MODULATION INSTABILITY AND OPTICAL SOLITONS OF RADHAKRISHNAN-KUNDU-LAKSHMANAN MODEL

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Abstract   This paper studies the solitons of Radhakrishnan-Kundu-Lakshmanan (RKL) model with power law nonlinearity. The modified simple equation method and $\exp(-\varphi(q))$ method are presented as integration mechanisms. Dark, bright, singular and periodic soliton solutions are extracted as well as the constraint conditions for their existence. A prized discussion on the stability of these soliton profiles on the basis of index of the power law nonlinearity is also carried out with the help of physical description of solutions. The integration techniques have been proved to be extremely efficient and robust to find new optical solitary wave solutions for various nonlinear evolution equations describing optical pulse propagation. Moreover, using linear stability analysis, modulation instability of the RKL model is studied. Different effects contributing to the modulation instability spectrum gain are analyzed.

Keywords   Optical solitons, constraints, modified simple equation method, $\exp(-\varphi(q))$ method, modulation instability.

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1. Introduction

Self localized waves or solitons have received a lot of attention in the context of numerous engineering and physical sciences in recent times [15, 22, 25, 30]. Despite of being the reality of the modern world, research in the theory of optical solitons have not slowed down. Researchers have been reporting new results on regular basis. The advancement of telecom business completely depends on the dynamical behaviour of these solitons in optical wave guides. The dynamics of soliton propagation have been forged through a number of mathematical models [1, 5–10, 14, 16, 17, 21, 23, 31–33]. The (RKL) model has been the part of many recent studies to narrate the propagation of optical pulses through optical fibres. Optical fibre is a new kind of medium consisting of two concentric cylindrical layers of glass. The light propagates through the inner layer known as core with refractive index n_1 while the exterior layer known as cladding with refractive index n_2 lower than n_1 guaranteeing that the light pulses are reflected back to the core. Optical fibres communication systems function between the micrometer wavelength zone of frequency spectrum. In this work, the

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RKL model is considered with the power law nonlinearity. Such kind of nonlinear response is observed in semiconductors. The modified simple equation and $\exp(-\varphi(q))$ method are used to carry out the integration of the RKL model. A variety of exact solutions including, dark, bright, singular and periodic solutions are retrieved. Also a valuable discussion on the index of power law nonlinearity have been made and stability of the soliton solutions is discussed through graphical depictions.

The generalized RKL equation is studied to inspect the passage of optical solitons in different kinds of irregular media [4,12,26]. In dimensionless form, it is given as

$$ip_t + ap_{xx} + bF(|p|^2)p = i\lambda\{F(|p|^2)p\}_x - i\gamma p_{xxx}. \tag{1.1}$$

In Eq. (1.1), a is the coefficient of group velocity dispersion and b represents the coefficient of the generalized type of irregularity. F represents the kind of nonlinear medium under investigation. On the right hand side of Eq. (1.1), third order dispersion coefficient is denoted by γ and the self-steepening coefficient is λ. The theme of the current work is to solve the Eq. (1.1) for the specific nonlinear medium through two integration tools.

The power law nonlinearity crops up in nonlinear plasmas in weak turbulence theory as well as in nonlinear optics. Several kinds of materials including semi conductors exhibit power law nonlinearity [3]. So, the RKL equation incorporating of power law nonlinearity which is given by the functional $F(q) = q^m$. It is mandatory to have $0 < m < 2$ and also $m \neq 2$ [11,18,19] for the stability of the soliton solutions. Thus the governing equation (1.1) takes the form

$$ip_t + ap_{xx} + b(|p|^{2m})p = i\lambda\{(|p|^{2m})p\}_x - i\gamma p_{xxx}. \tag{1.2}$$

Consider

$$p(x,t) = U(q)e^{i\phi(x,t)}, \tag{1.3}$$

where

$$q = x - vt. \tag{1.4}$$

In (1.3), $U(q)$ stands for the amplitude while velocity is denoted by v whereas the phase of the soliton is $\phi(x,t)$ which is defined as

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{1.5}$$

Here θ, ω and κ, denotes the phase constant, wave number and frequency respectively. Putting Eqs. (1.3)-(1.5) into Eq. (1.2) and separating the real and imaginary parts, we retrieve

$$(\omega + a\kappa^2 + \gamma\kappa^3)U - (b - \lambda\kappa)U^{2m+1} - (a + 3\gamma\kappa)U'' = 0, \tag{1.6}$$

and

$$(v + 2a\kappa^2 + 3\gamma\kappa^2)U' + \lambda(2m+1)U'U^{2m} + \gamma U''' = 0. \tag{1.7}$$

Integrating Eq. (1.12), we get

$$(v + 2a\kappa + 3\gamma\kappa^2)U + \lambda U^{2m+1} + \gamma U'' = 0. \tag{1.8}$$

Balancing U^{2m+1} and U'', we get

$$(2m+1)N = N + 2 \quad \text{implies} \quad N = \frac{1}{m}. \tag{1.9}$$

In order to get solution in closed form, we introduce a new transformation

$$U(x,t) = \phi^{1/2m}(x,t). \qquad (1.10)$$

Eq. (1.10) transforms Eqs. (1.6) and (1.8) into

$$4m^2(\omega + a\kappa^2 + \gamma\kappa^3)\phi^2 - 4m^2(b - \lambda\kappa)\phi^3 - (a + 3\gamma\kappa)\{(1 - 2m)(\phi')^2 + 2m\phi\phi''\} = 0, \qquad (1.11)$$

and

$$4m^2(v + 2a\kappa + 3\gamma\kappa^2)\phi^2 + 4m^2\lambda\phi^3 + \gamma\{(1 - 2m)(\phi')^2 + 2m\phi\phi''\} = 0. \qquad (1.12)$$

The rest of the paper is systemized as follow: In "The modified simple equation method" subsection, the modified simple equation method is discussed and applied on Eqs. (1.11) and (1.12). In "$\exp(-\varphi(q))$ method" subsection the $\exp(-\varphi(q))$ method and its application on Eqs. (1.11) and (1.12) are inspected. Modulation instability analysis is carried out in the third section while the results obtained are presented and discussed in the section "Results and Discussion". Finally, the paper is concluded in the "Conclusions" Section.

2. Methods

2.1. Modified simple equation approach

In this subsection, we integrate the RKL equation through modified simple equation method [13,24] incorporating power law media. To start of, it is assumed that Eqs. (1.11) and (1.12) has the following solution form

$$\phi(q) = \sum_{i=0}^{N} a_i \left(\frac{\psi'(q)}{\psi(q)} \right)^i, \qquad (2.1)$$

where a_i and $\psi(q)$ are the unknowns, provided $a_N \neq 0$.

Balancing ϕ^3 and $\phi\phi''$ in Eqs. (1.11) and (1.12), we have

$$N = 2. \qquad (2.2)$$

Thus, from Eq. (2.1), Eqs. (1.11) and (1.12) accepts the subsequent form of solution

$$U(q) = a_0 + a_1 \left(\frac{\psi'(q)}{\psi(q)} \right) + a_2 \left(\frac{\psi'(q)}{\psi(q)} \right)^2, \qquad (2.3)$$

where a_0, a_1 and a_2 are the constants to be determined such that $a_2 \neq 0$. By putting (2.3) into Eqs. (1.6) and (1.8), a system is extracted by comparing the coefficients of ψ^{-j}, $j = 0, 1, 2, 3, 4, 5, 6$ to zero, a bunch of algebraic equations is fetched as follows:
ψ^{-0} coeff:

$$4\,m^2\gamma\,\kappa^3 a_0{}^2 + 4\,m^2\lambda\,\kappa\,a_0{}^3 + 4\,m^2 a\kappa^2 a_0{}^2 - 4\,m^2 b a_0{}^3 + 4\,m^2\omega\,a_0{}^2 = 0,$$
$$4\,m^2 v a_0{}^2 + 12\,m^2\gamma\,\kappa^2 a_0{}^2 + 8\,m^2 a\kappa\,a_0{}^2 + 4\,m^2\lambda\,a_0{}^3 = 0,$$

ψ^{-1} coeff:

$$\left(-12\,m^2ba_0{}^2a_1 + 8\,m^2\gamma\,\kappa^3a_0a_1 + 8\,m^2\omega\,a_0a_1 + 12\,m^2\lambda\,\kappa\,a_0{}^2a_1\right.$$
$$\left. + 8\,m^2a\kappa^2a_0a_1\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right) - \left(2\,ama_0a_1 + 6\,\gamma\,\kappa\,ma_0a_1\right)\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = 0,$$

$$\left(8\,m^2va_0a_1 + 24\,m^2\gamma\,\kappa^2a_0a_1 + 12\,m^2\lambda\,a_0{}^2a_1 + 16\,m^2a\kappa\,a_0a_1\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)$$
$$+ 2\,\gamma\,ma_0a_1\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = 0,$$

ψ^{-2} coeff:

$$\left(-12\,\gamma\,\kappa\,ma_0a_2 - 4\,ama_0a_2 - 2\,ama_1{}^2 - 6\,\gamma\,\kappa\,ma_1{}^2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right)$$

$$+\left(6\,\gamma\,\kappa\,ma_1{}^2 - 3\,\gamma\,\kappa\,a_1{}^2 - 4\,ama_0a_2 - aa_1{}^2 - 12\,\gamma\,\kappa\,ma_0a_2 + 2\,ama_1{}^2\right)\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2$$

$$+\left(18\,\gamma\,\kappa\,ma_0a_1 + 6\,ama_0a_1\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) + \left(8\,m^2\gamma\,\kappa^3a_0a_2 + 8\,m^2\omega\,a_0a_2\right.$$
$$+ 8\,m^2a\kappa^2a_0a_2 + 4\,m^2\gamma\,\kappa^3a_1{}^2 + 4\,m^2a\kappa^2a_1{}^2 + 12\,m^2\lambda\,\kappa\,a_0a_1{}^2$$
$$\left. - 12\,m^2ba_0{}^2a_2 - 12\,m^2ba_0a_1{}^2 + 4\,m^2\omega\,a_1{}^2 + 12\,m^2\lambda\,\kappa\,a_0{}^2a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2 = 0,$$

$$\left(4\,\gamma\,ma_0a_2 + 2\,\gamma\,ma_1{}^2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) + \left(-2\,\gamma\,ma_1{}^2 + \gamma\,a_1{}^2\right.$$
$$+ 4\,\gamma\,ma_0a_2\right)\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2 - 6\,\gamma\,ma_0a_1\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right) + \left(12\,m^2\lambda\,a_0{}^2a_2\right.$$
$$+ 12\,m^2\lambda\,a_0a_1{}^2 + 12\,m^2\gamma\,\kappa^2a_1{}^2 + 4\,m^2va_1{}^2 + 24\,m^2\gamma\,\kappa^2a_0a_2$$
$$\left. + 16\,m^2a\kappa\,a_0a_2 + 8\,m^2a\kappa\,a_1{}^2 + 8\,m^2va_0a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2 = 0,$$

ψ^{-3} coeff:

$$\left(-6\,ama_1a_2 - 18\,\gamma\,\kappa\,ma_1a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) + \left(4\,ama_1a_2 - 12\,\gamma\,\kappa\,a_1a_2\right.$$
$$\left. - 4\,aa_1a_2 + 12\,\gamma\,\kappa\,ma_1a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2 + \left(20\,ama_0a_2 + 2\,ama_1{}^2\right.$$
$$+ 6\,\gamma\,\kappa\,a_1{}^2 + 6\,\gamma\,\kappa\,ma_1{}^2 + 2\,aa_1{}^2 + 60\,\gamma\,\kappa\,ma_0a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) +$$
$$\left(4\,m^2\lambda\,\kappa\,a_1{}^3 + 8\,m^2\gamma\,\kappa^3a_1a_2 - 24\,m^2ba_0a_1a_2 - 4\,ama_0a_1 - 12\,\gamma\,\kappa\,ma_0a_1\right.$$
$$\left. + 8\,m^2a\kappa^2a_1a_2 + 8\,m^2\omega\,a_1a_2 + 24\,m^2\lambda\,\kappa\,a_0a_1a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3 = 0,$$

$$\left(4\,\gamma\,a_1a_2 - 4\,\gamma\,ma_1a_2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2 + \left(-20\,\gamma\,ma_0a_2 - 2\,\gamma\,ma_1{}^2\right.$$
$$\left. - 2\,\gamma\,a_1{}^2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) + \left(16\,m^2a\kappa\,a_1a_2 + 24\,m^2\lambda\,a_0a_1a_2\right.$$

$$+ 8\,m^2 v a_1 a_2 + 24\,m^2 \gamma\,\kappa^2 a_1 a_2 + 4\,m^2 \lambda\,a_1{}^3 + 4\,\gamma\,m a_0 a_1 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3$$

$$+ 6\,\gamma\,m a_1 \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2 a_2 \left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = 0,$$

ψ^{-4} coeff:

$$\left(-4\,a m a_2{}^2 - 12\,\gamma\,\kappa\,m a_2{}^2\right) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3 \left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) + \Big(-12\,\gamma\,\kappa\,a_2{}^2$$

$$+ 4\,a m a_2{}^2 - 4\,a a_2{}^2 + 12\,\gamma\,\kappa\,m a_2{}^2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2 + \Big(8\,a a_1 a_2$$

$$+ 10\,a m a_1 a_2 + 30\,\gamma\,\kappa\,m a_1 a_2 + 24\,\gamma\,\kappa\,a_1 a_2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) + \Big($$

$$- 12\,a m a_0 a_2 - 12\,m^2 b a_0 a_2{}^2 + 4\,m^2 \omega\,a_2{}^2 - 3\,\gamma\,\kappa\,a_1{}^2 - 36\,\gamma\,\kappa\,m a_0 a_2 + 12\,m^2 \lambda\,\kappa\,a_1{}^2 a_2$$

$$- 12\,m^2 b a_1{}^2 a_2 - 6\,\gamma\,\kappa\,m a_1{}^2 + 12\,m^2 \lambda\,\kappa\,a_0 a_2{}^2 + 4\,m^2 \gamma\,\kappa^3 a_2{}^2$$

$$- 2\,a m a_1{}^2 - a a_1{}^2 + 4\,m^2 a \kappa^2 a_2{}^2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^4 = 0,$$

$$\left(4\,\gamma\,a_2{}^2 - 4\,\gamma\,m a_2{}^2\right) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^2 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)^2 + \Big(- 8\,\gamma\,a_1 a_2 - 10\,\gamma\,m a_1 a_2 \Big)$$

$$\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) + \Big(12\,m^2 \gamma\,\kappa^2 a_2{}^2 + 12\,\gamma\,m a_0 a_2 + 12\,m^2 \lambda\,a_1{}^2 a_2$$

$$+ \gamma\,a_1{}^2 + 12\,m^2 \lambda\,a_0 a_2{}^2 + 4\,m^2 v a_2{}^2 + 2\,\gamma\,m a_1{}^2 + 8\,m^2 a \kappa\,a_2{}^2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^4$$

$$+ 4\,\gamma\,m a_2{}^2 \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^3 \left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = 0,$$

ψ^{-5} coeff:

$$\left(12\,\gamma\,\kappa\,m a_2{}^2 + 24\,\gamma\,\kappa\,a_2{}^2 + 8\,a a_2{}^2 + 4\,a m a_2{}^2\right) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^4 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) +$$

$$\Big(- 12\,\gamma\,\kappa\,a_1 a_2 - 8\,a m a_1 a_2 - 12\,m^2 b a_1 a_2{}^2 - 24\,\gamma\,\kappa\,m a_1 a_2 + 12\,m^2 \lambda\,\kappa\,a_1 a_2{}^2$$

$$- 4\,a a_1 a_2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^5 = 0,$$

$$\left(-4\,\gamma\,m a_2{}^2 - 8\,\gamma\,a_2{}^2\right) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^4 \left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right)$$

$$+ \left(12\,m^2 \lambda\,a_1 a_2{}^2 + 4\,\gamma\,a_1 a_2 + 8\,\gamma\,m a_1 a_2\right) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^5 = 0,$$

ψ^{-6} coeff:

$$\Big(- 4\,m^2 b a_2{}^3 - 12\,\gamma\,\kappa\,m a_2{}^2 - 4\,a a_2{}^2 - 12\,\gamma\,\kappa\,a_2{}^2 + 4\,m^2 \lambda\,\kappa\,a_2{}^3$$

$$- 4\,a m a_2{}^2 \Big) \left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^6 = 0,$$

$$\left(4\,\gamma\,m{a_2}^2 + 4\,m^2\lambda\,{a_2}^3 + 4\,\gamma\,{a_2}^2\right)\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)^6 = 0,$$

Solving the aforementioned system of algebraic equations, we have
Case1:

$$\kappa = \frac{1}{4}\frac{-\lambda\,a + \gamma\,b}{\gamma\,\lambda},$$

$$\omega = -\frac{1}{32}\frac{-a^3\lambda^3 - 9\,\lambda^2 a^2\gamma\,b + 5\,\lambda\,a\gamma^2 b^2 + 8\,\lambda^3 av\gamma + 5\,\gamma^3 b^3 + 24\,b\gamma^2 v\lambda^2}{\gamma^2\lambda^3},$$

$$a_0 = 0,\ a_1 = \pm\frac{\gamma\,(m+1)\sqrt{-\frac{1}{4}\frac{-5\,\lambda^2 r^2 + 2\,\lambda\,rb\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\gamma^2}}}{\lambda^2 m},\ a_2 = -\frac{\gamma\,(m+1)}{m^2\lambda},$$

$$(2.4)$$

$$\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = -\frac{1}{4}\frac{\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)m^2\left(-5\,\lambda^2 r^2 + 2\,\lambda\,rb\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2\right)}{\lambda^2\gamma^2}, \quad (2.5)$$

$$\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) = \pm\frac{\sqrt{-\frac{1}{4}\frac{-5\,\lambda^2 r^2 + 2\,\lambda\,rb\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\gamma^2}}\,m\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)}{\lambda}, \quad (2.6)$$

From Eqs. (2.5) and (2.6), we have

$$\psi\left(\xi\right) = c_1 + c_2\mathrm{e}^{\pm\frac{1}{2}\frac{\sqrt{-16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2}\,m\xi}{\gamma\,\lambda}}, \quad (2.7)$$

and

$$\psi'(\xi) = \pm\frac{1}{2}\frac{c_2\sqrt{-16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2}\,mH}{\gamma\lambda}, \quad (2.8)$$

where c_1 and c_2 are arbitrary constants. So, using Eqs. (2.7) and (2.8) in Eq. (2.3) and simplification gives

$$p(x,t) = \left\{\mp\frac{1}{4}\frac{(m+1)\left(16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2\right)c_2\,H}{\gamma\lambda^3\left(c_1 + c_2\,H\right)^2}\right\}^{\frac{1}{2m}}$$
$$\times\,e^{i(-\kappa x + \omega t + \theta)}, \quad (2.9)$$

where κ and γ are given above and $H = \mathrm{e}^{\pm\frac{1}{2}\frac{\sqrt{-16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2}\,m\xi}{\gamma\,\lambda}}$.

Setting $c_1 = \pm 1$ and $c_2 = \mathrm{e}^{\pm\frac{1}{2}\frac{\sqrt{-16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2}\,m}{\gamma\,\lambda}\xi_0}$

$$p(x,t) = \left\{\pm\frac{1}{16}\frac{(m+1)\left(16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2\right)}{\gamma\lambda^3}\right.$$
$$\times\,\sec h^2\left[\frac{1}{4}\frac{\sqrt{-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2}\,m\left(\xi + \xi_0\right)}{\gamma\,\lambda}\right]\right\}^{\frac{1}{2m}}$$
$$\times\,e^{i(-\kappa x + \omega t + \theta)}, \quad (2.10)$$

$$p(x,t) = \left\{\pm\frac{1}{16}\frac{(m+1)\left(16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2\right)}{\gamma\lambda^3}\right.$$

$$\times \csc h^2 \left[\frac{1}{4} \frac{\sqrt{-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2}\,m\,(\xi + \xi_0)}{\gamma\,\lambda} \right] \right\}^{\frac{1}{2m}}$$

$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{2.11}$$

where Eqs. (2.10) and (2.11) respectively denotes the bright and singular soliton solutions which hold for $-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2 > 0$.

$$p(x,t) = \left\{ \pm \frac{1}{16} \frac{(m+1)\left(16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2\right)}{\gamma\lambda^3} \right.$$

$$\times \sec^2 \left[\frac{1}{4} \frac{\sqrt{-(-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2)}\,m\,(\xi + \xi_0)}{\gamma\,\lambda} \right] \right\}^{\frac{1}{2m}}$$

$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{2.12}$$

$$p(x,t) = \left\{ \pm \frac{1}{16} \frac{(m+1)\left(16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2\right)}{\gamma\lambda^3} \right.$$

$$\times \csc^2 \left[\frac{1}{4} \frac{\sqrt{-(-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2)}\,m\,(\xi + \xi_0)}{\gamma\,\lambda} \right] \right\}^{\frac{1}{2m}}$$

$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{2.13}$$

where Eqs. (2.12) and (2.13) denotes the periodic singular soliton solutions. These solutions remain valid until $-3\,b^2\gamma^2 + (-16\,v\lambda^2 - 2\,\lambda\,ab)\,\gamma + 5\,\lambda^2 a^2 < 0$.
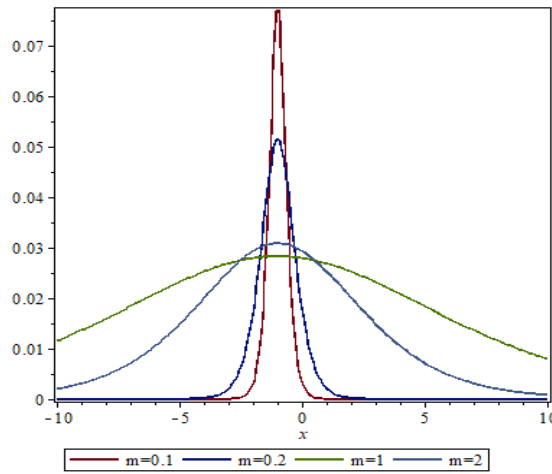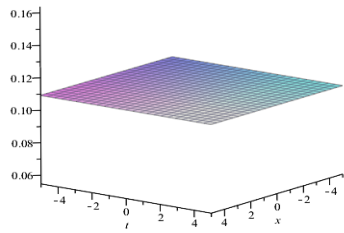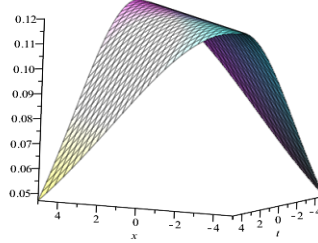

Figure 2. 2-D plot of solution given in Eq. (2.10) for different value of index m.
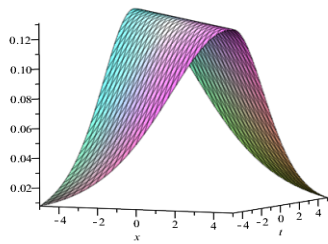
Case2:

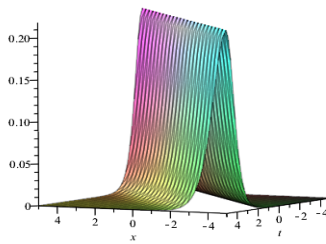$$\kappa = \frac{1}{4} \frac{-\lambda\,a + \gamma\,b}{\gamma\,\lambda}, \quad m = 1,$$

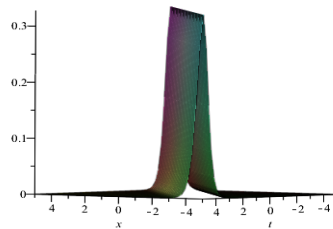(a) $\gamma = 1$, $b = 8$, $v = -1$, $\lambda = 10$, $a = 1$, $m = 0$

(b) $\gamma = 1$, $b = 8$, $v = -1$, $\lambda = 10$, $a = 1$, $m = 0.1$

(c) $\gamma = 1$, $b = 8$, $v = -1$, $\lambda = 10$, $a = 1$, $m = 0.2$

(d) $\gamma = 1$, $b = 8$, $v = -1$, $\lambda = 10$, $a = 1$, $m = 1$

(e) $\gamma = 1$, $b = 8$, $v = -1$, $\lambda = 10$, $a = 1$, $m = 2$

Figure 1. Graphical depiction of solutions given in Eqs. (2.10) respectively with suitable choice of parameters.

$$\omega = -\frac{1}{32}\frac{-a^3\lambda^3 - 9\lambda^2 a^2\gamma b + 5\lambda a\gamma^2 b^2 + 8\lambda^3 av\gamma + 5\gamma^3 b^3 + 24\,b\gamma^2 v\lambda^2}{\gamma^2\lambda^3}$$

$$a_0 = -\frac{1}{16}\frac{-5\lambda^2 a^2 + 2\lambda ab\gamma + 3b^2\gamma^2 + 16\,v\gamma\lambda^2}{\lambda^3\gamma}, \quad a_2 = -2\frac{\gamma}{\lambda} \tag{2.14}$$

$$a_1 = 2\gamma\frac{\sqrt{-\frac{1}{8}\frac{-3b^2\gamma^2 - 16\,v\gamma\lambda^2 + 5\lambda^2 a^2 - 2\lambda ab\gamma}{\gamma^2}}}{\lambda^2},$$

$$\left(\frac{d^3}{d\xi^3}\psi(\xi)\right) = \frac{1}{8}\frac{\left(\frac{d}{d\xi}\psi(\xi)\right)\left(-5\lambda^2 a^2 + 2\lambda ab\gamma + 3b^2\gamma^2 + 16\,v\gamma\lambda^2\right)}{\gamma^2\lambda^2}, \tag{2.15}$$

and

$$\left(\frac{d^2}{d\xi^2}\psi(\xi)\right) = \frac{\sqrt{-\frac{1}{8}\frac{-3b^2\gamma^2 - 16\,v\gamma\lambda^2 + 5\lambda^2 a^2 - 2\lambda ab\gamma}{\gamma^2}}\left(\frac{d}{d\xi}\psi(\xi)\right)}{\lambda}. \tag{2.16}$$

From Eqs. (2.15) and (2.16), we have

$$\psi(\xi) = c_1 + c_2 e^{\frac{1}{4}\frac{\sqrt{6b^2\gamma^2 + 32\,v\gamma\lambda^2 - 10\lambda^2 a^2 + 4\lambda ab\gamma}\,\xi}{\gamma\lambda}}, \tag{2.17}$$

and

$$\psi' = \frac{1}{4}\frac{c_2\sqrt{6b^2\gamma^2 + 32\,v\gamma\lambda^2 - 10\lambda^2 a^2 + 4\lambda ab\gamma}\,e^{\frac{1}{4}\frac{\sqrt{6b^2\gamma^2 + 32\,v\gamma\lambda^2 - 10\lambda^2 a^2 + 4\lambda ab\gamma}\,\xi}{\gamma\lambda}}}{\gamma\lambda}, \tag{2.18}$$

where c_1 and c_2 are arbitrary constants. So, by using Eqs. (2.17) and (2.16) in Eq. (2.3) gives

$$
\begin{aligned}
&p(x,t)\\
&=\left\{-\frac{1}{16}\frac{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}{\lambda^3\gamma}\right.\\
&\quad\left.+\frac{1}{4}\frac{\left(3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma\right)c_2 e^{\frac{1}{4}\frac{\sqrt{2}\sqrt{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}(\xi)}{\gamma\lambda}}}{\lambda^3\gamma\left(c_1 + c_2 e^{\frac{1}{4}\frac{\sqrt{2}\sqrt{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}(\xi)}{\gamma\lambda}}\right)^2}\right\}^{\frac{1}{2m}}\\
&\quad\times e^{i(-\kappa x + \omega t + \theta)}.
\end{aligned} \tag{2.19}
$$

By setting $c_1 = \pm 1$ and $c_2 = e^{\frac{1}{4}\frac{\sqrt{2}\sqrt{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}(\xi_0)}{\gamma\lambda}}$, we have

$$
\begin{aligned}
p(x,t) =&\left\{-\frac{1}{16}\frac{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}{\lambda^3\gamma}\right.\\
&\left(1 - \sec h^2\left[\frac{1}{8}\frac{\sqrt{2}\sqrt{3b^2\gamma^2 + (2\lambda ab + 16\,v\lambda^2)\gamma - 5\lambda^2 a^2}}{\gamma\lambda}(\xi + \xi_0)\right]\right)^{\frac{1}{2m}}\\
&\times e^{i(-\kappa x + \omega t + \theta)},
\end{aligned} \tag{2.20}
$$

$$p(x,t) =\left\{-\frac{1}{16}\frac{3b^2\gamma^2 + 16\,v\gamma\lambda^2 - 5\lambda^2 a^2 + 2\lambda ab\gamma}{\lambda^3\gamma}\right.$$

$$\left(1 - \csc h^2 \left[\frac{1}{8} \frac{\sqrt{2}\sqrt{3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,v\lambda^2)\,\gamma - 5\,\lambda^2 a^2}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}}$$
$$\times\, e^{i(-\kappa x + \omega t + \theta)}, \tag{2.21}$$

where Eqs. (2.20) and (2.21) represent bright and singular solitons respectively following the constraint $3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,v\lambda^2)\,\gamma - 5\,\lambda^2 a^2 > 0$. Similarly,

$$p(x,t) = \left\{ -\frac{1}{16}\frac{3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma}{\lambda^3\gamma} \right.$$
$$\left(1 - \sec^2\left[\frac{1}{8}\frac{\sqrt{2}\sqrt{-(3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,v\lambda^2)\,\gamma - 5\,\lambda^2 a^2)}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}}$$
$$\times\, e^{i(-\kappa x + \omega t + \theta)}, \tag{2.22}$$

$$p(x,t) = \left\{ -\frac{1}{16}\frac{3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma}{\lambda^3\gamma} \right.$$
$$\left(1 - \csc^2\left[\frac{1}{8}\frac{\sqrt{2}\sqrt{-(3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,v\lambda^2)\,\gamma - 5\,\lambda^2 a^2)}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}}$$
$$\times\, e^{i(-\kappa x + \omega t + \theta)}, \tag{2.23}$$

where Eqs. (2.22) and (2.23) represent periodic singular solitons respectively. These solutions remain valid until $3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,v\lambda^2)\,\gamma - 5\,\lambda^2 a^2 < 0$.

Case 3:

$$\kappa = \frac{1}{4}\frac{-\lambda\,a + \gamma\,b}{\gamma\,\lambda}, \quad m = \frac{1}{2},$$
$$\omega = -\frac{1}{32}\frac{-a^3\lambda^3 - 9\,\lambda^2 a^2\gamma\,b + 5\,\lambda\,a\gamma^2 b^2 + 8\,\lambda^3 av\gamma + 5\,\gamma^3 b^3 + 24\,b\gamma^2 v\lambda^2}{\gamma^2\lambda^3},$$
$$a_0 = -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma}, \quad a_2 = -6\frac{\gamma}{\lambda} \tag{2.24}$$
$$a_1 = 6\,\gamma\frac{\sqrt{-\frac{1}{16}\frac{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}{\gamma^2}}}{\lambda^2}$$
$$\left(\frac{d^3}{d\xi^3}\psi(\xi)\right) = \frac{1}{16}\frac{\left(\frac{d}{d\xi}\psi(\xi)\right)\left(-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2\right)}{\gamma^2\lambda^2}, \tag{2.25}$$

and

$$\left(\frac{d^2}{d\xi^2}\psi(\xi)\right) = \frac{\sqrt{-\frac{1}{16}\frac{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}{\gamma^2}}\left(\frac{d}{d\xi}\psi(\xi)\right)}{\lambda}. \tag{2.26}$$

From Eqs. (2.25) and (2.26), we have

$$\psi(\xi) = c_1 + c_2 e^{\frac{1}{4}\frac{\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\xi}{\gamma\,\lambda}}, \tag{2.27}$$

and

$$\psi' = \frac{1}{4}\frac{c_2\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\,\mathrm{e}^{\frac{1}{4}\frac{\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\,\xi}{\gamma\,\lambda}}}{\gamma\lambda},$$

$$(2.28)$$

where c_1 and c_2 are arbitrary constants. So, by using Eqs. (2.27) and (2.28), we have

$$p(x,t)$$

$$= \left\{ -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma} \right.$$

$$\left. + \frac{3}{8}\frac{\left(-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2\right)c_2\mathrm{e}^{\frac{1}{4}\frac{\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\,\xi}{\gamma\,\lambda}}}{\lambda^3\gamma\left(c_1 + c_2\mathrm{e}^{\frac{1}{4}\frac{\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\,\xi}{\gamma\,\lambda}}\right)^2} \right\}^{\frac{1}{2m}}$$

$$\times\, e^{i(-\kappa x + \omega t + \theta)}. \qquad (2.29)$$

By setting $c_1 = \pm 1$ and $c_2 = \mathrm{e}^{\frac{1}{4}\frac{\sqrt{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}\,\xi_0}{\gamma\,\lambda}}$, we have

$$p(x,t) = \left\{ -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma} \right.$$

$$\left. \left(1 - \frac{3}{2}\sec h^2\left[\frac{1}{8}\frac{\sqrt{3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,\lambda^2 v)\,\gamma - 5\,\lambda^2 a^2}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}} \right.$$

$$\times\, e^{i(-\kappa x + \omega t + \theta)}. \qquad (2.30)$$

$$p(x,t) = \left\{ -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma} \right.$$

$$\left. \left(1 - \frac{3}{2}\csc h^2\left[\frac{1}{8}\frac{\sqrt{3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,\lambda^2 v)\,\gamma - 5\,\lambda^2 a^2}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}} \right.$$

$$\times\, e^{i(-\kappa x + \omega t + \theta)}. \qquad (2.31)$$

Here Eqs. (2.30) and (2.31) respectively denotes the bright and singular solitons satisfying the constraint condition $3\,b^2\gamma^2 + \left(2\,\lambda\,ab + 16\,\lambda^2 v\right)\gamma - 5\,\lambda^2 a^2 > 0$. Similarly,

$$p(x,t) = \left\{ -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma} \right.$$

$$\left. \left(1 - \frac{3}{2}\sec^2\left[\frac{1}{8}\frac{\sqrt{-(3\,b^2\gamma^2 + (2\,\lambda\,ab + 16\,\lambda^2 v)\,\gamma - 5\,\lambda^2 a^2)}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}} \right.$$

$$\times\, e^{i(-\kappa x + \omega t + \theta)}. \qquad (2.32)$$

$$p(x,t) = \left\{ -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma} \right.$$

$$\left(1 - \frac{3}{2}\csc^2\left[\frac{1}{8}\frac{\sqrt{-\left(3\,b^2\gamma^2 + \left(2\,\lambda\,ab + 16\,\lambda^2 v\right)\gamma - 5\,\lambda^2 a^2\right)}\,(\xi + \xi_0)}{\gamma\,\lambda}\right]\right)^{\frac{1}{2m}}$$
$$\times\,e^{i(-\kappa x + \omega t + \theta)}, \tag{2.33}$$

where κ and ω are given in the solution set Case 3. Eqs. (2.32) and (2.33) denotes periodic singular solitons which hold for the constraint $3\,b^2\gamma^2 + \left(2\,\lambda\,ab + 16\,\lambda^2 v\right)\gamma - 5\,\lambda^2 a^2 < 0$.

Case 4:

$$\kappa = \frac{1}{4}\frac{-\lambda\,a + \gamma\,b}{\gamma\,\lambda}, \quad m = -2,$$

$$\omega = -\frac{1}{32}\frac{-a^3\lambda^3 - 9\,\lambda^2 a^2\gamma\,b + 5\,\lambda\,a\gamma^2 b^2 + 8\,\lambda^3 av\gamma + 5\,\gamma^3 b^3 + 24\,b\gamma^2 v\lambda^2}{\gamma^2\lambda^3},$$

$$a_0 = -\frac{1}{16}\frac{-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2}{\lambda^3\gamma}, \quad a_2 = \frac{1}{4}\frac{\gamma}{\lambda}, \tag{2.34}$$

$$a_1 = \frac{1}{4}\frac{\sqrt{5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2}}{\lambda^2}.$$

$$\left(\frac{d^3}{d\xi^3}\psi\left(\xi\right)\right) = -\frac{1}{4}\frac{\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)\left(-5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma + 3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2\right)}{\gamma^2\lambda^2}, \tag{2.35}$$

and

$$\left(\frac{d^2}{d\xi^2}\psi\left(\xi\right)\right) = -\frac{1}{2}\frac{\sqrt{5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma - 3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2}\left(\frac{d}{d\xi}\psi\left(\xi\right)\right)}{\gamma\,\lambda}. \tag{2.36}$$

From Eqs. (2.36) and (2.35), we have

$$\psi\left(\xi\right) = c_1 + c_2 e^{-\frac{1}{2}\frac{\sqrt{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}\,\xi}{\lambda\,\gamma}}, \tag{2.37}$$

and

$$\psi' = -\frac{1}{2}\frac{c_2\sqrt{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}\,e^{-\frac{1}{2}\frac{\sqrt{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}\,\xi}{\lambda\,\gamma}}}{\lambda\gamma}, \tag{2.38}$$

where c_1 and c_2 are arbitrary constants.

By setting $c_1 = \pm 1$ and $c_2 = e^{-\frac{1}{2}\frac{\sqrt{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}\,\xi_0}{\lambda\,\gamma}}$, we have

$$p(x,t) = \left\{-\frac{1}{16}\frac{3\,b^2\gamma^2 + 16\,v\gamma\,\lambda^2 - 5\,\lambda^2 a^2 + 2\,\lambda\,ab\gamma}{\gamma\,\lambda^3\left(1 + e^{-\frac{1}{2}\frac{\sqrt{-3\,b^2\gamma^2 - 16\,v\gamma\,\lambda^2 + 5\,\lambda^2 a^2 - 2\,\lambda\,ab\gamma}\,(\xi + \xi_0)}{\lambda\,\gamma}}\right)^2}\right\}^{\frac{1}{2m}}\times\,e^{i(-\kappa x + \omega t + \theta)}. \tag{2.39}$$

which is not a stable soliton because $m = -2$. It suggests that the soliton solutions exists when $0 < m < 2$.

2.2. The $\exp(-\varphi(q))$ method

In this section, the extraction of solitons for the RKL equation is carried out. The power law nonlinearity is considered. To start with, we assume the solution of Eqs.

(1.6) and (1.8) is of the following form

$$U(q) = \sum_{i=0}^{N} a_i (e^{-\varphi(q)})^i, a_N \neq 0, \tag{2.40}$$

where $\varphi = \varphi(q)$ satisfies the following ODE:

$$\varphi^{'}(q) + \zeta \exp(\varphi(q)) + \eta \exp(-\varphi(q)) = 0 \tag{2.41}$$

ζ and η are arbitrary constants.

Eq. (2.41) enjoys the subsequent solutions forms

$$\varphi(q) = -\ln\left(\sqrt{\frac{\zeta}{\eta}} \tan[\sqrt{\zeta\eta}(q+q_0)]\right), \ \zeta\eta > 0 \tag{2.42}$$

$$\varphi(q) = -\ln\left(-\sqrt{\frac{\zeta}{\eta}} \cot[\sqrt{\zeta\eta}(q+q_0)]\right), \ \zeta\eta > 0$$

$$\varphi(q) = -\ln\left(-\sqrt{\frac{\zeta}{-\eta}} \tanh[\sqrt{-\zeta\eta}(q+q_0)]\right), \ \zeta\eta < 0$$

$$\varphi(q) = -\ln\left(\sqrt{\frac{\zeta}{-\eta}} \coth[\sqrt{-\zeta\eta}(q+q_0)]\right), \ \zeta\eta < 0$$

$$\varphi(q) = \ln\left(-\frac{1}{\eta(q+q_0)}\right), \ \zeta = 0, \ \eta > 0,$$

where q_0 is an integration constant and $\zeta\eta > 0$ or $\zeta\eta < 0$ depends $\lambda's$ sign. The value of N determined as earlier is

$$N = 2. \tag{2.43}$$

Therefore,

$$\phi(q) = a_0 + a_1\exp(-\varphi(q)) + a_2\exp(-\varphi(q))^2. \tag{2.44}$$

Now, Plugging the values of U and U'' in Eqs. (1.6) and (1.8) using Eq. (2.41), and comparing coefficients of $\exp(-i\varphi(q))$, $i = 0, 1, 2, 3, 4, 5$ and 6, an nonlinear algebraic system of equations is gained

$\exp(-0 \ \varphi(q))$ coeff:

$$4 m^2 \left(\omega + a\kappa^2 + \gamma \kappa^3\right) a_0{}^2 - 4 m^2 \left(b - \lambda \kappa\right) a_0{}^3 - \left(a + 3\gamma\kappa\right)\left((1 - 2m) a_1{}^2\eta^2\right.$$
$$\left. + 2 m a_0 \left(\zeta \eta a_1 + 2 a_2\eta^2\right)\right) = 0,$$
$$4 m^2 \left(v + 2 a\kappa + 3\gamma\kappa^2\right) a_0{}^2 + 4 m^2\lambda a_0{}^3 + \gamma\left((1 - 2m) a_1{}^2\eta^2\right.$$
$$\left. + 2 m a_0 \left(\zeta \eta a_1 + 2 a_2\eta^2\right)\right) = 0.$$

$\exp(-1 \ \varphi(q))$ coeff:

$$-12 m^2 \left(b - \lambda \kappa\right) a_0{}^2 a_1 + 8 m^2 \left(\omega + a\kappa^2 + \gamma \kappa^3\right) a_0 a_1 - \left(a + 3\gamma\kappa\right)\left(-2(1 - 2m)\right.$$
$$a_1\eta\left(-\zeta a_1 - 2 a_2\eta\right)$$
$$\left. + 2 m a_0 \left(6 a_2\zeta \eta + 2 a_1\eta + \lambda^2 a_1\right) + 2 m a_1\left(\zeta \eta a_1 + 2 a_2\eta^2\right)\right) = 0,$$

$$12\,m^2\lambda\,{a_0}^2 a_1 + 8\,m^2\left(v + 2\,a\kappa + 3\,\gamma\,\kappa^2\right)a_0 a_1 + \gamma\left(-2\left(1 - 2\,m\right)a_1\eta\left(-\zeta\,a_1 - 2\,a_2\eta\right)\right.$$
$$\left. + 2\,m a_0\left(6\,a_2\zeta\,\eta + 2\,a_1\eta + \lambda^2 a_1\right) + 2\,m a_1\left(\zeta\,\eta\,a_1 + 2\,a_2\eta^2\right)\right) = 0.$$

$\exp(-2\,\varphi(q))$ coeff:

$$-4\,m^2\left(b - \lambda\,\kappa\right)\left(a_0\left(2\,a_0 a_2 + {a_1}^2\right) + 2\,{a_1}^2 a_0 + a_2{a_0}^2\right) + 4\,m^2\left(\omega + a\kappa^2 + \gamma\,\kappa^3\right)$$
$$\left(2\,a_0 a_2 + {a_1}^2\right) - \left(a + 3\,\gamma\,\kappa\right)\left(\left(1 - 2\,m\right)\left(-2\,a_1\eta\left(-2\,a_2\zeta - a_1\right) + \left(-\zeta\,a_1 - 2\,a_2\eta\right)^2\right)\right.$$
$$+ 2\,m a_0\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right) + 2\,m a_1\left(6\,a_2\zeta\,\eta + 2\,a_1\eta + \lambda^2 a_1\right)$$
$$\left. + 2\,m a_2\left(\zeta\,\eta\,a_1 + 2\,a_2\eta^2\right)\right) = 0,$$
$$4\,m^2\lambda\left(a_0\left(2\,a_0 a_2 + {a_1}^2\right) + 2\,{a_1}^2 a_0 + a_2{a_0}^2\right) + 4\,m^2\left(v + 2\,a\kappa + 3\,\gamma\,\kappa^2\right)\left(2\,a_0 a_2 + {a_1}^2\right)$$
$$+ \gamma\left(\left(1 - 2\,m\right)\left(-2\,a_1\eta\left(-2\,a_2\zeta - a_1\right) + \left(-\zeta\,a_1 - 2\,a_2\eta\right)^2\right)\right.$$
$$+ 2\,m a_0\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right) + 2\,m a_1\left(6\,a_2\zeta\,\eta + 2\,a_1\eta + \lambda^2 a_1\right)$$
$$\left. + 2\,m a_2\left(\zeta\,\eta\,a_1 + 2\,a_2\eta^2\right)\right) = 0.$$

$\exp(-3\,\varphi(q))$ coeff:

$$8\,m^2\left(\omega + a\kappa^2 + \gamma\,\kappa^3\right)a_1 a_2 - 4\,m^2\left(b - \lambda\,\kappa\right)\left(4\,a_0 a_1 a_2 + a_1\left(2\,a_0 a_2 + {a_1}^2\right)\right)$$
$$- \left(a + 3\,\gamma\,\kappa\right)\left(2\,m a_0\left(10\,\zeta\,a_2 + 2\,a_1\right) + 2\,m a_1\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right)\right.$$
$$+ 2\,m a_2\left(6\,\zeta\,a_2\eta + 2\,a_1\eta + \lambda^2 a_1\right) + \left(1 - 2\,m\right)\left(4\,a_1\eta\,a_2\right.$$
$$\left.\left. + 2\left(-\zeta\,a_1 - 2\,a_2\eta\right)\left(-2\,\zeta\,a_2 - a_1\right)\right)\right) = 0,$$
$$8\,m^2\left(v + 2\,a\kappa + 3\,\gamma\,\kappa^2\right)a_1 a_2 + 4\,m^2\lambda\left(4\,a_0 a_1 a_2 + a_1\left(2\,a_0 a_2 + {a_1}^2\right)\right)$$
$$+ \gamma\left(2\,m a_0\left(10\,\zeta\,a_2 + 2\,a_1\right) + 2\,m a_1\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right)\right.$$
$$+ 2\,m a_2\left(6\,\zeta\,a_2\eta + 2\,a_1\eta + \lambda^2 a_1\right) + \left(1 - 2\,m\right)\left(4\,a_1\eta\,a_2\right.$$
$$\left.\left. + 2\left(-\zeta\,a_1 - 2\,a_2\eta\right)\left(-2\,\zeta\,a_2 - a_1\right)\right)\right) = 0.$$

$\exp(-4\,\varphi(q))$ coeff:

$$4\,m^2\left(\omega + a\kappa^2 + \gamma\,\kappa^3\right){a_2}^2 - 4\,m^2\left(b - \lambda\,\kappa\right)\left(a_0{a_2}^2 + 2\,{a_1}^2 a_2 + a_2\left(2\,a_0 a_2 + {a_1}^2\right)\right)$$
$$- \left(a + 3\,\gamma\,\kappa\right)\left(\left(1 - 2\,m\right)\left(-4\left(-\zeta\,a_1 - 2\,a_2\eta\right)a_2 + \left(-2\,\zeta\,a_2 - a_1\right)^2\right)\right.$$
$$\left. + 12\,m a_0 a_2 + 2\,m a_1\left(10\,\zeta\,a_2 + 2\,a_1\right) + 2\,m a_2\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right)\right) = 0,$$
$$4\,m^2\left(v + 2\,a\kappa + 3\,\gamma\,\kappa^2\right){a_2}^2 + 4\,m^2\lambda\left(a_0{a_2}^2 + 2\,{a_1}^2 a_2 + a_2\left(2\,a_0 a_2 + {a_1}^2\right)\right)$$
$$+ \gamma\left(\left(1 - 2\,m\right)\left(-4\left(-\zeta\,a_1 - 2\,a_2\eta\right)a_2 + \left(-2\,\zeta\,a_2 - a_1\right)^2\right) + 12\,m a_0 a_2\right.$$
$$\left. + 2\,m a_1\left(10\,\zeta\,a_2 + 2\,a_1\right) + 2\,m a_2\left(4\,\lambda^2 a_2 + 3\,\zeta\,a_1 + 8\,a_2\eta\right)\right) = 0.$$

$\exp(-5\,\varphi(q))$ coeff:

$$-12\,m^2\left(b - \lambda\,\kappa\right)a_1{a_2}^2 - \left(a + 3\,\gamma\,\kappa\right)\left(-4\left(1 - 2\,m\right)\left(-2\,\zeta\,a_2 - a_1\right)a_2 + 12\,m a_1 a_2\right.$$

$$+ 2\,m a_2 \left(10\,\zeta\,a_2 + 2\,a_1\right)\Big) = 0,$$

$$12\,m^2\lambda\,a_1 a_2{}^2 + \gamma\left(-4\,(1-2\,m)\,(-2\,\zeta\,a_2 - a_1)\,a_2 + 12\,m a_1 a_2\right.$$

$$\left. + 2\,m a_2 \left(10\,\zeta\,a_2 + 2\,a_1\right)\right) = 0.$$

$\exp(-6\,\varphi(q))$ coeff:

$$-4\,m^2\,(b - \lambda\,\kappa)\,a_2{}^3 - (a + 3\,\gamma\,\kappa)\left(4\,(1-2\,m)\,a_2{}^2 + 12\,m a_2{}^2\right) = 0,$$

$$4\,m^2\lambda\,a_2{}^3 + \gamma\left(4\,(1-2\,m)\,a_2{}^2 + 12\,m a_2{}^2\right) = 0.$$

Solving this algebraic system of equations gives

$$v = -\frac{1}{16}\frac{2\,m^2 a\lambda\,b\gamma + 3\,m^2 b^2\gamma^2 - 5\,m^2 a^2\lambda^2 - 16\,\gamma^2\zeta\,\eta\,\lambda^2}{\gamma\,\lambda^2 m^2}$$

$$\omega = -\frac{1}{64}\frac{3\,m^2 a^3\lambda^3 - 5\,m^2 a^2\lambda^2 b\gamma + m^2 a\lambda\,b^2\gamma^2 + m^2 b^3\gamma^3 + 16\,\gamma^2 a\zeta\,\eta\,\lambda^3 + 48\,\gamma^3\zeta\,\eta\,\lambda^2 b}{\lambda^3 m^2\gamma^2}$$

$$\kappa = \frac{1}{4}\frac{-a\lambda + b\gamma}{\gamma\,\lambda}, \quad a_0 = -\frac{\gamma\,\zeta\,\eta\,(m+1)}{m^2\lambda},$$

$$a_1 = 0, \quad a_2 = -\frac{\gamma\,\eta^2\,(m+1)}{m^2\lambda},$$

Hence, Periodic, dark, singular and plain wave solutions along with their validity conditions are given below respectively as

$$p_{1,2}(x,t) = -\frac{\gamma\zeta\eta(m+1)}{m^2\lambda} - \frac{\gamma\eta^2(m+1)}{m^2\lambda}\left[\sqrt{\frac{\zeta}{\eta}}\tan(\sqrt{\zeta\eta}(q+q_0))\right]^2$$

$$\times\,\mathrm{e}^{i(-\kappa x + \omega t + \gamma)}, \text{ where } \zeta\eta > 0. \tag{2.45}$$

$$p_{3,4}(x,t) = -\frac{\gamma\zeta\eta(m+1)}{m^2\lambda} - \frac{\gamma\eta^2(m+1)}{m^2\lambda}\left[-\sqrt{\frac{\zeta}{\eta}}\cot(\sqrt{\zeta\eta}(q+q_0))\right]^2$$

$$\times\,\mathrm{e}^{i(-\kappa x + \omega t + \gamma)}, \text{ where } \zeta\eta > 0. \tag{2.46}$$

$$p_{5,6}(x,t) = -\frac{\gamma\zeta\eta(m+1)}{m^2\lambda} - \frac{\gamma\eta^2(m+1)}{m^2\lambda}\left[-\sqrt{\frac{\zeta}{-\eta}}\tanh(\sqrt{-\zeta\eta}(q+q_0))\right]^2$$

$$\times\,\mathrm{e}^{i(-\kappa x + \omega t + \gamma)}, \text{ where } \zeta\eta < 0. \tag{2.47}$$

$$p_{7,8}(x,t) = -\frac{\gamma\zeta\eta(m+1)}{m^2\lambda} - \frac{\gamma\eta^2(m+1)}{m^2\lambda}\left[\sqrt{\frac{\zeta}{-\eta}}\coth(\sqrt{-\zeta\eta}(q+q_0))\right]^2$$

$$\times\,\mathrm{e}^{i(-\kappa x + \omega t + \gamma)}, \text{ where } \zeta\eta < 0. \tag{2.48}$$

(a) $\gamma = -5, \ b = 0.5, \ \lambda = 0.19, \ \eta = 0.5, \ m = 0.1$

(b) $\gamma = -5, \ b = 0.5, \ \lambda = 0.19, \eta = 0.5, \ m = 0.2$

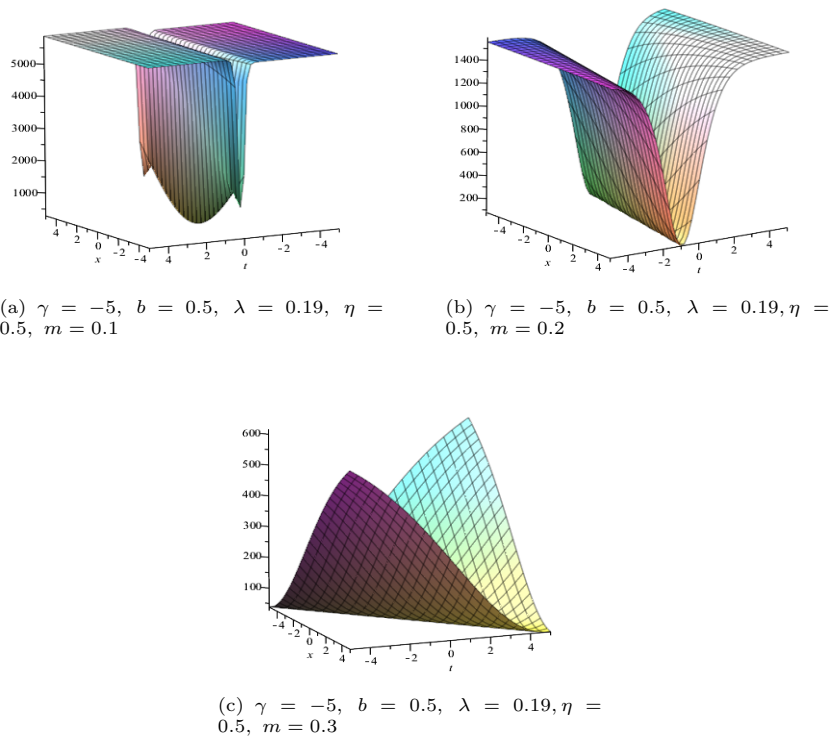(c) $\gamma = -5, \ b = 0.5, \ \lambda = 0.19, \eta = 0.5, \ m = 0.3$

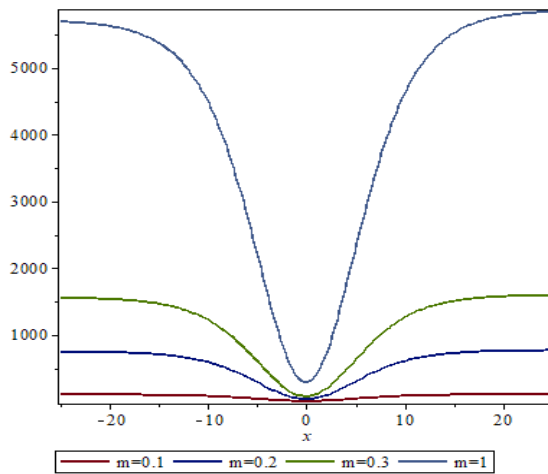**Figure 3.** Dark soliton given in Eqs. (2.47).

Figure 4. 2-D plot of solution given in Eq. (2.47) for different value of index m.

3. Modulation Instability

Modulation instability based on linear stability analysis [2] is carried out for Eq. (1.2) with $m = 1$ which has the subsequent steady-state solution [2]

$$p(x,t) = [\sqrt{I_0} + q(x,t)] \times \mathrm{e}^{\mathrm{ibI_0t}}, \qquad (3.1)$$

where I_0 is the incident power. To study the evolution of the perturbation $q(x,t)$, Eq. (3.1) is substituted into Eq. (1.2) and is linearized in q has the following form

$$iq_t + aq_{xx} + bI_0\,(q + q^*) = i\lambda I_0\,(2q_x + q_x^*) - i\gamma q_{xxx} \qquad (3.2)$$

where q^* is complex conjugate of q. Due to the presentation q^*, we assume

$$q(x,t) = q_1 \mathrm{e}^{\mathrm{i(Kx-\Omega t)}} + q_2 \mathrm{e}^{-\mathrm{i(Kx-\Omega t)}}, \qquad (3.3)$$

where K is the wave number and Ω is the frequency. Substituting Eq. (3.3) in Eq. (3.2), a system of two homogeneous equations in q_1 and q_2 is retrieved which gives the following dispersion relation

$$\Omega = \left(\gamma K^2 - 2\lambda I_0 \pm \sqrt{\lambda^2 I_0{}^2 + K^2 a^2 - 2\,abI_0}\right) K. \qquad (3.4)$$

It is evident from the dispersion relation (3.4) that the steady-state stability depends on self-steepening, GVD as well as the incident power. The steady state is stable whenever $\lambda^2 I_0{}^2 + K^2 a^2 - 2\,abI_0 > 0$ but the modulation instability comes into play when $\lambda^2 I_0{}^2 + K^2 a^2 - 2\,abI_0 < 0$ because in such a case the perturbation grows exponentially. Modulation instability gain is given by

$$G = |\mathrm{Im}\Omega| = |\mathrm{Im}\left(\gamma K^2 - 2\lambda I_0 \pm \sqrt{\lambda^2 I_0{}^2 + K^2 a^2 - 2\,abI_0}\right) K|. \qquad (3.5)$$
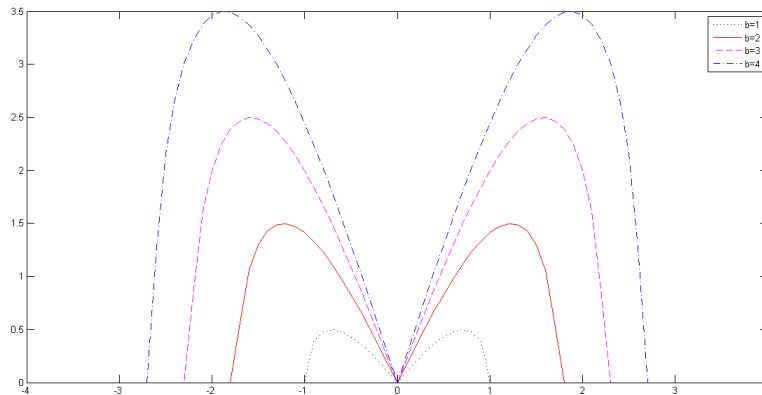

Figure 5. Modulation instability gain in terms of wave number for different values of nonlinear coefficient.
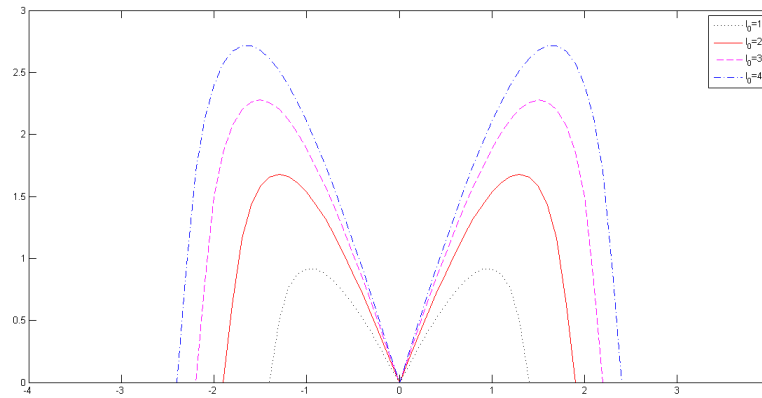
Figure 6. Modulation instability gain in terms of wave number for different incident powers I_0 with $\lambda = 0.4$, $a = 1$, $b = 1$ and $\gamma = 1$.
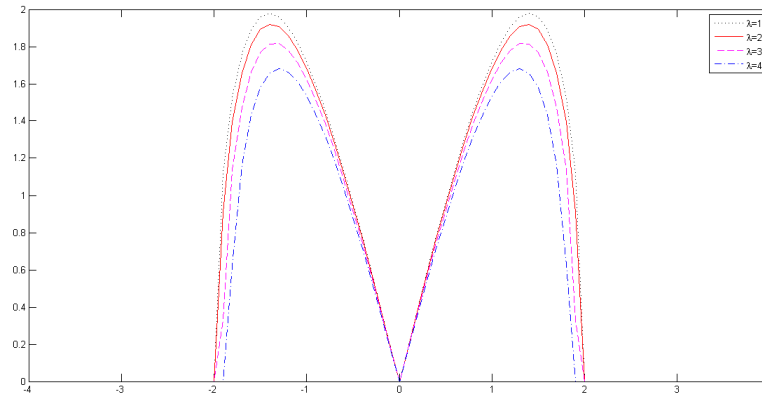

Figure 7. Modulation instability gain in terms of wave number for different values of λ with $I_0 = 0.2$, $a = 1$, $b = 10$ and $\gamma = 1$.

4. Results and Discussion

The results retrieved here are bright, dark, singular, periodic and plane wave solutions. Solutions in (2.10) retrieved through modified simple equation method are visualized through 3-D and 2-D graphical depictions portrayed in Figure 1 and Figure 2 respectively. As we know that the power law nonlinearity induces the same shape profile as given in Eq. (2.10). Also the width and height of the soliton depends on the index of nonlinearity m [27, 28]. It can easily be observed that for the infinitesimal value of index requires a gigantic width. So, if $m \to 0$ the soliton becomes a plane wave as shown in Figure 1(a). It is also observed that the soliton concentration is highly sensitive to the value of nonlinearity near $m = 2$ and the soliton width decreases drastically for $1 < m < 2$. So for $m \geq 2$, the soliton pro-

file becomes unstable and portrays a self focusing singularity which is highlighted through Figure 1 and 2.

Similarly, the 3-D and 2-D graphs of soliton solution given in Eq.(2.47) are portrayed in Figure 3 and Figure 4 respectively which are obtained through $\exp(-\varphi(q))$. From Eq. (2.47) the soliton solution represents a self focusing singularity near $m \to 0$. Unlike the bright soliton given in Eq. (2.10), the width of the soliton profile given in Eq. (2.47) increases as the index of nonlinearity increases and represents a plane wave near $m \to \infty$. The argument is supported by the graphs in Figure 3 and Figure 4.

In section 3, the modulation instability analysis of Eq. (1.2) is carried out. A dispersion relation (3.4) is obtained for the frequency in terms of wave number. The role of different parameters on the gain spectrum $G(K)$ is studied in Fig 5, 6 and 7. In Fig 5, it is clearly observed that the gain spectrum $G(K)$ is symmetric with $K = 0$. It is deduced that the gain spectrum increases with increase in b for given fixed values of $\lambda = 1$ (self-steepening) and $I_0 = 1$ (Incident power). From Fig 6, we can conclude that the instability gain spectrum increases with an increase in incident power I_0 by fixing $\lambda = 0.4$ and $b = 1$. On the other hand, we see that the modulation instability gain spectrum decreases with an increase in self-steepening λ. It is noticed that the third order dispersion plays no role in modulation instability which is quite obvious from Eq. (3.5) and [29, 32].

5. Conclusion

In this work, we have extracted soliton solutions of RKL model incorporating the power law nonlinearity. The extraction process is aided through reliable integration tools namely, the modified simple equation method and the $\exp(-\varphi(q))$ method. The bright, dark, singular, periodic, plane wave and other solutions are retrieved. The solutions extracted through modified simple equation method are catagorized in four different cases on the basis of values of nonlinearity index m. The bright soliton profile in Eq. (2.10) is highlighted through 2 and 3-dimensional graphs. It is observed graphically that the soliton profile is stable only for $0 < m < 2$ and the case 4 assists this argument. Similarly the solutions retrieved through $\exp(-\varphi(q))$ method are given. The physical aspects of the stability of the dark soliton profile given in Eq. (2.47) are highlighted through 2 and 3-dimensional graphs. The modulation instability analysis for the RKL model is carried out to extract a dispersion relation between K and Ω. The effect of different parameters on the Modulation instability gain spectrum is also studied. Therefore, the research provides an excellent additive to the existing literature and gives a valuable insight towards the spatio-temporal solitons of the power law nonlinearity.

6. Conflict of interest

The authors declare that there is no conflict of interest.

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