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POSITIVE SOLUTIONS FOR A *P*-LAPLACIAN TYPE SYSTEM OF IMPULSIVE FRACTIONAL BOUNDARY VALUE PROBLEM*

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Abstract In this paper, the aim is to discuss a class of p-Laplacian type fractional Dirichlet's boundary value problem involving impulsive impacts. Based on the approaches of variational method and the properties of fractional derivatives on the reflexive Banach spaces, the existence results of positive solutions for our equations are established. Two examples are given at the end of each main result.

Keywords Fractional differential system, p-Laplacian operator, positive solution, variational method.

MSC(2010) 26A33, 34B15, 35A15.

1. Introduction

This paper deals with a class of p-Laplacian type impulsive fractional differential system with Dirichlet's boundary value conditions

$$\begin{cases} {}_tD_T^{\alpha_i}\Phi_p(a_i(t){}_0D_t^{\alpha_i}u^i(t)) = \lambda(t)f_{u^i}(t,u(t)),\ t\in[0,T],\ t\neq t_j, \\ \Delta({}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i))(t_j) = I_{ij}(u^i(t_j)),\ j=1,2,...,m, \\ u^i(0)=u^i(T)=0, \end{cases} \tag{1.1}$$

for $i=1,2,...,n$, where $n\geq 1$, $u=(u^1,...,u^n)$, ${}_0D_t^{\alpha_i}$ and ${}_tD_T^{\alpha_i}$ are the left and right standard Riemann-Liouville derivatives with $0<\alpha_i\leq 1$, respectively, $i=1,2,...,n$. $\Phi_p(s)=|s|^{p-2}s$ $(s\neq 0, p>1)$. $\lambda(t)$ and $a_i(t)$ belong to $L^\infty([0,T],\mathrm{R}^+)$ with $\lambda^0=\operatorname{ess\,sup}_{[0,T]}\lambda(t)$, $\lambda_0=\operatorname{ess\,inf}_{[0,T]}\lambda(t)>0$, $a_i^0=\operatorname{ess\,sup}_{[0,T]}a_i(t)$, $a_{i_0}=\operatorname{ess\,inf}_{[0,T]}a_i(t)>0$, for $i=1,2,...,n$. $f(t,u):[0,T]\times\mathrm{R}^n\to\mathrm{R}$ is measurable with respect to t in $[0,T]$ for every $u\in\mathrm{R}^n$, and continuously differentiable in u for every $t\in[0,T]$, f_s denotes the partial derivative of f with respect to s. $I_{ij}\in C(\mathrm{R},\mathrm{R})$

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for $i = 1, 2, ..., n,\ j = 1, 2, ..., m,\ 0 = t_0 < t_1 < ... < t_{m+1} = T$, the operator Δ is defined as

$$\Delta({}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i))(t_j) = {}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^+) - {}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^-),$$

where

$$\begin{aligned}{}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^+) &= \lim_{t\to t_j^+} {}_tD_T^{\alpha_i-1}\Phi_p(a_i(t){}_0^cD_t^{\alpha_i}u^i(t)),\\ {}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^-) &= \lim_{t\to t_j^-} {}_tD_T^{\alpha_i-1}\Phi_p(a_i(t){}_0^cD_t^{\alpha_i}u^i(t)),\end{aligned}$$

and ${}_0^cD_t^{\alpha_i}$ denotes the left Caputo fractional derivative with $0 < \alpha_i \le 1$, $i = 1, 2, ..., n$.

Fractional differential boundary value problems (BVPs for short) can describe many interesting nonlinear phenomena due to the fact that they have wild application background in multifarious fields of science, for instance, mathematics, biological processes, chemical engineering, underground water flow, thermo-elasticity and plasma physics ([8, 11, 16]), etc. For this reason, many researchers have been attracted to focus on this kind of problems, and a large number of meaningful results have been obtained in resent years, (we refer the reader to the papers [5,22,29]). The classical approaches, such as the method of mixed monotone iterative, topological degree theory, fixed-point theorems and upper and lower solutions method, etc, are always used to investigate the existence results of positive solutions for nonlinear BVPs, and those methods have been developed maturely. Since the formulation of ordinary p-Laplacian operator was put forward by Leibenson in 1983 [21], there are numerous applications in nonlinear elastic mechanics, non-Newtonian fluid theory, and so on. Based on some classical metheds, many relevant existence results for fractional differential equations with generalized p-Laplacian operator have been established ([12, 13, 23, 31]). In [13], the following eigenvalue problem of nonlinear fractional differential equation with p-Laplacian operator was given

$$\begin{cases} D_{0^+}^{\beta}(\Phi_p(D_{0^+}^{\alpha}u(t))) = \lambda f(u(t)),\ 0 < t < 1,\\ u(0) = u'(0) = u'(1) = 0,\ \Phi_p(D_{0^+}^{\alpha}u(0)) = (\Phi_p(D_{0^+}^{\alpha}u(1)))' = 0,\end{cases} \tag{1.2}$$

where $\lambda > 0$ is a parameter, $2 < \alpha \le 3$, $1 < \beta \le 2$, Φ_p is the generalized p-Laplacian operator, $f : (0, +\infty) \to (0, +\infty)$ is continuous. The authors discussed the existence of at least one or two positive solutions for (1.2) by the Guo-Krasnosel'skii fixed point theorem in cones. Moreover, relying on the generalization of Leggett-Williams fixed point theorem, the existence of at least three positive solutions was obtained for the p-Laplacian type fractional equation involving both the Riemann-Liouville fractional derivatives and Caputo fractional derivatives in [23]. By virtue of some methods from nonlinear functional analysis including the contraction mapping theorem and the Brouwer fixed point theorem, the authors presented the existence and uniqueness of solution for a discrete fractional BVP with p-Laplacian operator in [31].

One the other hand, since Ambrosetti and Rabinowitz proposed Mountain pass theorem in 1973 [1], variational methods together with critical point theory have become useful and practical tools in dealing with the existence results for fractional differential equations in recent years [2, 10]. The problem of p-Laplacian type fractional differential equation is also studied based on variational approachs [6, 17–19].

For example, Li et al. in reference [17] considered a class of ordinary p-Laplacian type equation with the form of

$$\begin{cases} (\mid u'(t) \mid^{p-2} u'(t)) + \nabla F(t, u(t)) = 0, \\ u(0) - u(T) = u'(0) - u'(T) = 0, \end{cases} \tag{1.3}$$

where $p > 1$, $T > 0$, $F : [0, T] \times \mathrm{R}^{\mathrm{N}} \to \mathrm{R}$ is T-periodic in t for all $u \in \mathrm{R}^{\mathrm{N}}$. Through the generalized Mountain pass theorem of Rabinowitz, the existence of periodic solutions was studied for problem (1.3). In [18], the existence of at least one nontrivial solution was discussed for the following fractional BVP with p-Laplacian operator by using the Mountain pass theorem and iterative technique

$$\begin{cases} {}_tD_T^{\alpha}\left(\frac{1}{\omega(t)^{p-2}}\varphi_p(\omega(t)_0D_t^{\alpha}u(t))\right) + \lambda u(t) = f(t, u, {}_0^cD_t^{\alpha}u(t)) + h(u(t)), \\ u(0) = u(T) = 0, \text{ a.e. } t \in [0, T], \end{cases}$$

where $f : [0, T] \times \mathrm{R} \times \mathrm{R} \to \mathrm{R}$ is continuous, $h : \mathrm{R} \to \mathrm{R}$ is a Lipschitz continuous function with the Lipschitz constant $L > 0$. Furthermore, by using critical point theory, the existence of at least one weak solution was studied for a fractional differential equation with generalized p-Laplacian operator in [6].

Additionally, it is well known that the normal characteristic of impulsive effects is the sudden changes at some certain moments owing to instantaneous disturbances. Because the impulses always appear in many actual systems, such as in multi-agent systems, signal processing systems, automatic control systems, etc., the research on the impulsive problem of fractional differential BVPs has received great progress. Recently, many existence results about impulsive fractional differential equations via variational methods have been studied (see [3, 4, 9, 10, 25]).

Inspired by the mentioned work above, this paper devotes to investigate a class of p-Laplacian type impulsive fractional system with Dirichlet's boundary value conditions. Based on the variational approaches and the properties of fractional derivatives defined on reflexive Banach spaces, the existence results for the problem (1.1) are established. The main features for our paper are stated as follows: Firstly, we discuss the existence of positive solutions for problem (1.1) in the general case $1 < p < \infty$, which is the generalization for some related results based on the particular case of $p = 2$. It's worth noting that the differential operator ${}_tD_T^{\alpha}\Phi_p({}_0D_t^{\alpha})$ $(\alpha > 0, p > 1)$ is nonlocal and nonlinear, and it can be reduced to the linear differential operator ${}_tD_T^{\alpha}{}_0D_t^{\alpha}$ under $p = 2$. Further, if take $\alpha = 1$, the operator ${}_tD_T^{\alpha}{}_0D_t^{\alpha}$ can be recovered the usual definition, i.e., a second-order differential operator $\frac{d^2}{dt^2}$. So that, BVPs (1.1) can be recovered a integer-order Dirichlet's BVPs with impulsive impacts in the particular case of $p = 2$, $\alpha_i = 1$, $i = 1, 2, ..., n$, i.e.,

$$\begin{cases} a_i(t)\ddot{u}^i(t) = \lambda(t)f_{u^i}(t, u(t)), \ t \in [0, T], \ t \neq t_j, \\ \Delta(a_i(t_j)\dot{u}^i(t_j)) = I_{ij}(u^i(t_j)), \ j = 1, 2, ..., m, \\ u^i(0) = u^i(T) = 0, \end{cases}$$

for $i = 1, 2, ..., n$. Secondly, we discuss a n-dimensional fractional differential equations rather than a single equation and consider the impulsive effects, which cause some aspects of the paper more complicated, such as the Euler-Lagrange functional related to the system (1.1), the procedure of applying the Mountain pass theorem. Finally, the positive solution is also studied in this paper. To the best of authors' knowledge, little work has been developed on studying the positive solutions for generalized p-Laplacian type impulsive fractional system by using the variational method. Therefore, it is worthwhile to be investigated. So that, our main results are different from those relevant literatures mentioned above, and moreover, complement the results in previous ones.

This paper's organization is stated as follows. In section 2, some basic definitions and preliminary facts for fractional calculus are introduced. In section 3, we establish appropriate function spaces and the variational framework for problem (1.1), which are necessary for the discussion of this paper. Then, applying critical point theorems, the main results are obtained, and meanwhile, tow examples are given to illustrate the applications of our results in section 4. Finally, a conclusion is presented in section 5.

2. Preliminaries

In this section, some basic definitions and properties are introduced for fractional calculus, and some important theorems are given, which shall be used throughout this paper.

For any $[a, b] \subseteq \mathrm{R}$, denote

$$\|x\|_\infty = \max_{t\in[a,b]} \mid x(t) \mid, \quad \forall\, x \in C([a, b], \mathrm{R});$$
$$\|x\|_{L^p} = (\int_a^b \mid x(t) \mid^p dt)^{\frac{1}{p}}, \quad \forall\, x \in L^p([a, b], \mathrm{R}).$$

Definition 2.1 ([16,26])**.** Let x be a function defined on $[a, b]$. Then the left and right Riemann-Liouville fractional integrals denoted by ${}_aD_t^{-\gamma}$ and ${}_tD_b^{-\gamma}$ with order $\gamma > 0$ are defined by

$${}_aD_t^{-\gamma}x(t) = \frac{1}{\Gamma(\gamma)} \int_a^t (t-s)^{\gamma-1}x(s)ds,$$
$${}_tD_b^{-\gamma}x(t) = \frac{1}{\Gamma(\gamma)} \int_t^b (s-t)^{\gamma-1}x(s)ds.$$

Definition 2.2 ([16,26])**.** Let x be a function defined on $[a, b]$. Define the left and right Riemann-Liouville fractional derivatives denoted by ${}_aD_t^{\gamma}$ and ${}_tD_b^{\gamma}$ with order $n-1 \leq \gamma < n$ and $n \in \mathrm{N}$ as follows

$${}_aD_t^{\gamma}x(t) = \frac{d^n}{dt^n}{}_aD_t^{\gamma-n}x(t) = \frac{1}{\Gamma(n-\gamma)}\frac{d^n}{dt^n} \int_a^t (t-s)^{n-\gamma-1}x(s)ds,$$
$${}_tD_b^{\gamma}x(t) = (-1)^n\frac{d^n}{dt^n}{}_tD_b^{\gamma-n}x(t) = \frac{(-1)^n}{\Gamma(n-\gamma)}\frac{d^n}{dt^n} \int_t^b (s-t)^{n-\gamma-1}x(s)ds,$$

in particular, if $\gamma = n-1$, then, the left and right Riemann-Liouville fractional derivatives ${}_aD_t^{n-1}x(t)$ and ${}_tD_b^{n-1}x(t)$ can be recovered the usual definitions, i.e.,

$${}_aD_t^{n-1}x(t) = x^{n-1}(t), \quad {}_tD_b^{n-1}x(t) = (-1)^{n-1}x^{n-1}(t), \ t \in [a, b].$$

Definition 2.3 ([16, 26]). Let $\gamma \geq 0$, $n \in \mathrm{N}$.
(I) If $x(t) \in AC^n([a,b],\mathrm{R})$ and $n-1<\gamma<n$. Then the left and right Caputo fractional derivatives of order γ for x denoted as ${}_a^cD_t^{\gamma}x(t)$ and ${}_t^cD_b^{\gamma}x(t)$ respectively, are given by

$$
\begin{aligned}
{}_a^cD_t^{\gamma}x(t) &= {}_aD_t^{\gamma-n}x^n(t) = \frac{1}{\Gamma(n-\gamma)}\int_a^t (t-s)^{n-\gamma-1}x^n(s)ds,\\
{}_t^cD_b^{\gamma}x(t) &= (-1)^n{}_tD_b^{\gamma-n}x^n(t) = \frac{(-1)^n}{\Gamma(n-\gamma)}\int_t^b (s-t)^{n-\gamma-1}x^n(s)ds.
\end{aligned}
$$

(II) If $x(t) \in AC^{n-1}([a,b],\mathrm{R})$ and $\gamma = n-1$, we can obtain the usual definitions for the Caputo fractional derivatives, i.e.,

$$
{}_a^cD_t^{n-1}x(t) = x^{n-1}(t),\ \ {}_t^cD_b^{n-1}x(t) = (-1)^{n-1}x^{n-1}(t),\ t \in [a,b],
$$

particularly, ${}_a^cD_t^0x(t) = {}_t^cD_b^0x(t) = x(t)$ for $t \in [a,b]$.

Lemma 2.1 ([16, 26]). *Let x be a function defined on $[a,b]$ and $n-1<\gamma<n$, $n \in \mathrm{N}$. The Caputo fractional derivatives ${}_a^cD_t^{\gamma}x(t)$ and ${}_t^cD_b^{\gamma}x(t)$ of order γ and the Riemann-Liouville fractional derivatives ${}_aD_t^{\gamma}x(t)$ and ${}_tD_b^{\gamma}x(t)$ have the following characteristics to contact with each other:*

$$
\begin{aligned}
{}_a^cD_t^{\gamma}x(t) &= {}_aD_t^{\gamma}x(t) - \sum_{i=0}^{n-1}\frac{x^i(a)}{\Gamma(i-\gamma+1)}(t-a)^{i-\gamma},\\
{}_t^cD_b^{\gamma}x(t) &= {}_tD_b^{\gamma}x(t) - \sum_{i=0}^{n-1}\frac{x^i(b)}{\Gamma(i-\gamma+1)}(b-t)^{i-\gamma},
\end{aligned}
$$

in addition, when $0<\gamma<1$, one has

$$
{}_a^cD_t^{\gamma}x(t) = {}_aD_t^{\gamma}x(t) - \frac{x(a)}{\Gamma(1-\gamma)}(t-a)^{-\gamma}, \tag{2.1}
$$

$$
{}_t^cD_b^{\gamma}x(t) = {}_tD_b^{\gamma}x(t) - \frac{x(b)}{\Gamma(1-\gamma)}(b-t)^{-\gamma}. \tag{2.2}
$$

Lemma 2.2. *If there exist $C>0$, $\mu_i>p$, $i=1,2,...,n$, such that $0<f(t,x)\leq \sum\limits_{i=1}^{n}\frac{1}{\mu_i}f_{x^i}(t,x)x^i$, for any $x=(x^1,x^2,...,x^n)\in\mathrm{R}^{\mathrm{n}}$ with $\sum\limits_{i=1}^{n}\mid x^i\mid^{\mu_i}\geq 2C$. Then, the following inequality:*

$$
f(t,x) \geq c_1(\sum_{i=1}^{n}\mid x^i\mid^{\mu_i}) - c_2,\ \textit{for } c_1,c_2>0
$$

holds. (The proof immediately follows from Theorem 5 of [7].)

Nextly, we point out some fundamental definitions and theorems, which will be used to obtain our main results.

Definition 2.4. Let E be a real Banach space and functional $\varphi \in C^1(E,\mathrm{R})$. If any sequence $\{u_k\}_{k=1}^{\infty}\subset E$ for which $\{\varphi(u_k)\}_{k=1}^{\infty}$ is bounded and $\varphi'(u_k)\to 0$ as $k\to\infty$ possesses a convergent subsequence, then we say that φ satisfies the Palais-Smale condition (*P.S.* condition for short).

Theorem 2.1 ([24]). *Let E be a real reflexive Banach space. If the functional $\varphi : E \to \mathrm{R}$ is weakly lower semicontinuous and coercive, i.e., $\lim\limits_{\|z\|\to\infty} \varphi(z) = +\infty$, then there exists $z_0 \in E$ such that $\varphi(z_0) = \inf\limits_{z\in E} \varphi(z)$. Moreover, if φ is also Fréchet differentiable on E, then $\varphi'(z_0) = 0$.*

Theorem 2.2 ([20,27]). *Let E be a real Banach space and functional $\varphi \in C^1(E, \mathrm{R})$ satisfies the P.S. condition. Suppose that*
(C_1) $\varphi(0) = 0$;
(C_2) There exist $\rho > 0$ and $\sigma > 0$ such that $\varphi(z) \geq \sigma$ for all $z \in E$ with $\| z \| = \rho$;
(C_3) There exists $z_1 \in E$ with $\| z_1 \| > \rho$ such that $\varphi(z_1) < \sigma$.

Then φ possesses a critical value $c \geq \sigma$. Moreover, c can be characterized as

$$c = \inf_{g\in\Omega} \max_{z\in g([0,1])} \varphi(z),$$

where $\overline{\Omega} = \{g \in C([0, 1], E) \mid g(0) = 0, g(1) = z_1\}$.

3. Fractional derivative spaces and variational setting

In what follows, the appropriate function spaces and the variational framework for problem (1.1) are established.

Definition 3.1. Let $1 < p < \infty$, $0 < \alpha_i \leq 1$, $i = 1, 2, ..., n$. The fractional derivative space $E_0^{\alpha_i,p}$ is defined as

$$E_0^{\alpha_i,p} = \{u^i(t) \in L^p([0,T], \mathrm{R}) |{}_0D_t^{\alpha_i} u^i(t) \in L^p([0,T], \mathrm{R}), u^i(0) = u^i(T) = 0\},$$

with the norm

$$\|u^i\|_{\alpha_i,p} = \left(\int_0^T \mid u^i(t) \mid^p dt + \int_0^T a_i(t) \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt \right)^{\frac{1}{p}}, \tag{3.1}$$

for any $u^i(t) \in E_0^{\alpha_i,p}$, $i = 1, 2, ..., n$.

Remark 3.1. For any $u^i(t) \in E_0^{\alpha_i,p}$ with $u^i(0) = u^i(T) = 0$, $i = 1, 2, ..., n$, owing to (2.1) and (2.2) yields

$${}_0D_t^{\alpha_i} u^i(t) = {}_0^cD_t^{\alpha_i} u^i(t), \quad {}_tD_T^{\alpha_i} u^i(t) = {}_t^cD_T^{\alpha_i} u^i(t), \ \forall t \in [0,T], \ i = 1, 2, ..., n.$$

Lemma 3.1 ([14]). *Let $1 < p < \infty$, $0 < \alpha_i \leq 1$, $i = 1, 2, ..., n$. For any $u^i(t) \in E_0^{\alpha_i,p}$, we have*

$$\| u^i \|_{L^p} \leq \frac{T^{\alpha_i}}{\Gamma(\alpha_i + 1)} \| {}_0D_t^{\alpha_i} u^i \|_{L^p}, \ \textit{for } i = 1, 2, ..., n, \tag{3.2}$$

when $\alpha_i > \frac{1}{p}$ and $\frac{1}{p} + \frac{1}{q} = 1$, one has

$$\| u^i \|_\infty \leq \frac{T^{\alpha_i - \frac{1}{p}}}{\Gamma(\alpha_i)((\alpha_i - 1)q + 1)^{\frac{1}{q}}} \| {}_0D_t^{\alpha_i} u^i \|_{L^p}, \ \textit{for } i = 1, 2, ..., n. \tag{3.3}$$

Remark 3.2. For any $u^i(t) \in E_0^{\alpha_i,p}$, $i = 1, 2, ..., n$, based on lemma 3.1, one has

$$\| u^i \|_{L^p} \leq \mathbf{a}_i \bigg(\int_0^T a_i(t) \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt \bigg)^{\frac{1}{p}}, \ \forall\ 0 < \alpha_i \leq 1, \tag{3.4}$$

$$\| u^i \|_{\infty} \leq \mathbf{b}_i \bigg(\int_0^T a_i(t) \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt \bigg)^{\frac{1}{p}}, \ \ \forall\ \alpha_i > \frac{1}{p}, \frac{1}{p} + \frac{1}{q} = 1, \tag{3.5}$$

where

$$\mathbf{a}_i = \frac{T^{\alpha_i}}{\Gamma(\alpha_i + 1) a_{i_0}^{\frac{1}{p}}}, \quad \mathbf{b}_i = \frac{T^{\alpha_i - \frac{1}{p}}}{\Gamma(\alpha_i) a_{i_0}^{\frac{1}{p}} ((\alpha_i - 1)q + 1)^{\frac{1}{q}}}, \ i = 1, 2, ..., n. \tag{3.6}$$

Therefore, from (3.4), the norm of (3.1) can be replaced with the following norm

$$\|u^i\|_{\alpha_i,p} = \bigg(\int_0^T a_i(t) \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt \bigg)^{\frac{1}{p}}, \ \ \forall u^i(t) \in E_0^{\alpha_i,p}, \ i = 1, 2, ..., n. \tag{3.7}$$

Then, from (3.4) and (3.5), we also have

$$\sum_{i=1}^n \| u^i \|_{L^p}^p \leq \mathcal{A} (\sum_{i=1}^n \| u^i \|_{\alpha_i,p}^p), \ \forall\ 0 < \alpha_i \leq 1, \ i = 1, 2, ..., n, \tag{3.8}$$

$$\sum_{i=1}^n \| u^i \|_{\infty}^p \leq \mathcal{B} (\sum_{i=1}^n \| u^i \|_{\alpha_i,p}^p), \ \forall\ \alpha_i > \frac{1}{p}, \ i = 1, 2, ..., n, \tag{3.9}$$

where

$$\mathcal{A} = \max_{1 \leq i \leq n} \left\{ \frac{T^{p\alpha_i}}{(\Gamma(\alpha_i + 1))^p a_{i_0}} \right\}, \ \mathcal{B} = \max_{1 \leq i \leq n} \left\{ \frac{T^{p\alpha_i - 1}}{(\Gamma(\alpha_i))^p a_{i_0} ((\alpha_i - 1)q + 1)^{\frac{p}{q}}} \right\}. \tag{3.10}$$

Define X is a Cartesian product of n fractional derivative spaces, i.e., $X = E_0^{\alpha_1,p} \times E_0^{\alpha_2,p} \times ... \times E_0^{\alpha_n,p}$, and furnished with the norm $\| u \|_X = (\sum\limits_{i=1}^n \| u^i \|_{\alpha_i,p}^p)^{\frac{1}{p}}$ for any $u(t) = (u^1(t), u^2(t), ..., u^n(t)) \in X$.

For any $x(t) = (x^1(t), x^2(t), ..., x^n(t)) \in X$, denote $\| x \|_{\infty} = \sum\limits_{i=1}^n \max\limits_{t \in [0,T]} |x^i(t)|$. Then, we obtain

$$\begin{aligned} \| x \|_{\infty} &\leq \sum_{i=1}^n \frac{T^{\alpha_i - \frac{1}{p}}}{\Gamma(\alpha_i)((\alpha_i - 1)q + 1)^{\frac{1}{q}} a_{i_0}^{\frac{1}{p}}} \ \| x^i \|_{\alpha_i,p} \\ &\leq d (\sum_{i=1}^n \| x^i \|_{\alpha_i,p})^{p \cdot \frac{1}{p}} \leq nd (\sum_{i=1}^n \| x^i \|_{\alpha_i,p}^p)^{\frac{1}{p}} = nd \| x \|_X, \end{aligned} \tag{3.11}$$

where $\alpha_i > \frac{1}{p}$, $i = 1, 2, ..., n$, $d = \max\limits_{1 \leq i \leq n} \left\{ \frac{T^{\alpha_i - \frac{1}{p}}}{\Gamma(\alpha_i)((\alpha_i - 1)q + 1)^{\frac{1}{q}} a_{i_0}^{\frac{1}{p}}} \right\}$ and the inequality $(\sum\limits_{i=1}^n b_i)^p \leq n^p \sum\limits_{i=1}^n b_i^p$ ($b_i \in \mathrm{R}^+$, $i = 1, 2, ..., n$) is used.

Lemma 3.2. *The fractional derivative space X is a reflexive and separable Banach space.*

Proof. The proof is similar to Lemma 9 of [19], we omit it here. □

Lemma 3.3 ([14]). *Let $\frac{1}{p} < \gamma \leq 1$ and $1 < p < \infty$. Assume that the sequence $\{u_k\}$ converges to u in $E_0^{\gamma,p}$ weakly as $k \to \infty$, then $u_k \to u$ in $C([0,T],\mathrm{R})$.*

Here, we give the definition of weak solution for BVPs (1.1).

Definition 3.2. If $u(t) = (u^1(t), u^2(t), ..., u^n(t)) \in X$ is a weak solution of the system (1.1), we mean that for any $v(t) = (v^1(t), v^2(t), ..., v^n(t)) \in X$, such that the following relationship holds

$$\begin{aligned}&\sum_{i=1}^{n}\int_0^T \Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t))_0D_t^{\alpha_i}v^i(t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u^i(t_j))v^i(t_j)\\ =&\sum_{i=1}^{n}\int_0^T \lambda(t)f_{u^i}(t,u^1(t),u^2(t),...,u^n(t))v^i(t)dt.\end{aligned}$$

Lemma 3.4. *For any $u^i, v^i \in E_0^{\alpha_i,p}$, $i = 1,2,...,n$, the following identity holds:*

$$\begin{aligned}&\int_0^T \Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t))_0D_t^{\alpha_i}v^i(t)dt\\ =&\int_0^T {}_tD_T^{\alpha_i}(\Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t)))v^i(t)dt - \sum_{j=1}^{m}\Delta({}_tD_T^{\alpha_i-1}\Phi_p(a_i(t_j){}_0^cD_t^{\alpha_i}u^i(t_j)))v^i(t_j),\end{aligned}$$

(The proof immediately follows from Proposition 2.5 in [32].).

Remark 3.3. We say that $u(t) \in X$ is a classical solution of BVPs (1.1) if $u(t)$ is a weak solution of (1.1).

Proof. Indeed, if $u(t)$ is a weak solution of BVPs (1.1), then, the Definition 3.2 holds. Choose a function $v(t) = (v^1(t), v^2(t), ..., v^n(t)) \in X$ satisfying $v(t) = 0$ for any $t \in [0,t_j] \cup [t_{j+1},T]$, $j \in \{1,...,m\}$. By using Definition 3.2 and Lemma 3.4, we get

$$\begin{aligned}&\sum_{i=1}^{n}\int_{t_j}^{t_{j+1}} \Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t))_0D_t^{\alpha_i}v^i(t)dt\\ =&\sum_{i=1}^{n}\int_{t_j}^{t_{j+1}} {}_tD_T^{\alpha_i}(\Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t)))v^i(t)dt,\end{aligned}\tag{3.12}$$

and

$$\begin{aligned}&\sum_{i=1}^{n}\int_{t_j}^{t_{j+1}} \Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t))_0D_t^{\alpha_i}v^i(t)dt\\ =&\sum_{i=1}^{n}\int_{t_j}^{t_{j+1}} \lambda(t)f_{u^i}(t,u^1(t),u^2(t),...,u^n(t))v^i(t)dt.\end{aligned}\tag{3.13}$$

Hence, we can obtain from (3.12) and (3.13) that

$${}_tD_T^{\alpha_i}\Phi_p(a_i(t)_0D_t^{\alpha_i}u^i(t)) = \lambda(t)f_{u^i}(t,u^1(t),u^2(t),...,u^n(t)), \quad \forall\, t \in (t_j,t_{j+1}), \tag{3.14}$$

which shows that $u(t)$ satisfies the equation of BVPs (1.1) for every $t \in [0,T]$, $t \neq t_j$, $j = 1,2,...,m$.

Moreover, according to Proposition 2.6 in [32], we know that the following limits

$$
\begin{aligned}
{}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^+) &= \lim_{t\to t_j^+} {}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t),\\
{}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t_j^-) &= \lim_{t\to t_j^-} {}_tD_T^{\alpha_i-1}\Phi_p(a_i{}_0^cD_t^{\alpha_i}u^i)(t),
\end{aligned}
$$

exist. From Lemma 3.4, multiplying (3.14) by $v^i \in E_0^{\alpha_i,p}$ and integrating between 0 and T before summing i from 1 to n, yields

$$
\sum_{i=1}^{n}\sum_{j=1}^{m}\Delta({}_tD_T^{\alpha_i-1}\Phi_p(a_i(t_j){}_0^cD_t^{\alpha_i}u^i(t_j)))v^i(t_j) = \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u^i(t_j))v^i(t_j).
$$

Therefore, $\Delta({}_tD_T^{\alpha_i-1}\Phi_p(a_i(t_j){}_0^cD_t^{\alpha_i}u^i(t_j))) = I_{ij}(u^i(t_j))$, for $i = 1,2,...,n$, $j = 1,2,...,m$, which means that u satisfies the impulsive conditions of BVPs (1.1). Meanwhile, according to the definition of X that u also satisfies the boundary conditions of (1.1). So that, u is a classical solution of (1.1).

Obviously, u is a weak solution of (1.1) if u is a classical one. □

Define the Euler-Lagrange functional $I : X \to \mathrm{R}$ related to BVPs (1.1) by

$$
\begin{aligned}
I(u(t)) =& \frac{1}{p}\sum_{i=1}^{n}\int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i}u^i(t) \mid^p dt + \sum_{i=1}^{n}\sum_{j=1}^{m}\int_0^{u^i(t_j)} I_{ij}(s)ds\\
&- \int_0^T \lambda(t)f(t,u^1(t),u^2(t),...,u^n(t))dt, \quad \forall\ u(t) \in X.
\end{aligned}
$$

Lemma 3.5. *We claim that $I \in C^1(X,\mathrm{R})$.*

Proof. Denote

$$
\begin{aligned}
I_1(u(t)) &= \frac{1}{p}\sum_{i=1}^{n}\int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i}u^i(t) \mid^p dt + \sum_{i=1}^{n}\sum_{j=1}^{m}\int_0^{u^i(t_j)} I_{ij}(s)ds,\\
I_2(u(t)) &= \int_0^T \lambda(t)f(t,u^1(t),u^2(t),...,u^n(t))dt, \quad \forall\ u(t) = (u^1(t),...,u^n(t)) \in X.
\end{aligned}
$$

Note that f is continuously differentiable and I_{ij} are continuous for $j = 1,2,...,m$, $i = 1,2,...,n$. Thus, $I_1(u)$ and $I_2(u)$ are clearly continuous and differentiable on X, and we present the derivative of I at the point $u(t) \in X$ as

$$
\begin{aligned}
I'(u(t))(v(t)) =& I_1'(u(t))(v(t)) - I_2'(u(t))(v(t))\\
=& \sum_{i=1}^{n}\int_0^T \Phi_p(a_i(t){}_0D_t^{\alpha_i}u^i(t)){}_0D_t^{\alpha_i}v^i(t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u^i(t_j))v^i(t_j)\\
&- \sum_{i=1}^{n}\int_0^T \lambda(t)f_{u^i}(t,u^1(t),u^2(t),...,u^n(t))v^i(t)dt, \ \forall\ v(t) \in X.
\end{aligned}
$$

Using a standard argument in Theorem 4.1 of [15], we have $I_1(u(t)) \in C^1(X,\mathrm{R})$. Nextly, we claim that I_2' is continuous. Suppose that $(u^1,u^2,...,u^n) \to (u_0^1,u_0^2,...,u_0^n)$

in X. According to Lemma 3.3, we have $(u^1, u^2, ..., u^n) \to (u_0^1, u_0^2, ..., u_0^n)$ on $[0, T]$. Then, for any $u = (u^1, u^2, ..., u^n), x = (x^1, x^2, ..., x^n) \in X$ with $\| x \|_X = 1$, one has

$$\begin{aligned}
&\sup_{\|x\|_X=1} | I_2'(u)(x) - I_2'(u_0)(x) | \\
&\leq \sup_{\|x\|_X=1} \lambda^0 \int_0^T | (\nabla f(t, u^1, u^2, ..., u^n) - \nabla f(t, u_0^1, u_0^2, ..., u_0^n))(x^1, x^2, ..., x^n) | \, dt \\
&\leq \sup_{\|x\|_X=1} \lambda^0 \int_0^T | \nabla f(t, u^1, u^2, ..., u^n) - \nabla f(t, u_0^1, u_0^2, ..., u_0^n) || (x^1, x^2, ..., x^n) | \, dt \\
&\leq \sup_{\|x\|_X=1} \lambda^0 T \| (\nabla f(t, u^1, u^2, ..., u^n) - \nabla f(t, u_0^1, u_0^2, ..., u_0^n) \|_\infty \| x \|_\infty \\
&\leq \lambda^0 T n d \| (\nabla f(t, u^1, u^2, ..., u^n) - \nabla f(t, u_0^1, u_0^2, ..., u_0^n)) \|_\infty,
\end{aligned}$$

where (3.11) is used and $\nabla f(t, u^1, u^2, ..., u^n)$ is the gradient of f at $(u^1, u^2, ..., u^n)$. Recalling that $(u^1, u^2, ..., u^n) \to (u_0^1, u_0^2, ..., u_0^n)$ on $[0, T]$ and f is continuously differentiable in $(u^1, u^2, ..., u^n)$ for every $t \in [0, T]$, we have

$$\| \nabla f(t, u^1, u^2, ..., u^n) - \nabla f(t, u_0^1, u_0^2, ..., u_0^n) \|_\infty \to 0 \text{ as } (u^1, u^2, ..., u^n) \to (u_0^1, u_0^2, ..., u_0^n).$$

Therefore, we obtain $I_2'(u)(x) - I_2'(u_0)(x) \to 0$ with $\| x \|_X = 1$, which means that I_2' is continuous. So that $I \in C^1(X, \mathrm{R})$. □

In order to simplify the description of further discussion, some notations are stated here. Denote

$$\omega_* = \begin{cases} \min\limits_{1 \leq i \leq n} \{(a_i^0)^{p-2}\}, & 1 < p \leq 2, \\ \min\limits_{1 \leq i \leq n} \{a_{i_0}^{p-2}\}, & p \geq 2, \end{cases} \quad \omega^* = \begin{cases} \max\limits_{1 \leq i \leq n} \{a_{i_0}^{p-2}\}, & 1 < p \leq 2, \\ \max\limits_{1 \leq i \leq n} \{(a_i^0)^{p-2}\}, & p \geq 2. \end{cases} \tag{3.15}$$

4. Main Results

In this section, the existence results of positive solutions for BVPs (1.1) are considered.

Define $f^+(t, u^1, u^2, ..., u^n) = f(t, \theta(u^1), \theta(u^2), ..., \theta(u^n))$, where $\theta(u^i) := \max\{0, u^i\}$, $i = 1, 2, ..., n$. From [30], it is easy to know that $f^+(t, u^1, u^2, ..., u^n) \in C^1([0, T] \times \mathrm{R}^n, \mathrm{R})$ and $f_{u^i}^+(t, u^1, u^2, ..., u^n) = f_{u^i}(t, \theta(u^1), \theta(u^2), ..., \theta(u^n))$, $i = 1, 2, ..., n$.

In order to obtain the positive solution of problem (1.1), we define

$$\begin{aligned}
I_+(u(t)) =& \frac{1}{p} \sum_{i=1}^n \int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt + \sum_{i=1}^n \sum_{j=1}^m \int_0^{u^i(t_j)} I_{ij}(s) ds \\
& - \int_0^T \lambda(t) f^+(t, u^1(t), ..., u^n(t)) dt, \quad \forall\ u(t) \in X.
\end{aligned} \tag{4.1}$$

Then, we have

$$\begin{aligned}
I_+'(u(t))(v(t)) =& \sum_{i=1}^n \int_0^T \Phi_p(a_i(t) {}_0D_t^{\alpha_i} u^i(t)) {}_0D_t^{\alpha_i} v^i(t) dt + \sum_{i=1}^n \sum_{j=1}^m I_{ij}(u^i(t_j)) v^i(t_j) \\
& - \sum_{i=1}^n \int_0^T \lambda(t) f_{u^i}(t, \theta(u^1(t)), ..., \theta(u^n(t))) v^i(t) dt, \ \forall\ v(t) \in X.
\end{aligned} \tag{4.2}$$

Firstly, we apply the Theorem 2.1 to discuss the existence of at least one positive solution for BVPs (1.1).

Theorem 4.1. *Let $p > 2$, $\frac{1}{p} < \alpha_i \leq 1$, $i = 1, 2, ..., n$. Assume that*
(A_1) *There exist nonnegative constants K_i, $i = 1, 2, ..., n$, such that*

$$\frac{\omega_*}{p} - \lambda^0 \max_{1 \leq i \leq n} \{K_i\} \mathcal{A} > 0;$$

(A_2) *There exist functions $h_i(t) \in L^1([0, T], \mathrm{R}^+)$, $i = 1, 2, ..., n$, such that*

$$f(t, x) \leq \sum_{i=1}^{n} K_i \mid x^i \mid^p + h_i(t), \ \forall \ x \in \mathrm{R^n};$$

(A_3) *$I_{ij}(0) = 0$ and there exist constants $L_{ij} > 0$ such that $\mid I_{ij}(s_1) - I_{ij}(s_2) \mid \leq L_{ij} \mid s_1 - s_2 \mid$, for any $s_1, s_2 \in \mathrm{R}$, $i = 1, 2, ..., n$, $j = 1, 2, ..., m$;*
(A_4) *$f_{x^i}(t, x^1, ..., x^{i-1}, 0, x^{i+1}, ..., x^n) = 0$, $\forall x \in \mathrm{R^n}$ with $x^i \leq 0$, $i = 1, 2, ..., n$;*
(A_5) *$\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} I_{ij}(s)s \geq 0$ for any $s < 0$, $i = 1, 2, ..., n$, $j = 1, 2, ..., m$.*

Then, BVPs (1.1) admits at least one positive solution that minimizes I on X.

Proof. Lemma 3.2 shows the fact that fractional derivative space X is a reflexive and separable Banach space. For any sequence $\{u_k = (u_k^1, u_k^2, ..., u_k^n)\}_{k=1}^{\infty} \subset X$, assume that $u_k(t) \rightharpoonup u(t) = (u^1(t), u^2(t), ..., u^n(t))$ in X as $k \to \infty$. Then, $\| u^i \|_{\alpha_i, p} \leq \lim\limits_{k \to \infty} \inf \| u_k^i \|_{\alpha_i, p}$ and $u_k^i(t)$ uniform converges to $u^i(t)$ on $[0, T]$, $i = 1, 2, ..., n$. Hence

$$\begin{aligned}
&\lim_{k \to \infty} \inf I_+(u_k(t)) \\
=& \lim_{k \to \infty} \left\{ \frac{1}{p} \sum_{i=1}^{n} \int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i} u_k^i(t) \mid^p dt + \sum_{i=1}^{n} \sum_{j=1}^{m} \int_0^{u_k^i(t_j)} I_{ij}(s) ds \right. \\
&\left. - \int_0^T \lambda(t) f^+(t, u_k(t)) dt \right\} \\
\geq& \frac{1}{p} \sum_{i=1}^{n} \int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i} u^i(t) \mid^p dt + \sum_{i=1}^{n} \sum_{j=1}^{m} \int_0^{u^i(t_j)} I_{ij}(s) ds \\
&- \int_0^T \lambda(t) f^+(t, u(t)) dt = I_+(u(t)),
\end{aligned}$$

which implies that functional I_+ is weakly lower semi-continuous on X. So that, in order to accomplish Theorem 4.1, we need to guarantee that functional I_+ is coercive on X.

Indeed, for any $x(t) \in X$, combining (4.1), (3.15), (3.4), (A_2) and (A_3), one has

$$\begin{aligned}
I_+(x(t)) \geq& \frac{\omega_*}{p} \sum_{i=1}^{n} \| x^i \|_{\alpha_i, p}^p - \sum_{i=1}^{n} \sum_{j=1}^{m} \int_0^{x^i(t_j)} \mid I_{ij}(s) \mid ds \\
&- \lambda^0 \int_0^T \sum_{i=1}^{n} K_i \mid x^i(t) \mid^p + h_i(t) dt
\end{aligned}$$

$$
\begin{aligned}
&\geq \frac{\omega_*}{p}\sum_{i=1}^{n} \| x^i \|_{\alpha_i,p}^{p} - \sum_{i=1}^{n}\sum_{j=1}^{m} L_{ij}\int_0^{x^i(t_j)} | s | ds - \lambda^0 \sum_{i=1}^{n} K_i \| x^i \|_{L_p}^{p} \\
&\quad - \lambda^0 \sum_{i=1}^{n} \| h_i \|_{L_1} \\
&\geq \Big(\frac{\omega_*}{p} - \lambda^0 \max_{1\leq i\leq n}\{K_i\}\mathcal{A}\Big)\sum_{i=1}^{n} \| x^i \|_{\alpha_i,p}^{p} - \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{L_{ij}\mathbf{b}_i^2}{2} \| x^i \|_{\alpha_i,p}^{2} \\
&\quad - \lambda^0 \sum_{i=1}^{n} \| h_i \|_{L_1} .
\end{aligned}
$$

Recalling that $p > 2$, $\frac{\omega_*}{p} - \lambda^0 \max\limits_{1\leq i\leq n}\{K_i\}\mathcal{A} > 0$. Then, we obtain $\lim_{\|x\|_X\to\infty} I_+(x) = +\infty$, which means that the functional I_+ is coercive. This fact together with Theorem 2.1 guarantee that there exists $u_0(t) = (u_0^1(t), u_0^2(t), ..., u_0^n(t)) \in X$ such that $I_+(u_0(t)) = \inf\limits_{u(t)\in X} I_+(u(t))$. Since I_+ is differentiable, we have $I'_+(u_0(t))(v(t)) = 0$, i.e.,

$$
\begin{aligned}
&\sum_{i=1}^{n}\int_0^T \Phi_p(a_i(t)_0D_t^{\alpha_i}u_0^i(t))_0D_t^{\alpha_i}v^i(t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u_0^i(t_j))v^i(t_j) \\
&= \sum_{i=1}^{n}\int_0^T \lambda(t)f_{u^i}(t,\theta(u_0^1(t)),...,\theta(u_0^n(t)))v^i(t)dt, \quad \forall\ v(t)\in X. \qquad (4.3)
\end{aligned}
$$

Define $u^- = \min\{0, u\}$. Then, due to (A_4) and (A_5), one has

$$
\sum_{i=1}^{n}\int_0^T \lambda(t)f_{u^i}(t,\theta(u_0^1(t)),...,\theta(u_0^n(t)))u_0^{i-}(t)dt = 0, \ \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u_0^{i-}(t_j))u_0^{i-}(t_j) \geq 0. \tag{4.4}
$$

Take $v = (u_0^{1-}, u_0^{2-}, ..., u_0^{n-})$, then, combining (4.3) with (4.4), we obtain

$$
0 = \sum_{i=1}^{n}\int_0^T \Phi_p(a_i(t)_0D_t^{\alpha_i}u_0^{i-}(t))_0D_t^{\alpha_i}u_0^{i-}(t)dt \geq \omega_* \sum_{i=1}^{n} \| u_0^{i-} \|_{\alpha_i,p}^{p},
$$

which implies that $u_0^{i-} \equiv 0$, $i = 1, 2, ..., n$. Hence, we have $u_0^i \geq 0$, $1 \leq i \leq n$. Namely, $I_+(u_0) = I(u_0)$ and $u_0 = (u_0^1, u_0^2, ..., u_0^n)$ is a positive solution of problem (1.1). $\square$

Example 4.1. Let $n = 2, m = 1$. Define $f(t, x, y) = \sin t \cos y \sin x^p + \sin t \cos x \sin y^p$ and $I_{i1}(s) = c_i \arctan s$, $c_i > 0$, $i = 1, 2$. By the direct computation, we derive that $f(t, x, y) \leq | x |^p + | y |^p$, $f_x(t, 0, y) = f_y(t, x, 0) = 0$, $I_{11}(s)s + I_{21}(s)s \geq 0$ and $L_{i1} = c_i$, $K_i = h_i(t) = 0$, $i = 1, 2$. Obviously, we can choose appropriate functions and parameters $\lambda(t), a_i(t), \alpha_i$, $i = 1, 2$, such that (A_1) holds. So that, the conditions of Theorem 4.1 are satisfied.

In what follows, the existence of mountain pass solutions of BVPs (1.1) is established by Theorem 2.2.

Theorem 4.2. *Let $p > 1$, $\frac{1}{p} < \alpha_i \leq 1$, $i = 1, 2, ..., n$. Assume that*

(A_6) *There exist nonnegative constants M and μ with $\mu > p$ such that*

$$0 < \mu f(t,x) \leq \sum_{i=1}^{n} f_{x^i}(t,x)x^i, \ \forall\ x \in \mathrm{R^n} \ \textit{with} \ \mid x \mid > M, \ t \in [0,T];$$

(A_7) *There exists a constant $\upsilon \in (p, \mu]$ such that $0 \leq I_{ij}(s)s \leq \upsilon \int_0^s I_{ij}(\xi)d\xi$ for any $s \in \mathrm{R}$, $i = 1,2,...,n$, $j = 1,2,...,m$;*
(A_8) *$f_{x^i}(t, x^1, ..., x^{i-1}, 0, x^{i+1}, ..., x^n) = 0$, $\forall x \in \mathrm{R^n}$ with $x^i \leq 0$, $i = 1,2,...,n$.*
(A_9)

$$\limsup_{\sum_{i=1}^{n} |x^i| \to 0} \frac{f(t,x)}{\sum_{i=1}^{n} \mid x^i \mid^p} < \frac{1}{C_0},$$

for any $x = (x^1, x^2, ..., x^n) \in \mathrm{R^n}$, where $C_0 = p\mathcal{B}$ and $\mathcal{B}$ is introduced in (3.10);
(A_{10}) *There exist constants $M_{ij} \geq 0$, $0 < w_{ij} \leq p-1$ such that $I_{ij}(s) \leq M_{ij}s^{w_{ij}}$, for any $s \in \mathrm{R}$, $i = 1,2,...,n$, $j = 1,2,...,m$.*

Then, BVPs (1.1) admits at least one positive solution on X.

Proof. First of all, we claim that the functional I_+ satisfies the *P.S.* condition.

Assume $\{u_k\}_{k\in\mathrm{N}} \subset X$ is a *P.S.* sequence associated with I_+, i.e.,

$$\mid I_+(u_k) \mid \leq K, \quad I'_+(u_k) \to 0 \text{ as } k \to \infty,$$

where K is a nonnegative constant. From (4.2) we have

$$\begin{aligned} I'_+(u_k(t))(u_k(t)) =& \sum_{i=1}^{n} \int_0^T \Phi_p(a_i(t) {}_0D_t^{\alpha_i} u_k^i(t)) {}_0D_t^{\alpha_i} u_k^i(t)dt + \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u_k^i(t_j))u_k^i(t_j) \\ & - \sum_{i=1}^{n} \int_0^T \lambda(t) f_{u_k^i}(t, \theta(u_k^1(t)), ..., \theta(u_k^n(t)))u_k^i(t)dt. \end{aligned} \tag{4.5}$$

Since $I'_+(u_k) \to 0$ as $k \to \infty$, then, there exists $\varepsilon_k \to 0$ such that

$$\mid I'_+(u_k(t))(v_k(t)) \mid \leq \varepsilon_k, \ \forall\ v_k(t) \in X, \ k \in \mathrm{N}, \ t \in [0,T]. \tag{4.6}$$

Based on (A_7) and (A_8), one has

$$\sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u_k^{i-}(t_j))u_k^{i-}(t_j) \geq 0, \ \sum_{i=1}^{n} \int_0^T \lambda(t) f_{u_k^i}(t, \theta(u_k^1(t)), ..., \theta(u_k^n(t)))u_k^{i-}(t)dt = 0,$$

that is

$$\begin{aligned} I'_+(u_k(t))(u_k^-(t)) =& \sum_{i=1}^{n} \int_0^T a_i(t)^{p-1} \mid {}_0D_t^{\alpha_i} u_k^{i-}(t) \mid^p dt + \sum_{i=1}^{n}\sum_{j=1}^{m} I_{ij}(u_k^{i-}(t_j))u_k^{i-}(t_j) \\ \geq & \ \omega_* \sum_{i=1}^{n} \parallel u_k^{i-} \parallel_{\alpha_i,p}^p . \end{aligned} \tag{4.7}$$

So that, combining (4.6) and (4.7), we obtain

$$u_k^{i-} \to 0 \text{ as } k \to \infty, \ 1 \leq i \leq n. \tag{4.8}$$

On the other hand, condition (A_7) implies that

$$\mu \int_0^{u_k^i(t_j)} I_{ij}(s)ds - I_{ij}(u_k^i(t_j))u_k^i(t_j) \geq (\mu - \upsilon) \int_0^{u_k^i(t_j)} I_{ij}(s)ds \geq 0, \ (\mu \geq \upsilon), \quad (4.9)$$

for $i = 1, 2, ..., n,\ j = 1, 2, ..., m$. Then, from (4.1), (4.2), (4.5), (4.9) and (A_6), we have

$$\begin{aligned}
&\mu I_+(\theta(u_k(t))) - I'_+(\theta(u_k(t)))\theta(u_k(t)) \\
\geq &(\frac{\mu}{p} - 1)\omega_* \sum_{i=1}^{n} \parallel \theta(u_k^i) \parallel_{\alpha_i,p}^p + \sum_{i=1}^{n}\sum_{j=1}^{m}(\mu - \upsilon) \int_0^{\theta(u_k^i(t_j))} I_{ij}(s)ds \\
&+ \sum_{i=1}^{n} \int_0^T \lambda(t) f_{u_k^i}(t, \theta(u_k^1(t)), ..., \theta(u_k^n(t)))\theta(u_k^i(t))dt \\
&- \mu \int_0^T \lambda(t) f^+(t, u_k^1(t), ..., u_k^n(t))dt \\
\geq &(\frac{\mu}{p} - 1)\omega_* \sum_{i=1}^{n} \parallel \theta(u_k^i) \parallel_{\alpha_i,p}^p + \lambda_0 \int_0^T \sum_{i=1}^{n} f_{u_k^i}(t, \theta(u_k(t)))\theta(u_k^i(t)) - \mu f(t, \theta(u_k(t)))dt \\
\geq &(\frac{\mu}{p} - 1)\omega_* \sum_{i=1}^{n} \parallel \theta(u_k^i) \parallel_{\alpha_i,p}^p .
\end{aligned}$$

Noting that $\mu > p$, $\mid I_+(\theta(u_k)) \mid \leq K$ and $I'_+(\theta(u_k)) \to 0$ as $k \to \infty$, we conclude that $\{\theta(u_k)\}$ is bounded in X. Hence, in view of (4.8), we get that the sequence $\{u_k\}_{k \in \mathrm{N}}$ is bounded in X.

Taking into account that X is a reflexive Banach space, assume $u_k \rightharpoonup u_*$ in X, $u_* = (u_*^1, u_*^2, ..., u_*^n) \in X$. From Lemma 3.3, we get that $u_k \to u_*$ on [0, T]. Then, we deduce

$$\begin{aligned}
(I'_+(u_k) - I'_+(u_*))(u_k - u_*) =& I'_+(u_k)(u_k - u_*) - I'_+(u_*)(u_k - u_*) \\
\leq& \parallel I'_+(u_k) \parallel \cdot \parallel u_k - u_* \parallel_X - I'_+(u_*)(u_k - u_*) \\
\to& 0, \quad \text{as} \quad k \to \infty. \qquad (4.10)
\end{aligned}$$

Moreover, since function f continuously differentiable in u and I_{ij} are continuous, $i = 1, 2, ..., n,\ j = 1, 2, ..., m$, we have

$$\begin{cases}
\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{m}(I_{ij}(u_k^i(t_j)) - I_{ij}(u_*^i(t_j)))(u_k^i(t_j) - u_*^i(t_j)) \to 0, \ k \to \infty, \\
\sum\limits_{i=1}^{n} \int_0^T \lambda(t)(f_{u_k^i}(t, u_k(t)) - f_{u_*^i}(t, u_*(t)))(u_k^i(t) - u_*^i(t))dt \to 0, \ k \to \infty.
\end{cases} \quad (4.11)$$

Recalling that

$$\begin{aligned}
&(I'_+(u_k(t)) - I'_+(u_*(t)))(u_k(t) - u_*(t)) \\
=&\sum_{i=1}^{n} \int_0^T (\Phi_p(a_i(t)_0D_t^{\alpha_i}u_k^i(t)) - \Phi_p(a_i(t)_0D_t^{\alpha_i}u_*^i(t)))_0D_t^{\alpha_i}(u_k^i(t) - u_*^i(t))dt \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{m}(I_{ij}(u_k^i(t_j)) - I_{ij}(u_*^i(t_j)))(u_k^i(t_j) - u_*^i(t_j))
\end{aligned}$$

$$-\sum_{i=1}^{n}\int_0^T \lambda(t)(f_{u_k^i}(t,\theta(u_k(t)))-f_{u_*^i}(t,\theta(u_*(t))))(u_k^i(t)-u_*^i(t))dt,$$

then, this fact combining with (4.11) and (4.10) imply that

$$\sum_{i=1}^{n}\int_0^T (\Phi_p(a_i(t)_0D_t^{\alpha_i}u_k^i(t))-\Phi_p(a_i(t)_0D_t^{\alpha_i}u_*^i(t)))_0D_t^{\alpha_i}(u_k^i(t)-u_*^i(t))dt\to 0, \tag{4.12}$$

as $k\to\infty$.

Further, literature [28] shows a well-known inequality, which is given by

$$\langle\mid s_1\mid^{p-2} s_1-\mid s_2\mid^{p-2} s_2, s_1-s_2\rangle\geq\begin{cases} a_1\mid s_1-s_2\mid^p, & p\geq 2,\\ a_1\frac{|s_1-s_2|^2}{(|s_1|+|s_2|)^{2-p}}, & 1<p\leq 2,\end{cases} \tag{4.13}$$

for every $s_1,s_2\in\mathrm{R}^{\mathrm{N}}$, where a_1 is a nonnegative constant. Then, base on (4.13), we obtain that there exist some positive numbers j_i, j_i', $i=1,2,...,n$, such that

$$\begin{aligned}&\int_0^T (\Phi_p(a_i(t)_0D_t^{\alpha_i}u_k^i(t))-\Phi_p(a_i(t)_0D_t^{\alpha_i}u_*^i(t)))_0D_t^{\alpha_i}(u_k^i(t)-u_*^i(t))dt\\ &\geq\begin{cases} j_i\int_0^T\frac{1}{a_i(t)}\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^p dt, & p\geq 2,\\ j_i'\int_0^T\frac{1}{a_i(t)}\frac{|a_i(t)_0D_t^{\alpha_i}u_k^i(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)|^2}{(|a_i(t)_0D_t^{\alpha_i}u_k^i(t)|+|a_i(t)_0D_t^{\alpha_i}u_*^i(t)|)^{2-p}}dt, & 1<p<2.\end{cases}\end{aligned}$$

When $1<p<2$, one has

$$\begin{aligned}&\frac{1}{(a_i^0)^{\frac{p}{2}}}\int_0^T\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^p dt\\
&\leq\frac{1}{(a_i^0)^{\frac{p}{2}}}\left(\int_0^T\left(\frac{\mid a_i(t)_0D_t^{\alpha_i}u_k(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^p}{(a_i(t))^{\frac{p}{2}}(\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid)^{\frac{(2-p)p}{2}}}\right)^{\frac{2}{p}}dt\right)^{\frac{p}{2}}\cdot\\
&\left(\int_0^T\left((a_i(t))^{\frac{p}{2}}(\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid)^{\frac{(2-p)p}{2}}\right)^{\frac{2}{2-p}}dt\right)^{\frac{2-p}{2}}\\
&\leq\left(\int_0^T\frac{\mid a_i(t)_0D_t^{\alpha_i}u_k(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^2}{a_i(t)(\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid)^{2-p}}dt\right)^{\frac{p}{2}}\\
&2^{\frac{p(2-p)}{2}}\left(\int_0^T\frac{(a_i(t))^{\frac{p}{2-p}}}{(a_i^0)^{\frac{p}{2-p}}}\left(\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid^p+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^p\right)dt\right)^{\frac{2-p}{2}}\\
&\leq\left(\int_0^T\frac{\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)-a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^2}{a_i(t)(\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid)^{2-p}}dt\right)^{\frac{p}{2}}\\
&2^{\frac{p(2-p)}{2}}\left(\int_0^T\mid a_i(t)_0D_t^{\alpha_i}u_k^i(t)\mid^p+\mid a_i(t)_0D_t^{\alpha_i}u_*^i(t)\mid^p dt\right)^{\frac{2-p}{2}},\end{aligned}$$

where the inequality $(b_1+b_2)^p\leq 2^p(b_1^p+b_2^p)$, $b_1,b_2\in\mathrm{R}^+$, is used. Therefore, we obtain

$$\begin{aligned}&\int_0^T (\Phi_p(a_i(t)_0D_t^{\alpha_i}u_k^i(t))-\Phi_p(a_i(t)_0D_t^{\alpha_i}u_*^i(t)))_0D_t^{\alpha_i}(u_k^i(t)-u_*^i(t))dt\\ &\geq J_i'\parallel u_k^i-u_*^i\parallel_{\alpha_i,p}^2(\parallel u_k^i\parallel_{\alpha_i,p}^p+\parallel u_*^i\parallel_{\alpha_i,p}^p)^{\frac{p-2}{p}},\quad i=1,2,...,n,\end{aligned} \tag{4.14}$$

where J_i' is a nonnegative constant, $i = 1, 2, ..., n$. When $p \geq 2$, we can easy to observe that

$$\int_0^T (\Phi_p(a_i(t)_0D_t^{\alpha_i}u_k^i(t)) - \Phi_p(a_i(t)_0D_t^{\alpha_i}u_*^i(t)))_0D_t^{\alpha_i}(u_k^i(t) - u_*^i(t))dt \\ \geq j_i\omega_* \parallel u_k^i - u_*^i \parallel_{\alpha_i,p}^p, \quad i = 1, 2, ..., n. \tag{4.15}$$

Then, combining (4.12), (4.14) and (4.15) yields

$$\sum_{i=1}^n \parallel u_k^i - u_*^i \parallel_{\alpha_i,p}^p = \parallel u_k - u_* \parallel_X^p \to 0, \quad \text{as} \quad k \to \infty.$$

That is, the sequence $\{u_k\}_{k\in \mathrm{N}}$ converges to u_* strongly in X. Therefore, we assert that the functional I_+ satisfies the $P.S.$ condition.

Nextly, we show that the fuctional I_+ satisfies the geometry conditions of the Mountain pass theorem.

Obviously, from the definition of I_+, we have $I_+(0) = 0$. In view of (A_9), there exist $\delta > 0$ and $0 < \varepsilon < \min\left\{1, \frac{\omega_*}{\lambda^0 T}\right\}$ such that

$$f(t, u) \leq \frac{\sum\limits_{i=1}^n \mid u^i \mid^p}{C_0}\varepsilon, \tag{4.16}$$

for any $u = (u^1, u^2, ..., u^n) \in \mathrm{R}^\mathrm{n}$ with $\mid u^i \mid < \delta$, $i = 1, 2, ...n$.

Choose $\rho = \frac{1}{nd}\delta > 0$ and $\sigma = \frac{\omega_* - \lambda^0 T\varepsilon}{p}\rho^p > 0$. Then, the inequality (3.11) implies that

$$\parallel u \parallel_\infty \leq nd \parallel u \parallel_X = \delta, \quad \forall u \in X, \quad \parallel u \parallel_X = \rho.$$

Combining (4.1), (3.4), (A_7) and (4.16) yields

$$\begin{aligned} I_+(u(t)) &\geq \frac{\omega_*}{p}\sum_{i=1}^n \parallel u^i \parallel_{\alpha_i,p}^p + \sum_{i=1}^n\sum_{j=1}^m \int_0^{u^i(t_j)} I_{ij}(s)ds - \frac{\lambda^0}{C_0}\varepsilon\int_0^T \sum_{i=1}^n \mid u^i(t) \mid^p dt \\ &\geq \frac{\omega_*}{p}\sum_{i=1}^n \parallel u^i \parallel_{\alpha_i,p}^p - \frac{\lambda^0}{C_0}\varepsilon T\sum_{i=1}^n \parallel u^i \parallel_\infty^p \\ &\geq \frac{\omega_*}{p} \parallel u \parallel_X^p - \frac{\lambda^0 T}{p}\varepsilon \parallel u \parallel_X^p \\ &= \frac{\omega_* - \lambda^0 T\varepsilon}{p} \parallel u \parallel_X^p = \sigma, \end{aligned}$$

for $u = (u^1, u^2, ..., u^n) \in X$ with $\parallel u \parallel_X = \rho$, which means that the condition (C_2) of Theorem 2.2 holds.

Moreover, Lemma 2.2 shows that the condition (A_6) yields the following result

$$f(t, u) \geq c_1(\sum_{i=1}^n \mid u^i \mid^\mu) - c_2, \text{ for } c_1, c_2 > 0. \tag{4.17}$$

Let B is a unit ball in R^{N}, $z = (z^1, z^2, ..., z^n) \in X$ and z^i is a positive function, $i = 1, 2, ...n$. Denote by z_0 the extension of z to zero out of B. Then, for any $\tau \in \mathrm{R}^+$, due to (4.1), (4.17) and (A_{10}), we infer

$$\begin{aligned}I_+(\tau z_0) \leq & \frac{\omega^*}{p} \parallel \tau z_0 \parallel_X^p + \sum_{i=1}^n \sum_{j=1}^m \int_0^{\tau z_0^i(t_j)} I_{ij}(s)ds - \lambda_0 c_1 (\sum_{i=1}^n \int_0^T \mid \tau z_0^i \mid^\mu dt) + \lambda^0 c_2 T \\ \leq & \frac{\omega^*}{p} \tau^p \parallel z_0 \parallel_X^p + \sum_{i=1}^n \sum_{j=1}^m \tau^{\omega_{ij}+1} \frac{M_{ij} \mathbf{b}_i^{\omega_{ij}+1}}{\omega_{ij}+1} \parallel z_0^i \parallel_{\alpha_i,p}^{\omega_{ij}+1} \\ & - \lambda_0 c_1 \tau^\mu (\sum_{i=1}^n \parallel z_0^i \parallel_{L^\mu}^\mu) + \lambda^0 c_2 T.\end{aligned}$$

Note that $\mu > p$, $0 < \omega_{ij} \leq p-1$, $1 \leq i \leq n$, $1 \leq j \leq m$, then, we have

$$I_+(\tau z_0) \to -\infty, \quad \text{as} \quad \tau \to +\infty,$$

which implies that there exists a enough large number τ_* such that $I_+(\tau_* z_0) \leq 0$ and $\parallel \tau_* z_0 \parallel > \rho$. Namely, the condition (C_3) of Theorem 2.2 holds. Therefore, we have shown that the functional I_+ satisfies the geometry conditions of the Mountain pass theorem. This fact guarantees that the functional I_+ possesses a critical value $z_*(t) \in X$ such that $I'_+(z_*(t))(v(t)) = 0$ for any $v(t) \in X$, i.e.,

$$\begin{aligned}& \sum_{i=1}^n \int_0^T \Phi_p(a_i(t)_0 D_t^{\alpha_i} z_*^i(t))_0 D_t^{\alpha_i} v^i(t)dt + \sum_{i=1}^n \sum_{j=1}^m I_{ij}(z_*^i(t_j))v^i(t_j) \\ = & \sum_{i=1}^n \int_0^T \lambda(t) f_{z_*^i}(t, \theta(z_*^1(t)), \theta(z_*^2(t)), ..., \theta(z_*^n(t)))v^i(t)dt, \end{aligned} \tag{4.18}$$

then, by using (A_7) and (A_8), and taking $v^i = z_*^{i-}$ for $i = 1, 2, ..., n$, we deduce

$$\begin{aligned}0 = & \sum_{i=1}^n \int_0^T \Phi_p(a_i(t)_0 D_t^{\alpha_i} z_*^{i-}(t))_0 D_t^{\alpha_i} z_*^{i-}(t)dt + \sum_{i=1}^n \sum_{j=1}^m I_{ij}(z_*^{i-}(t_j)) z_*^{i-}(t_j) \\ \geq & \omega_* \sum_{i=1}^n \parallel z_*^{i-} \parallel_{\alpha_i,p}^p .\end{aligned}$$

Hence, we obtain $z_*^{i-} \equiv 0$, $i = 1, 2, ..., n$, which means that $z_*^i \geq 0$, $i = 1, 2, ..., n$, and $f^+(t, z_*) = f(t, z_*)$, i.e., z_* is also a critical value for functional I. Then, we say z_* is a positive solution of BVPs (1.1). □

Example 4.2. Let $n = 2$, $m = 1$. Define $f(t, x, y) = x^{p+1}y^2 + y^{p+1}x^2$ and

$$I_{i1}(s) = \begin{cases} d_i s^{p-1}, & s \geq 0, \quad d_i > 0, \quad 1 \leq i \leq 2, \\ -d_i(-s)^{p-1}, & s < 0, \quad d_i > 0, \quad 1 \leq i \leq 2, \end{cases}$$

where p is an arbitrary odd number with $p > 1$. It is easy to observe that $f(t, x, y)$ and $I_{i1}(s)$ $(1 \leq i \leq 2)$ are satisfy all the conditions of Theorem 4.2 by taking $\mu = p + 3$, $\upsilon = p + 1$ and $w_{i1} = p - 1$, $i = 1, 2$.

5. Conclusion

In this paper, by the methods of a critical point theorem and the Mountain pass theorem, the existence of at least one positive solution has been addressed for a class of p-Laplacian type fractional Dirichlet's boundary value problem involving impulsive impacts. Two examples have been given to illustrate the applications of our results.

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