sum, X)$-continuous, if, for any fixed T and τ, $T \geq \tau$, projection $(u_\tau, \sigma) \to U_\sigma(T,\tau)u_\tau$ is continuous from $X \times \sum$ to X.

Definition 2.5. The space $L^p(0,T;X)$ represents all measurable functions $f : [0,T] \to X$ with the norm

$$\|f\|_{L^p(0,T;X)} = \left(\int_0^T \|f(t)\|_X^p dt\right)^{1/p} < \infty, \tag{2.5}$$

for $1 \leq p < \infty$, and

$$\|f\|_{L^\infty(0,T;X)} = \operatorname*{ess\,sup}_{0\leq t\leq T} \|f(t)\|_X < \infty, \tag{2.6}$$

for $p = \infty$.

Lemma 2.1. *Let $\sum$ be a compact metric space and suppose $\{T(h)|h \geq 0\}$ is a family of operators defined on $\sum$, satisfying*

(i)

$$T(h)\sum = \sum, \ \forall h \in \mathbb{R}_+; \tag{2.7}$$

(ii) translation identity:

$$\begin{aligned} &U_\sigma(t+h,\tau+h) = U_{T(h)\sigma}(t,\tau),\\ &\forall \sigma \in \sum,\ t \geq \tau,\ \tau \in \mathbb{R},\ h \geq 0, \end{aligned} \tag{2.8}$$

where $U_\sigma(T,\tau)$ is an arbitrary process in compact metric space E.

Note that if the family of processes $\{U_{\sigma\in\sum}(T,\tau)\}$ is $(E\times\sum, E)$-continuous and it has a uniform compact attracting set, then the skew product flow corresponding to it has a global attractor $\mathcal{A}$ on $E \times \sum$. And the projection of $\mathcal{A}$ on $\sum$, $\mathcal{A}_\sum$, is the compact uniform attractor of $\{U_{\sigma\in\sum}(T,\tau)\}$.

Remark 2.1. Assumption (2.8) holds if the system has a unique solution.

Lemma 2.2. *Let $(X, \|\cdot\|_X)$ be a uniform convex Banach space (particularly, a Hilbert space), and let $\{x_k\}_{k\geq 0}$ be a sequence in X. If $x_k \rightharpoonup x_0$ and $\|x_k\|_X \to \|x_0\|_X$, then $x_k \to x_0$.*

Lemma 2.3. *Let $\{x_k\}_{k\geq 0}$ be a sequence in the uniform convex Banach space X. If $x_k \rightharpoonup x_0$, then*

$$\sup_{k\geq 1}\|x_k\|_X < \infty,$$
$$\|x_0\|_X \leq \liminf_{k\to\infty}\|x_k\|_X. \tag{2.9}$$

Next, we introduce $W(x,t) = (u(x,t), v(x,t))$ and $Y(x,t) = (f(x,t), g(x,t))$. We denote the space of $W(x,t) = (u(x,t), v(x,t))$ by $E_0 = H^2(\Omega)\bigcap H^1_{per}(\Omega) \times H^1_{per}(\Omega)$ with norm

$$\|W\|_{E_0} = \left(\|u\|^2_{H^2} + \|v\|^2_{H^1}\right)^{1/2}. \tag{2.10}$$

Similarly, we denote the space of $Y(x,t)$ by $\sum_0$ with norm

$$\|Y\|_{\sum_0} = \left(\|f\|^2_{H^2} + \|g\|^2_{H^1}\right)^{1/2}. \tag{2.11}$$

Definition 2.6. Suppose that the symbol $Y(x,t)$ belongs to the symbol space $\sum$, defined by

$$\sum = \overline{\{Y_0(x, s+r) \mid r \in \mathbb{R}_+\}}, \tag{2.12}$$

where $Y_0 = (f_0(x,t), g_0(x,t)) \in L^2_c(\mathbb{R}; E_0)$ and the closure is taken in the sense of local quadratic mean convergence topology in the topological space $L^2_{loc}(\mathbb{R}; \sum_0)$. Moreover, we assume $f_{0t}(x,t) \in L^2_b(\mathbb{R}; H^1)$.

Remark 2.2. Due to the conception of translation compact/boundedness, we remark that

(i) $\forall Y_1 \in \sum$, $\|Y_1\|^2_{L^2_b(\mathbb{R};\sum_0)} \leq \|Y_0\|^2_{L^2_b(\mathbb{R};\sum_0)}$;

(ii) $T(t)\sum = \sum$, $\forall t \in \mathbb{R}$, where $T(t)f(s) = f(s+t)$ is a translation operator.

3. A priori estimates

In order to obtain the existence and uniqueness of weak solutions in the next section, in the following, we establish some uniform a priori estimate of the solutions both in time t and in symbol space ($Y \in \Sigma$).

Lemma 3.1. *If $u_\tau \in L^2(\Omega)$, $Y(x,t)$ satisfy Definition 2.6, then for the weak solution of the problem (1.1)-(1.4), we have*

$$\|u\|^2 \leq C_1, \ \forall t \geq t_1, \tag{3.1}$$

where $C_1 = C_1(\gamma, f_0)$, $t_1 = C(\gamma, f_0, \|u_\tau\|)$.

Proof. Taking the inner product of (1.1) with u and taking the imaginary part, we get

$$\frac{1}{2}\frac{d}{dt}\|u\|^2 + \gamma\|u\|^2 = \operatorname{Im}\int_\Omega f\overline{u}dx. \tag{3.2}$$

By using Young's inequality, we get

$$\frac{d}{dt}\|u\|^2 + \frac{\gamma}{2}\|u\|^2 \leq \frac{1}{\gamma}\|f\|^2_{L^2_b(\mathbb{R};H^1)} \leq \frac{1}{\gamma}\|f_0\|^2_{L^2_b(\mathbb{R};H^1)}. \tag{3.3}$$

And then by Gronwall's inequality, we can complete the proof. □

Lemma 3.2. *If $W_\tau \in H^1 \times H$, $2\gamma \geq \alpha$ and $Y(x,t)$ satisfy Definition 2.6, then for the weak solution of the problem (1.1)-(1.4), we have*

$$\|W(t)\|^2_{H^1\times H} \leq C_2, \quad \forall t \geq t_2, \tag{3.4}$$

where $C_2 = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t})$, $t_2 = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t}, \|W_\tau\|_{H^1\times H})$.

Proof. Taking the inner product of (1.1) with u and taking the real part, we get that

$$\|u_x\|^2 + \int_\Omega v|u|^2 dx - \mathrm{Re}\int_\Omega |u|^{\beta+1} dx = -\mathrm{Re}(f,u). \tag{3.5}$$

Taking the inner product of (1.1) with u_t and taking the real part, we get that

$$\frac{d}{dt}\|u_x\|^2 + 2\mathrm{Re}\int_\Omega uv\overline{u}_t dx - 2\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_t dx = -2\mathrm{Re}(f,u_t). \tag{3.6}$$

Since

$$\frac{d}{dt}(\mathrm{Re}(f,u)) = \mathrm{Re}(f_t,u) + \mathrm{Re}(f,u_t), \tag{3.7}$$

$$\frac{d}{dt}\mathrm{Re}\int_\Omega v|u|^2 dx = \int_\Omega v_t|u|^2 dx + 2\mathrm{Re}\int_\Omega vu\overline{u}_t dx, \tag{3.8}$$

$$\frac{d}{dt}\mathrm{Re}\int_\Omega |u|^{\beta+1} dx = (\beta+1)\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_t dx. \tag{3.9}$$

(3.6) can be written as follows

$$\begin{aligned}&\frac{d}{dt}\|u_x\|^2 + \frac{d}{dt}\int_\Omega |u|^2 v dx - \frac{2}{\beta+1}\frac{d}{dt}\mathrm{Re}\int_\Omega |u|^{\beta+1}dx - \int_\Omega |u|^2 v_t dx \\ &= -2\frac{d}{dt}(\mathrm{Re}(f,u)) + 2\mathrm{Re}(f_t,u).\end{aligned} \tag{3.10}$$

Combining (3.5) and (3.10), we get

$$\begin{aligned}&\frac{d}{dt}(\|u_x\|^2 + \int_\Omega |u|^2 v dx - \frac{2}{\beta+1}\mathrm{Re}\int_\Omega |u|^{\beta+1}dx + 2\mathrm{Re}(f,u)) \\ &+2\alpha\|u_x\|^2 + 2\alpha\int_\Omega v|u|^2 dx - 2\alpha\mathrm{Re}\int_\Omega |u|^{\beta+1}dx + 2\alpha\mathrm{Re}(f,u) \\ &-\int_\Omega |u|^2 v_t dx - 2\mathrm{Re}(f_t,u) = 0.\end{aligned} \tag{3.11}$$

Note that, by (1.1),

$$\begin{aligned}\mathrm{Re}(f_t,u) &= \mathrm{Re}\int_\Omega f_t\overline{u}dx \\ &= \mathrm{Re}\int_\Omega (iu_{tt} + u_{xxt} - u_t v - uv_t + \beta|u|^{\beta-1}u_t + i\gamma u_t)\overline{u}dx \\ &= \int_\Omega (\mathrm{Re}u_{xxt}\overline{u} - \mathrm{Re}u_t v\overline{u} - |u|^2 v_t + \mathrm{Re}\beta|u|^{\beta-1}u_t\overline{u})dx \\ &= -\frac{1}{2}\frac{d}{dt}\|u_x\|^2 - \frac{1}{2}\int_\Omega |u|^2 v_t dx - \frac{1}{2}\frac{d}{dt}\int_\Omega |u|^2 v dx + \frac{\beta}{\beta+1}\frac{d}{dt}\mathrm{Re}\int_\Omega |u|^{\beta+1}dx.\end{aligned} \tag{3.12}$$

So (3.11) can be written that

$$\begin{aligned}&\frac{d}{dt}(2\|u_x\|^2+2\int_\Omega |u|^2 v dx-2\mathrm{Re}\int_\Omega |u|^{\beta+1}dx+2\mathrm{Re}(f,u))\\&+2\alpha\|u_x\|^2+2\alpha\int_\Omega v|u|^2dx-2\alpha\mathrm{Re}\int_\Omega |u|^{\beta+1}dx+2\alpha\mathrm{Re}(f,u)=0.\end{aligned}\tag{3.13}$$

Taking the inner product of (1.2) with v, we get that

$$\frac{d}{dt}\|v\|^2+2\alpha\|v\|^2+2\gamma\int_\Omega |u|_x^2 v dx=2(g,v).\tag{3.14}$$

Note that, by (1.1)

$$\begin{aligned}&\int_\Omega |u|_x^2 v dx\\=&2\mathrm{Re}\int_\Omega (uv)\overline{u}_x dx\\=&2\mathrm{Re}\int_\Omega (iu_t+u_{xx}+|u|^{\beta-1}u+i\gamma u-f)\overline{u}_x dx\\=&-\frac{d}{dt}\mathrm{Im}\int_\Omega u\overline{u}_x dx-2\gamma\mathrm{Im}\int_\Omega u\overline{u}_x dx-2\mathrm{Re}\int_\Omega f\overline{u}_x dx.\end{aligned}\tag{3.15}$$

Without loss of generality, we may assume that $2\gamma\ge\alpha$. Applying (3.13)-(3.15), we get

$$\begin{aligned}&\frac{d}{dt}(2\|u_x\|^2+\|v\|^2+2\int_\Omega |u|^2 v dx-2\gamma\mathrm{Im}\int_\Omega u\overline{u}_x dx-2\mathrm{Re}\int_\Omega |u|^{\beta+1}dx\\&+2\mathrm{Re}(f,u))+\alpha(2\|u_x\|^2+\|v\|^2+2\int_\Omega |u|^2 v dx-2\gamma\mathrm{Im}\int_\Omega u\overline{u}_x dx\\&-2\mathrm{Re}\int_\Omega |u|^{\beta+1}dx+2\mathrm{Re}(f,u))\\=&-\alpha\|v\|^2+2\gamma(2\gamma-\alpha)\mathrm{Im}\int_\Omega u\overline{u}_x dx+4\gamma\mathrm{Re}\int_\Omega f\overline{u}_x dx+2(g,v).\end{aligned}\tag{3.16}$$

Let

$$\begin{aligned}\varphi(u,v)=&2\|u_x\|^2+\|v\|^2+2\int_\Omega |u|^2 v dx-2\gamma\mathrm{Im}\int_\Omega u\overline{u}_x dx\\&-2\mathrm{Re}\int_\Omega |u|^{\beta+1}dx+2\mathrm{Re}(f,u),\\ \phi(u,v)=&-\alpha\|v\|^2+2\gamma(2\gamma-\alpha)\mathrm{Im}\int_\Omega u\overline{u}_x dx+4\gamma\mathrm{Re}\int_\Omega f\overline{u}_x dx+2(g,v),\end{aligned}$$

and then (3.16) can be written in the following form

$$\frac{d}{dt}\varphi(u,v)+\alpha\varphi(u,v)=\phi(u,v).\tag{3.17}$$

We choose some suitable $\varepsilon_i (i = 1, 2, 3)$ and use Young's inequality to obtain that

$$\begin{aligned}\phi(u,v) =& -\alpha\|v\|^2 + 2\gamma(2\gamma-\alpha)\text{Im}\int_\Omega u\overline{u}_x dx + 4\gamma\text{Re}\int_\Omega f\overline{u}_x dx + 2(g,v)\\ \le& 2\gamma(2\gamma-\alpha)\text{Im}\int_\Omega u\overline{u}_x dx + 4\gamma\text{Re}\int_\Omega f\overline{u}_x dx + 2(g,v)\\ \le& \varepsilon_1\|u_x\|^2 + C(\varepsilon_1)\|u\|^2 + \varepsilon_2\|u_x\|^2 + C(\varepsilon_2)\|f\|^2 + \varepsilon_3\|v\|^2 + C(\varepsilon_3)\|g\|^2\\ \le& \frac{\alpha}{2}(\|u_x\|^2 + \|v\|^2) + C\\ \le& \frac{\alpha}{2}\varphi(u,v) + C,\end{aligned} \tag{3.18}$$

where $C = C(\varepsilon_1, \varepsilon_2, \varepsilon_3, \|u\|, \|f\|, \|g\|)$. So (3.17) can be written as follows

$$\frac{d}{dt}\varphi(u,v) + \frac{\alpha}{2}\varphi(u,v) \le C. \tag{3.19}$$

By using the Gronwall inequality as follows

$$\varphi(u,v) \le \varphi(u_0,v_0)e^{-\frac{\alpha}{2}t} + \frac{2C}{\alpha}(1 - e^{-\frac{\alpha}{2}t}). \tag{3.20}$$

By using Hölder's inequality, we get

$$\begin{aligned}\int_\Omega |u|^2 dx \le (\int_\Omega |u|^2 dx)^{\frac{1}{2}} \le \ldots \le (\int_\Omega |u|^{2(\beta+1)} dx)^{\frac{1}{\beta+1}},\\ \|u\|^{2(\beta+1)} \le \int_\Omega |u|^{2(\beta+1)} dx.\end{aligned} \tag{3.21}$$

Now we estimate the value of $\varphi(u,v)$,

$$\begin{aligned}\varphi(u,v) =& 2\|u_x\|^2 + \|v\|^2 + 2\int_\Omega |u|^2 v dx - 2\gamma\text{Im}\int_\Omega u\overline{u}_x dx\\ &- 2\text{Re}\int_\Omega |u|^{\beta+1} dx + 2\text{Re}(f,u),\\ \ge& 2\|u_x\|^2 + \|v\|^2 - \varepsilon_1\|u_x\|^2 - \varepsilon_2\|v\|^2\\ &- C(\varepsilon_1,\varepsilon_2)(\|u\|^4 + \|u\|^6 + \|u\|^{\beta+1} + \|f\|^2)\\ \ge& \frac{1}{2}(\|u_x\|^2 + \|v\|^2) - C.\end{aligned} \tag{3.22}$$

By using (3.20) and (3.22) we have

$$\|u_x\|^2 + \|v\|^2 \le C, \quad \forall t \ge t_2, \tag{3.23}$$

where $C = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t})$, $t_2 = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t}, \|W_\tau\|_{H^1\times H})$. □

Lemma 3.3. *If $W_\tau \in E_0$, $\beta \ge \frac{3}{2}$ and $Y(x,t)$ satisfy Definition 2.6, then for the weak solution of the problem (1.1)-(1.4), we have*

$$\|W(t)\|^2_{H^2\times H^1} \le C_3, \quad \forall t \ge t_3, \tag{3.24}$$

where $C_3 = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t})$, $t_3 = C(\alpha, \beta, \gamma, f, g, Y_0, f_{0t}, \|W_\tau\|_{E_0})$.

Proof. Taking the inner product of (1.1) with u_{xx} and taking the real part, we get that

$$\|u_{xx}\|^2 - \mathrm{Re}\int_\Omega vu\overline{u}_{xx}dx + \mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx = \mathrm{Re}\int_\Omega f\overline{u}_{xx}dx. \tag{3.25}$$

Taking the inner product of (1.1) with u_{xxt} and taking the real part, we get that

$$\frac{1}{2}\frac{d}{dt}\|u_{xx}\|^2 - \mathrm{Re}\int_\Omega vu\overline{u}_{xxt}dx + \mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xxt}dx = \mathrm{Re}\int_\Omega f\overline{u}_{xxt}dx. \tag{3.26}$$

Since

$$\mathrm{Re}\frac{d}{dt}(f, u_{xx}) = \mathrm{Re}(f_t, u_{xx}) + \mathrm{Re}(f, u_{xxt}), \tag{3.27}$$

$$\mathrm{Re}\int_\Omega uv\overline{u}_{xxt}dx = \frac{d}{dt}\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx - \mathrm{Re}\int_\Omega (uv)_t\overline{u}_{xx}dx, \tag{3.28}$$

$$\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xxt}dx = \frac{d}{dt}\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx - \beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_t\overline{u}_{xx}dx. \tag{3.29}$$

Note that, by (1.1),

$$\begin{aligned}
&Re(f_t, u_{xx})\\
=&\mathrm{Re}\int_\Omega f_t\overline{u}_{xx}dx\\
=&\mathrm{Re}\int_\Omega (iu_{tt} + u_{xxt} - u_t v - uv_t + \beta|u|^{\beta-1}u_t + i\gamma u_t)\overline{u}_{xx}dx\\
=&\int_\Omega (\mathrm{Re}u_{xxt}\overline{u}_{xx} - \mathrm{Re}u_t v\overline{u}_{xx} - \mathrm{Re}uv_t\overline{u}_{xx} + \mathrm{Re}\beta|u|^{\beta-1}u_t\overline{u}_{xx})dx\\
=&\frac{1}{2}\frac{d}{dt}\|u_{xx}\| - \mathrm{Re}\int_\Omega (uv)_t\overline{u}_{xx}dx + \beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_t\overline{u}_{xx}dx.
\end{aligned} \tag{3.30}$$

From (3.25) and (3.26), we can infer that

$$\begin{aligned}
&\frac{d}{dt}(\|u_{xx}\|^2 - 2\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx + 2\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx - 2\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx)\\
&+2\alpha\|u_{xx}\|^2 - 2\alpha\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx + 2\alpha\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx - 2\alpha\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx\\
&+2\mathrm{Re}\int_\Omega u_t v\overline{u}_{xx}dx + 2\mathrm{Re}\int_\Omega uv_t\overline{u}_{xx}dx - 2\mathrm{Re}\int_\Omega f_t\overline{u}_{xx}dx = 0.
\end{aligned} \tag{3.31}$$

From (1.1) we can get

$$\begin{aligned}
&\mathrm{Re}\int_\Omega u_t v\overline{u}_{xx}dx\\
=&-\mathrm{Re}\int_\Omega i(f - u_{xx} + uv - |u|^{\beta-1}u - i\gamma u)v\overline{u}_{xx}dx\\
=&\mathrm{Im}\int_\Omega fv\overline{u}_{xx}dx + \mathrm{Im}\int_\Omega uv^2\overline{u}_{xx}dx - \gamma\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx - \mathrm{Im}\int_\Omega |u|^{\beta-1}uv\overline{u}_{xx}dx.
\end{aligned}$$

From (1.2) we can get

$$
\begin{aligned}
&\mathrm{Re}\int_\Omega uv_t\overline{u}_{xx}dx\\
=&\mathrm{Re}\int_\Omega u(-\alpha v-\gamma|u|_x^2+g)\overline{u}_{xx}dx\\
=&-\alpha\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx-\gamma\mathrm{Re}\int_\Omega u|u|_x^2\overline{u}_{xx}dx+\mathrm{Re}\int_\Omega gu\overline{u}_{xx}dx.
\end{aligned}
$$

Inserting the above two equalities into (3.31) we obtain

$$
\begin{aligned}
&\frac{d}{dt}(\|u_{xx}\|^2-2\mathrm{Re}\int_\Omega vu\overline{u}_{xx}dx+2\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx-2\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx)\\
&+2\alpha\|u_{xx}\|^2-(4\alpha+2\gamma)\mathrm{Re}\int_\Omega vu\overline{u}_{xx}dx+2\alpha\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx\\
&-2\alpha\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx+2\mathrm{Im}\int_\Omega fv\overline{u}_{xx}dx+2\mathrm{Im}\int_\Omega uv^2\overline{u}_{xx}dx\\
&-2\mathrm{Im}\int_\Omega |u|^{\beta-1}uv\overline{u}_{xx}dx-2\gamma\mathrm{Re}\int_\Omega u|u|_x^2\overline{u}_{xx}dx\\
&+2\mathrm{Re}\int_\Omega gu\overline{u}_{xx}dx-2\mathrm{Re}\int_\Omega f_t\overline{u}_{xx}dx=0. \qquad (3.32)
\end{aligned}
$$

We differentiate (1.2) with respect to x and take the inner product with v_x to get

$$
\frac{1}{2}\frac{d}{dt}\|v_x\|^2+\alpha\|v_x\|^2+2\gamma\mathrm{Re}\int_\Omega u\overline{u}_{xx}v_xdx+2\gamma\int_\Omega |u_x^2|v_xdx=\int_\Omega g_x\overline{v}_xdx. \qquad (3.33)
$$

Note that,

$$
\mathrm{Re}\int_\Omega u\overline{u}_{xx}v_xdx=\mathrm{Re}\int_\Omega (uv)_x\overline{u}_{xx}dx-\mathrm{Re}\int_\Omega u_x\overline{u}_{xx}vdx,
$$

by (1.1), we get

$$
\begin{aligned}
&\mathrm{Re}\int_\Omega (uv)_x\overline{u}_{xx}dx\\
=&\mathrm{Re}\int_\Omega (iu_{xt}+u_{xxx}-i\gamma u_x-f_x+\beta|u|^{\beta-1}u_x)\overline{u}_{xx}dx\\
=&-\mathrm{Im}\int_\Omega u_{xt}\overline{u}_{xx}dx-\gamma\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx-\mathrm{Re}\int_\Omega f_x\overline{u}_{xx}dx\\
&+\beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_x\overline{u}_{xx}dx,
\end{aligned}
$$

through integrating by parts yields, we have

$$
\mathrm{Im}\int_\Omega u_{xt}\overline{u}_{xx}dx=\frac{1}{2}\frac{d}{dt}\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx,
$$

and then, we obtain that

$$
\begin{aligned}
\mathrm{Re}\int_\Omega (uv)_x\overline{u}_{xx}dx=&-\frac{1}{2}\frac{d}{dt}\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx-\gamma\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx\\
&-\mathrm{Re}\int_\Omega f_x\overline{u}_{xx}dx+\beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_x\overline{u}_{xx}dx.
\end{aligned}
$$

Therefore, we have

$$\mathrm{Re}\int_\Omega u\overline{u}_{xx}v_x dx = -\frac{1}{2}\frac{d}{dt}\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx - \gamma\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx - \mathrm{Re}\int_\Omega f_x\overline{u}_{xx}dx + \beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_x\overline{u}_{xx}dx - \mathrm{Re}\int_\Omega u_x\overline{u}_{xx}vdx.$$

Inserting the above equalities into (3.33) we obtain

$$\frac{d}{dt}(\|v_x\|^2 - 2\alpha\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx) + 2\alpha\|v_x\|^2 - 4\alpha\gamma\mathrm{Im}\int_\Omega u_x\overline{u}_{xx}dx$$
$$-4\alpha\mathrm{Re}\int_\Omega u_x v\overline{u}_{xx}dx - 4\alpha\mathrm{Re}\int_\Omega f_x\overline{u}_{xx}dx + 4\alpha\beta\mathrm{Re}\int_\Omega |u|^{\beta-1}u_x\overline{u}_{xx}dx$$
$$+4\int_\Omega |u_x|^2\overline{v}_x dx - 2\int_\Omega g_x\overline{v}_x dx = 0. \tag{3.34}$$

By some basic calculation from (3.32) and (3.34), we have

$$\frac{d}{dt}\varphi_1(u,v) + \alpha\varphi_1(u,v) = \phi_1(u,v), \tag{3.35}$$

where

$$\varphi_1(u,v) = 2\|u_{xx}\|^2 + \|v_x\|^2 - 2\mathrm{Re}\int_\Omega vu\overline{u}_{xx}dx - 2\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx + 2\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx,$$
$$\phi_1(u,v) = -\alpha\|v_x\|^2 - 4\gamma\int_\Omega |u_x|^2 v_x dx - 4\gamma\mathrm{Re}\int_\Omega uv_x\overline{u}_{xx}dx + 2\int_\Omega g_x\overline{v}_x dx.$$

Next, Gagliardo-Nirenberg inequality, Young's inequality and Holder's inequality are used to estimate $\phi_1(u,v)$.

$$\begin{aligned}\int_\Omega |u_x|^2 v_x dx \le& \varepsilon\int_\Omega |u_x|^4 dx + C(\varepsilon)\|v_x\|^2\\ \le& \varepsilon\|u_{xx}\|^2 + C(\varepsilon)(\|u_x\|^4 + \|u_x\|^6) + C(\varepsilon)\|v_x\|^2\\ \le& \varepsilon\|u_{xx}\|^2 + C(\varepsilon)\|v_x\|^2 + C,\end{aligned} \tag{3.36}$$

$$\begin{aligned}\mathrm{Re}\int_\Omega g_x\overline{v}_x dx \le& \|g_x\|\|v_x\|\\ \le& \varepsilon\|v_x\|^2 + C(\varepsilon)\|g_x\|^2,\end{aligned} \tag{3.37}$$

$$\begin{aligned}\mathrm{Re}\int_\Omega uv_x\overline{u}_{xx}dx \le& |u\|_\infty\|v_x\|\|u_{xx}\|\\ \le& \varepsilon\|v_x\|^2 + C(\varepsilon)\|u_{xx}\|^2.\end{aligned} \tag{3.38}$$

From the above estimates, we can get

$$\begin{aligned}\phi_1(u,v) \le& \frac{\alpha}{2}(\|u_x\|^2 + \|v\|^2) + C\\ \le& \frac{\alpha}{2}\varphi_1(u,v) + C.\end{aligned} \tag{3.39}$$

So (3.35) can be written as follows:

$$\frac{d}{dt}\varphi_1(u,v)+\frac{\alpha}{2}\varphi_1(u,v)\le C, \tag{3.40}$$

where $C=C(\alpha,\beta,\gamma,\|u\|_{H^1},\|v\|_H,\|f\|_{H^1},\|g\|_{H^1})$. So using Gronwall inequality, we can get that

$$\varphi_1(u,v)\le e^{-\frac{\alpha}{2}t}\varphi_1(u_0,v_0)+\frac{2C}{\alpha}(1-e^{-\frac{\alpha}{2}t}). \tag{3.41}$$

Since $2\beta-3\ge 0$ for $\beta\ge\frac{3}{2}$, using the formula of integration by parts, we have

$$\begin{aligned}
&\mathrm{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx\\
=&-Re\int_\Omega |u|^{\beta-1}|u_x|^2dx-(\beta-1)\mathrm{Re}\int_\Omega |u|^{\beta-3}u|u_x|^2dx\\
\ge&-\|u\|^{\beta-1}_{2(\beta-1)}\|u_x\|_4^2-(\beta-1)\|u\|_\infty^{\beta-3}\|u\|\|u_x\|_4^2\\
\ge&-C\|u\|^{\frac{\beta}{2}}\|u_x\|^{\frac{2\beta-1}{4}}\|u_{xx}\|^{\frac{5}{4}}-C\|u\|^{\frac{\beta-1}{2}}\|u_x\|^{\frac{2\beta-3}{4}}\|u_{xx}\|^{\frac{5}{4}}\\
\ge&-\varepsilon_1\|u_{xx}\|^2-C(\varepsilon_1)\|u\|^{\frac{4\beta}{3}}\|u_x\|^{\frac{2(2\beta-1)}{3}}-\varepsilon_2\|u_{xx}\|^2-C(\varepsilon_2)\|u\|^{\frac{4(\beta-1)}{3}}\|u_x\|^{\frac{2(2\beta-3)}{3}}\\
\ge&-(\varepsilon_1+\varepsilon_2)\|u_{xx}\|^2-C(\varepsilon_1,\varepsilon_2).
\end{aligned} \tag{3.42}$$

And then, we have

$$\begin{aligned}
&\mathrm{Re}\int_\Omega uv\overline{u}_{xx}dx\\
\le&\varepsilon_3\|u_{xx}\|^2+C(\varepsilon_3)(\int_\Omega |v|^4dx+\int_\Omega |u|^4dx)\\
\le&\varepsilon_3\|u_{xx}\|^2+C(\varepsilon_3)(\|v\|^3\|v_x\|+\|u\|^3\|u_x\|),\\
&\mathrm{Re}\int_\Omega f\overline{u}_{xx}dx\\
\le&\varepsilon_3\|u_{xx}\|^2+C(\varepsilon_3)\|f\|^2.
\end{aligned} \tag{3.43}$$

By using the above estimates and choosing the appropriate $\varepsilon_i(i=1,2,3)$, we get

$$\varphi_1(u,v)\ge\frac{1}{2}(\|u_{xx}\|^2+\|v_x\|^2)-C. \tag{3.44}$$

And from the discussion of (3.41) and (3.44), we get

$$\|W(t)\|^2_{H^2\times H^1}\le C,\quad \forall t\ge t_3, \tag{3.45}$$

where $C=C(\alpha,\beta,\gamma,f,g,Y_0,f_{0t})$, $t_3=C(\alpha,\beta,\gamma,f,g,Y_0,f_{0t},\|W_\tau\|_{E_0})$. □

4. Existence and Uniqueness of the Solution

In this section, we will prove that the system (1.1)-(1.4) has a unique global weak solution. Since a prior estimate of the solution has been established in section 3, the existence of the solution can be easily obtained by Galërkin's method(see [16,17,26,29]). In this section, we will show the unique existence theorem and give a simple proof.

Theorem 4.1. *Assume that* $\beta \geq \frac{3}{2}$ *and* $Y(x,t)$ *satisfy Definition 2.6, for each* $W_\tau \in E_0$*, system (1.1)-(1.4) has a unique global weak solution* $W(x,t) \in L^\infty(\tau, t; E_0)$*,* $\forall T > \tau$.

Proof. We will prove the existence and uniqueness of a global solution respectively.

Step 1. The existence of solution

Using Galërkin's method, we have the following approximate solution to approach the solution of (1.1)-(1.4):

$$(u^l, v^l) = \sum_{j=1}^{l} \omega_j^l(t)\eta_j(x), \tag{4.1}$$

where $\{\eta_j\}_{j=1}^l$ is a orthogonal basis of $H(\Omega)$, and (u^l, v^l) satisfies

$$\begin{aligned} (iu_t^l + u_{xx}^l - u^l v^l + |u^l|^{\beta-1}u^l + i\gamma u^l, \eta_j) &= (f(x,t), \eta_j), \\ (v_t^l + \alpha v^l + \gamma |u^l|_x^2, \eta_j) &= (g(x,t), \eta_j), \\ (W^l(x,\tau)\eta_j) &= (W_\tau, \eta_j), \\ W^l(x+2D, t) &= W^l(x,t). \end{aligned} \tag{4.2}$$

We get that (4.2) is an initial-boundary value problem of ordinary differential equations. According to the standard existence of ordinary differential equations and the priori estimates in Section 3, we obtain the unique solution for (4.2). Similar to [13, 26], we get

$$\{W^l\}_{l=1}^\infty \stackrel{*}{\rightharpoonup} W(x,t) \text{ in } L^\infty(\tau, t; E_0), \quad \forall T > \tau,$$

where $\stackrel{*}{\rightharpoonup}$ means weak star convergence.

Step 2. The uniqueness of solution.

Suppose that $W_1(x,t) = (u_1, v_1)$ and $W_2(x,t) = (u_2, v_2)$ are two solutions of (1.1)-(1.4). Let $W(x,t) = (u,v) = (u_1,v_1) - (u_2,v_2)$, then $W(x,t)$ satisfy

$$\begin{aligned} &iu_t + u_{xx} + u_2 v_2 - u_1 v_1 \\ &+|u_1|^{\beta-1}u_1 - |u_2|^{\beta-1}u_2 + i\gamma u = 0, \\ &v_t + \alpha v + \gamma(|u_1|_x^2 - |u_2|_x^2) = 0, \\ &W|_{t=\tau} = 0, \quad W|_{\partial\Omega} = 0. \end{aligned} \tag{4.3}$$

Similar to [13, 26], we can get that $\|W\|_2 = 0$. Therefore, we complete the proof of the theorem. □

5. Uniform Absorbing Set and Uniform Attractor

In this section, we will prove the existence of the strong compact uniform attractor of problem (1.1)-(1.4) applying Ball et al.'s idea (see [3, 26]). Firstly, we construct a bounded uniformly absorbing set. Next, we prove the the existence of weakly compact uniform attractor of the system. Lastly, we derive that the weak uniform attractor is actually the strong one. In this section, "$\rightharpoonup$" represents weak convergence, "$\stackrel{*}{\rightharpoonup}$" represents weak star convergence, and $\{U_{\sigma\in\sum}(t,\tau)\}$ represents a family of processes in E_0 satisfying definition 2.2.

Theorem 5.1. *Suppose $\beta \geq \frac{3}{2}$ and $\{U_{\sigma\in\sum}(t,\tau)\}$ is a family of processes in E_0. Under assumptions of Theorem 4.1, then it admits a strong compact uniform attractor $\mathcal{A}_{\sum}$.*

Proof. We prove this theorem by three steps.
Step 1. $\{U_{\sigma\in\sum}(t,\tau)\}$ possess a bounded uniformly absorbing set in E_0.

Let $B_0 = \{W \in E_0;\ \|W\|^2_{E_0} \leq C(\alpha,\beta,\gamma,\|Y_0\|_{L^2_c(\mathbb{R};\sum_0)},\|f_{0t}\|_{L^2(\mathbb{R};H^1)},\|W_\tau\|_{E_0})\}$. By Theorem 4.1, B_0 is a bounded absorbing set of the process $U_{\sigma=Y_0}$.

By Definition 2.6, we know that $\|Y\|_{L^2_c(\mathbb{R};\sum_0)} \leq \|Y_0\|_{L^2_c(\mathbb{R};\sum_0)}$ for $\forall Y \in \sum$, hold. So the solution of(1.1)-(1.4) satisfies

$$\begin{aligned}\|W\|^2_{E_0} \leq & C(\alpha,\beta,\gamma,\|Y\|_{L^2_c(\mathbb{R};\sum_0)},\|f_{0t}\|_{L^2(\mathbb{R};H^1)},\|W_\tau\|_{E_0})\\ \leq & C(\alpha,\beta,\gamma,\|Y_0\|_{L^2_c(\mathbb{R};\sum_0)},\|f_{0t}\|_{L^2(R\mathbb{R};H^1)},\|W_\tau\|_{E_0}).\end{aligned} \tag{5.1}$$

Then we get the set B_0 is a bounded uniformly absorbing set of $\{U_{\sigma\in\sum}(t,\tau)\}$.
Step 2. We show the weak compact uniform attractor $\mathcal{A}_{\sum}$ in E_0.

We need to show that $\{U_{\sigma\in\sum}(t,\tau)\}$ is $(E_0\times\sum, E_0)$-continuous by Lemma 2.1, Theorem 4.1, and Step 1, i.e. for any fixed $t_1 \geq \tau \in \mathbb{R}$, let

$$(W_{\tau k},\sigma_k) \rightharpoonup (W_\tau,\sigma) \ \text{ in } E_0 \times \sum. \tag{5.2}$$

We need to get

$$W_{\sigma_k}(t_1) \rightharpoonup W_\sigma(t_1) \ \text{ in } E_0, \tag{5.3}$$

where $W_{\sigma_k}(t_1) = (u_k(t_1), v_k(t_1)) = U_{\sigma_k}(t_1,\tau)W_{\tau_k}$ and $W_\sigma(t_1) = (u(t_1),v(t_1)) = U_\sigma(t_1,\tau)W_\tau$. By Lemma 2.3, Theorem 4.1 and (5.2), we have

$$\|W_{\tau_k}\|_{E_0} \leq C, \tag{5.4}$$

$$\sup_{t\in[\tau,T]} \|W_{\sigma_k}\|_{E_0} \leq C. \tag{5.5}$$

Then by Lemmas 3.1-3.3, we can see that

$$\|W_{\sigma_k}(t)\|_\infty \leq C, \quad \forall 0 \leq t \leq T. \tag{5.6}$$

Note that

$$iu_{kt} = (-\Delta)u_k + u_k v_k - |u_k|^{\beta-1}u_k - i\gamma u_k + f_k(x,t), \tag{5.7}$$

$$v_{kt} = -\alpha v_k - \gamma|u_k|^2_x + g_k(x,t), \tag{5.8}$$

and $\sigma_k = (f_k(x,t), g_k(x,t)) \in \sum$. According to (5.5) and (5.6), we can see that $\partial_t W_{\sigma_k}(t) \in L^\infty(\tau,T;L^2(\Omega)\times H^1(\Omega))$ and

$$\|\partial_t W_{\sigma_k}(t)\|_{L^\infty(\tau,T;L^2(\Omega)\times H^1(\Omega))} \leq C. \tag{5.9}$$

Because of Theorem 4.1 and (5.9), we can see that there exist a subsequence $\{W_{\sigma_{k_l}}(t)\}$ of $\{W_{\sigma_k}(t)\}$ and $\widetilde{W}(t) \triangleq (\widetilde{u}(t),\widetilde{v}(t)) \in L^\infty(\tau,T;E_0)$, such that

$$W_{\sigma_{k_l}}(t) \overset{*}{\rightharpoonup} \widetilde{W}(t) \ \text{ in } L^\infty(\tau,T;E_0), \tag{5.10}$$

$$\partial_t W_{\sigma_{k_l}}(t) \overset{*}{\rightharpoonup} \partial_t\widetilde{W}(t) \ \text{ in } L^\infty(\tau,T;L^2(\Omega)\times H^1(\Omega)). \tag{5.11}$$

Besides, by (5.5), there exists $W^0 \triangleq (u^0(t_1), v^0(t_1)) \in E_0$ for any $t_1 \in [\tau, T]$, such that

$$W_{\sigma_k}(t_1) \rightharpoonup W^0 \text{ in } E_0. \tag{5.12}$$

By (5.10) and compactness embedding theorem, we get

$$u_{k_l}(t) \rightharpoonup \widetilde{u}(t) \text{ in } L^2(\tau, T; H^1). \tag{5.13}$$

Next, we will to prove $\widetilde{W}(t)$ is a solution of problem (1.1)-(1.4). For all $\nu \in L^2(\Omega)$, $\forall \psi \in C_0^\infty(\tau, T)$, by (5.10) we have that

$$\begin{aligned}
&\int_\tau^T (iu_{k_l t}, \psi(t)\nu)dt + \int_\tau^T (\Delta u_{k_l}, \psi(t)\nu)dt \\
&- \int_\tau^T (v_{k_l} u_{k_l}, \psi(t)\nu)dt + \int_\tau^T (|u_{k_l}|^{\beta-1} u_{k_l}, \psi(t)\nu)dt \\
&+ \int_\tau^T (i\gamma u_{k_l}, \psi(t)\nu)dt - \int_\tau^T (f(x,t), \psi(t)\nu)dt = 0.
\end{aligned} \tag{5.14}$$

Since

$$\begin{aligned}
&\int_\tau^T (v_{k_l} u_{k_l}, \psi(t)\nu)dt - \int_\tau^T (\widetilde{u}\widetilde{v}, \psi(t)\nu)dt \\
=&\int_\tau^T ((u_{k_l} - \widetilde{u})v_{k_l}, \psi(t)\nu)dt + \int_\tau^T (\widetilde{u}(v_{k_l} - \widetilde{v}), \psi(t)\nu)dt,
\end{aligned} \tag{5.15}$$

by (5.6), (5.10) and (5.13),

$$\begin{aligned}
\int_\tau^T ((u_{k_l} - \widetilde{u})v_{k_l}, \psi(t)\nu)dt \leq& \sup_{0\leq t\leq T} \|v_{k_l}\|_{L^\infty} \|\psi(t)\nu\|_{L^2(0,T;L^2(\Omega))} \\
&\times \|u_{k_l} - \widetilde{u}\|_{L^2(0,T;L^2(\Omega))} \to 0, \\
\int_\tau^T (\widetilde{u}(v_{k_l} - \widetilde{v}), \psi(t)\nu)dt =& \int_\tau^T ((v_{k_l} - \widetilde{v}), \psi(t)\nu\overline{\widetilde{u}})dt \to 0.
\end{aligned}$$

Then we have

$$\int_\tau^T (v_{k_l} u_{k_l}, \psi(t)\nu)dt \ \to \ \int_\tau^T (\widetilde{u}\widetilde{v}, \psi(t)\nu)dt. \tag{5.16}$$

And by (5.10), we get

$$\begin{aligned}
&\int_\tau^T (\Delta u_{k_l}, \psi(t)\nu)dt - \int_\tau^T (\Delta\widetilde{u}, \psi(t)\nu)dt \\
\leq&\|(\Delta)(u_{k_l} - \widetilde{u})\|_{L^2(0,T;L^2(\Omega))} \times \|\psi(t)\nu\|_{L^2(0,T;L^2(\Omega))} \to 0.
\end{aligned}$$

By using the similar methods to the other terms of (5.14), we have

$$\begin{aligned}
&\int_\tau^T (i\widetilde{u}_t, \nu)\psi(t)dt + \int_\tau^T (\Delta\widetilde{u}, \nu)\psi(t)dt \\
&- \int_\tau^T (\widetilde{u}\widetilde{v}, \nu)\psi(t)dt + \int_\tau^T (|\widetilde{u}|^{\beta-1}\widetilde{u}, \nu)\psi(t)dt \\
&+ \int_\tau^T (i\gamma\widetilde{u}, \nu)\psi(t)dt - \int_\tau^T (f(x,t), \nu)\psi(t)dt = 0.
\end{aligned} \tag{5.17}$$

So, we can get that

$$i\widetilde{u}_t + \widetilde{u}_{xx} - \widetilde{u}\widetilde{v} + |\widetilde{u}|^{\beta-1}\widetilde{u} + i\gamma\widetilde{u} = f(x,t), \tag{5.18}$$

which shows that $(\widetilde{u}, \widetilde{v})$ satisfies (1.1).

For any $\nu \in L^2(\Omega)$, $\forall\psi \in C_0^\infty(\tau, T)$ with $\psi(T) = 0$, $\psi(\tau) = 1$, by (5.7) we find that

$$\begin{aligned} &-\int_\tau^T (iu_{k_l}, \nu)\psi'(t)dt - \int_\tau^T ((-\Delta)u_{k_l}, \nu)\psi(t)dt \\ &-\int_\tau^T (u_{k_l}v_{k_l}, \nu)\psi(t)dt + \int_\tau^T (|u_{k_l}|^{\beta-1}u_{k_l}, \nu)\psi(t)dt \\ &+\int_\tau^T (i\gamma u_{k_l}, \nu)\psi(t)dt - \int_\tau^T (f(x,t), \nu)\psi(t)dt = i(u_{k_l}(\tau), \nu). \end{aligned} \tag{5.19}$$

We know that (5.2) implies that

$$u_{k_l}(\tau) = u_{\tau k_l} \rightharpoonup u_\tau \quad \text{in } H^2. \tag{5.20}$$

Then from (5.19) and (5.20), we have

$$\begin{aligned} &-\int_\tau^T (i\widetilde{u}, \nu)\psi'(t)dt - \int_\tau^T ((-\Delta)\widetilde{u}, \nu)\psi(t)dt \\ &-\int_\tau^T (\widetilde{u}\widetilde{v}, \nu)\psi(t)dt + \int_\tau^T (|\widetilde{u}|^{\beta-1}\widetilde{u}, \nu)\psi(t)dt \\ &+\int_\tau^T (i\gamma\widetilde{u}, \nu)\psi(t)dt - \int_\tau^T (f(x,t), \nu)\psi(t)dt = i(u_\tau, \nu), \end{aligned} \tag{5.21}$$

while by (5.18) we have that

$$\begin{aligned} &-\int_\tau^T (i\widetilde{u}, \nu)\psi'(t)dt - \int_\tau^T ((-\Delta)\widetilde{u}, \nu)\psi(t)dt \\ &-\int_\tau^T (\widetilde{u}\widetilde{v}, \nu)\psi(t)dt + \int_\tau^T (|\widetilde{u}|^{\beta-1}\widetilde{u}, \nu)\psi(t)dt \\ &+\int_\tau^T (i\gamma\widetilde{u}, \nu)\psi(t)dt - \int_\tau^T (f(x,t), \nu)\psi(t)dt = i(\widetilde{u}(\tau), \nu). \end{aligned} \tag{5.22}$$

So by (5.21) and (5.22), we have that

$$(u_\tau, \nu) = (\widetilde{u}(\tau), \nu), \quad \forall\nu \in L^2(\Omega), \tag{5.23}$$

$$u_\tau = \widetilde{u}(\tau). \tag{5.24}$$

By (5.18) and (5.23), we get

$$\widetilde{u}(t) = u(t). \tag{5.25}$$

For any $\nu \in L^2(\Omega)$, $\forall\psi \in C_0^\infty(\tau, t_1)$, with $\psi(\tau) = 0$, $\psi(t_1) = 1$, then repeating the procedure of proofs of (5.19)-(5.22) by (5.12) we have

$$u^0(t_1) = \widetilde{u}(t_1). \tag{5.26}$$

From (5.12), (5.25) and (5.26), we have that

$$u_k(t_1) \rightharpoonup u(t_1) \ \text{ in } H^2(\Omega). \tag{5.27}$$

Similarly, we can also derive that

$$v_k(t_1) \rightharpoonup v(t_1) \ \text{ in } H^1(\Omega). \tag{5.28}$$

From (5.27) and (5.28), we deduce (5.3). We complete the proof of the step.

Step 3. We show the weakly compact uniform attractor $\mathcal{A}_{\sum}$ is actually the strong one. From the proof of Lemma 3.3 , we know each solution for problem (1.1)-(1.4) satisfies

$$\frac{d}{dt}(\|u_{xx}\|^2 + F(u,v)) + 2\alpha(\|u_{xx}\|^2 + F(u,v)) = G(u,v), \tag{5.29}$$

$$\frac{d}{dt}(\|v_x\|^2 + F_1(u,v)) + 2\alpha(\|v_x\|^2 + F_1(u,v)) = G_1(u,v), \tag{5.30}$$

where

$$\begin{aligned}
F(u,v) =& \|u_{xx}\|^2 - 2\text{Re}\int_\Omega uv\overline{u}_{xx}dx + 2\text{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx - 2\text{Re}\int_\Omega f\overline{u}_{xx}dx,\\
G(u,v) =& 2\gamma\text{Re}\int_\Omega uv\overline{u}_{xx}dx + 2\gamma\text{Re}\int_\Omega u|u|_x^2\overline{u}_{xx}dx - 2\text{Re}\int_\Omega gu\overline{u}_{xx}dx\\
&- 2\text{Im}\int_\Omega fv\overline{u}_{xx}dx - 2\text{Im}\int_\Omega uv^2\overline{u}_{xx}dx + 2\text{Im}\int_\Omega |u|^{\beta-1}uv\overline{u}_{xx}dx\\
&- 2\alpha\text{Re}\int_\Omega |u|^{\beta-1}u\overline{u}_{xx}dx + 2\alpha\text{Re}\int_\Omega f\overline{u}_{xx}dx,\\
F_1(u,v) =& -2\gamma\text{Im}\int_\Omega u_x\overline{u}_{xx}dx,\\
G_1(u,v) =& 4\alpha\text{Re}\int_\Omega u_xv\overline{u}_{xx}dx + 4\alpha\text{Re}\int_\Omega f_x\overline{u}_{xx}dx - 4\alpha\beta\text{Re}\int_\Omega |u|^{\beta-1}u_x\overline{u}_{xx}dx\\
&- 4\gamma\int_\Omega |u_x|^2v_xdx + 2\int_\Omega g_xv_xdx.
\end{aligned}$$

By the uniform boundedness and the compactness embedding, we have that F, G, and F_1, G_1 are all weakly continuous in $E_0 \times \sum$.

From step 2, we know that the point $(\omega, m) \in \mathcal{A}$ if and only if there exist two sequences $\{\omega_k^0, m_k^0\}_{k\in N}$ and $\{t_k\}_{k\in N}$ such that for all $\sigma(t) \in \sum$, it uniformly satisfies that

$$U_\sigma(t_k,\tau)(\omega_k^0, m_k^0) \rightharpoonup (\omega, m) \ \text{ in } E_0,\ k \to \infty, \tag{5.31}$$

where $t_k \to \infty$ as $k \to \infty$. If the weak convergence implies strong one, we obtain $\mathcal{A}_{\sum}$ is the strong compact attractor. For each fixed $h > \tau$, because of $t_k \to \infty$, we consider it as $h < t_k$, $k \in N_+$. By Lemma 3.3 and Theorem 4.1, $U_\sigma(t_k - h,\tau)(\omega_{k_l}^0, m_{k_l}^0)$ is bounded in E_0. Then there exists a subsequence $U_\sigma(t_{k_l} - h,\tau)(\omega_{k_l}^0, m_{k_l}^0)$ of $U_\sigma(t_k - h,\tau)(\omega_{k_l}^0, m_{k_l}^0)$ and a point $(n,p) \in E_0$, such that

$$U_\sigma(t_{k_l} - h,\tau)(\omega_{k_l}^0, m_{k_l}^0) \rightharpoonup (n,p) \ \text{ in } E_0. \tag{5.32}$$

Let

$$
\begin{aligned}
&(\omega_{k_l}(t), m_{k_l}(t)) \\
=&U_{T(t_{k_l}-h-\tau)\sigma}(t+\tau,\tau)U_\sigma(t_{k_l}-h,\tau)(\omega^0_{k_l}, m^0_{k_l}) \\
=&U_\sigma(t+t_{k_l}-h, t_{k_l}-h)U_\sigma(t_{k_l}-h,\tau)(\omega^0_{k_l}, m^0_{k_l}) \\
=&U_\sigma(t+t_{k_l}-h,\tau)(\omega^0_{k_l}, m^0_{k_l}),
\end{aligned} \tag{5.33}
$$

where $T(\cdot)$ is the translation operator on $\sum$. Since $\sigma(t)$ is translation compact symbol, there exists a symbol $\sigma^* \in \sum$ such that

$$
T(t_{k_l}-h-\tau)\sigma \to \sigma^* \ \text{ in } \sum. \tag{5.34}
$$

Then by (5.31) and(5.32), and the weak $(E\times\sum)$-continuity of $U_{\sigma\in\sum}(t,\tau)$, we can get that

$$
(\omega_{k_l}(t), m_{k_l}(t)) \rightharpoonup U_{\sigma^*}(t,\tau)(n,p) = (\omega, m) \ \text{ in } E_0, \ \forall t>\tau. \tag{5.35}
$$

From (5.32), we can know that the solution trajectory $(\omega_{k_l}(t), m_{k_l}(t))$ is created by $U_{T(t_{k_l}-h-\tau)\sigma}\ (t+\tau,\tau)$ starting at $U_\sigma(t_{k_l}-h,\tau)(\omega^0_{k_l}, m^0_{k_l})$. By (5.29), (5.32), (5.34), we have that

$$
\begin{aligned}
&\|\omega_{k_l}(t)\|^2_{H^2} + F(\omega_{k_l}(t), m_{k_l}(t)) \\
=&e^{-2\alpha(t-\tau)}(\|U_\sigma(t_{k_l}-h,\tau)\omega^0_{k_l}\|^2_{H^2} \\
&+ F(U_\sigma(t_{k_l}-h,\tau)\omega^0_{k_l}, U_\sigma(t_{k_l}-h,\tau)m^0_{k_l})) \\
&+ \int_\tau^t e^{-2\alpha(t-\tau)}G(\omega^0_{k_l}(s), m^0_{k_l}(s))ds \\
=&e^{-2\alpha(t-\tau)}(\|U_\sigma(t_{k_l}-h,\tau)\omega^0_{k_l}\|^2_{H^2} + F(n,p)) \\
&+ \int_\tau^t e^{-2\alpha(t-\tau)}G(U_{\sigma^*}(s+\tau,\tau)(n,p))ds.
\end{aligned} \tag{5.36}
$$

Let $t=h$ in (5.35). Since F and G are weakly continuous in E_0, $\|U_\sigma(t_{k_l}-h,\tau)(\omega^0_{k_l}, m^0_{k_l})\|^2_{H^2} \le C$, and the Lebesgue dominated convergence theorem, we can obtain that

$$
\begin{aligned}
&\limsup_{k_l\to\infty} \|U_\sigma(t_{k_l}-h,\tau)\omega^0_{k_l}\|^2_{H^2} + F(U_{\sigma^*}(h+\tau,\tau)(n,p)) \\
\le& e^{-2\alpha(h-\tau)}(C+F(n,p)) + \int_\tau^h e^{-2\alpha(h-\tau)}G(U_{\sigma^*}(s+\tau,\tau)(n,p))ds.
\end{aligned} \tag{5.37}
$$

Since $(\omega, m) = U_{\sigma^*}(S+\tau,\tau)(n,p)$, we can see the solution (ω, m) about h corresponding to the initial data (n,p) and the symbol σ^*. Similarly to (5.35), we get

$$
\begin{aligned}
\|\omega\|^2_{H^2} + F(\omega, m) =& e^{-2\alpha(h-\tau)}(\|n\|^2_{H^2} + F(n,p)) \\
&+ \int_\tau^h e^{-2\alpha(h-\tau)}G(U_{\sigma^*}(s+\tau,\tau)(n,p))ds.
\end{aligned} \tag{5.38}
$$

Combining (5.37) and (5.38), we have

$$\begin{aligned}&\limsup_{k_l\to\infty}\|U_\sigma(t_{k_l},\tau)\omega_{k_l}^0\|_{H^2}^2\\ \le&\|\omega\|_{H^2}^2+Ce^{-2\alpha(h-t)}-e^{-2\alpha(h-t)}\|n\|_{H^2}^2\\ \le&\|\omega\|_{H^2}^2+Ce^{-2\alpha(h-t)}.\end{aligned}\tag{5.39}$$

As $h\to\infty$, we get

$$\limsup_{k_l\to\infty}\|U_\sigma(t_{k_l}-h,\tau)\omega_{k_l}^0\|_{H^2}^2\le\|\omega\|_{H^2}^2.\tag{5.40}$$

Similarly, the weak convergence $U_\sigma(t_k-h,\tau)\omega_k^0\rightharpoonup\omega$ implies that

$$\liminf_{k_l\to\infty}\|U_\sigma(t_{k_l}-h,\tau)\omega_{k_l}^0\|_{H^2}^2\ge\|\omega\|_{H^2}^2.\tag{5.41}$$

From the above two inequalities, we get that

$$\lim_{k_l\to\infty}\|U_\sigma(t_k-h,\tau)\omega_k^0\|_{H^2}^2=\|\omega\|_{H^2}^2.\tag{5.42}$$

Similar to the proof above, we can also derive that

$$\lim_{k_l\to\infty}\|U_\sigma(t_k-h,\tau)m_k^0\|_{H^2}^2=\|m\|_{H^2}^2.\tag{5.43}$$

Then, we get that $U_\sigma(t_k,\tau)(\omega_k^0,m_k^0)\to(\omega,m)$ in E_0. The proof of Theorem 5.1 is completed. □

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References

[1] X. Y. Abdallah, Y. Ahmed, R. Wannan and C. Y. Sun, *Second order non-autonomous lattice systems and their uniform attractors*, Commun. Pure Appl. Anal., 2019, 18(4), 1827–1846.

[2] G. S. Aragao, F. M. Bezerra, *Upper semicontinuity of the pullback attractors of non-autonomous damped wave equations with terms concentrating on the boundary*, J. Math. Anal. Appl., 2018, 462(1), 871–899.

[3] J. M. Ball, *Global attractors for damped semilinear wave equations*, Discrete Contin. Dyn. Syst., 2004, 10(1–2), 31–52.

[4] D. Bekiranov, T. Ogawa and G. Ponce, *On the well-posedess of Benney's interaction equation of short and long waves*, Adv. Difference Equ., 1996, 1(6), 919–937.

[5] D. Bekiranov, T. Ogawa and G. Ponce, *Interaction equations for short and long dispersive waves*, Journal of Functional Analysis, 1998, 158(2), 357–388.

[6] D. J. Benney, *A general theory for interactions between short and long waves*, Studies in Appl. Math., 1977, 56(1), 81–94.

[7] V. V. Chepyzhov and M. I. Vishik, *Non-autonomous evolutionary equations with translation-compact symbols and their attractors*, C. R. Acad. Sci. Paris Sér. I Math., 1995, 321(2), 153–158.

[8] V. V. Chepyzhov, M. I. Vishik, *Attractors of nonautonomous dynamical systems and their dimension*, J. Math. Pures Appl., 1994, 73(3), 279–333.

[9] H. Y. Cui, J. Xin, A. R. Li, *Weakly compact uniform attractor for the nonautonomous long-short wave equations*, Abstr. Appl. Anal., 2013, DOI: 10.1155/2013/601325.

[10] X. Y. Du, B. L. Guo, *The long time behavior of the long-short wave equation in* **R**, Acta Math. Appl. Sin., 2005, 28(4), 723–734.

[11] H. J. Gao and G. G. Lin, *Attractor for weakly damped driven long wave-short wave resonance equation*, Appl. Math. J. Chinese Univ. Ser. B, 1998, 13(4), 377–384.

[12] R. H. J. Grimshaw, *The modulation of an internal gravity-wave packet, and the resonance with the mean motion*, Studies in Appl. Math., 1976/77, 56(3), 241–266.

[13] B. L. Guo, *The global solution for one class of the system of* LS *nonlinear wave interaction*, J. Math. Res. Exposition, 1987, 7(1), 69–76.

[14] B. L. Guo and L. Chen, *Orbital stability of solitary waves of the long wave-short wave resonance equations*, Math. Methods Appl. Sci., 1998, 21(10), 883–894.

[15] B. L. Guo and B. X. Wang, *Attractors for the long-short wave equations*, J. Partial Diferential Equations, 1998, 11(4), 361–383.

[16] B. L. Guo and B. X. Wang, *The global solution and its long time behavior for a class of generalized LS type equations*, Progr. Natur. Sci., 1996, 6(5), 533–546.

[17] B. L. Guo, *Infinite Dimensional Dynamical Systems*, National Defene Industry Press, Beijing, 2000(in Chinese).

[18] L. T. Hoang, E. J. Olson and J. C. Robinson, *Continuity of pullback and uniform attractors*, J. Differential Equations, 2018, 264(6), 4067–4093.

[19] Y. S. Li, *Long time behavior for the weakly damped driven long-wave-short-wave resonance equations*, J. Differential Equations, 2006, 223(2), 261–289.

[20] H. Liu and H. J. Gao, *Ergodicity and dynamics for the stochastic 3D Navier-Stokes equations with damping*, Commun. Math. Sci., 2018, 16(1), 97–122.

[21] H. Lu and J. Xin, *The existence of a global attractor for the (2+1)-dimensional long wave-short wave resonance interaction equation*, Nonlinear Anal., 2010, 73(12), 3767–3778.

[22] D. R. Nicholson and M. V. Goldman, *Damped nonlinear Schrödinger equation*, Phys. Fluids, 1976, 19(10), 1621–1625.

[23] R. Samprogna and J. Simsen, *A selected pullback attractor*, J. Math. Anal. Appl., 2018, 468(1), 364–375.

[24] M. Tsutsumi and S. Hatano, *Well-posedness of the Cauchy problem for Benney's first equations of long wave short wave interactions*, Funkcial. Ekvac., 1994, 37(2), 289–316.

[25] M. Tsutsumi and S. Hatano, *Well-posedness of the Cauchy problem for the long wave-short wave resonance equations*, Nonlinear Anal., 1994, 22(2), 155–171.

[26] J. Xin, B. L. Guo, Y. Q. Han and D. W. Huang, *The global solution of the (2+1)-dimensional long wave-short wave resonance interaction equation*, J. Math. Phys., 2008, 49(7), 13 pp.

[27] M. Zgurovsky, M. Gluzman, N. Gorban, P. Kasyanov, L. Paliichuk, O. Khomenko, *Uniform global attractors for non-autonomous dissipative dynamical systems*, Discrete Contin. Dyn. Syst. Ser. B, 2017, 22(5), 2053–2065.

[28] R. F. Zhang, *Existence of global attractor for LS type equations*, J. Math. Res. Expositi§àn, 2006, 26(4), 708–714.

[29] R. F. Zhang and B. L. Guo, *Global solution and its long time behavior for the generalized long-short wave equations*, J. Partial Differential Equations, 2005, 18(3), 206–218.