Journal of Applied Analysis and Computation
Volume 9, Number 6, December 2019, 2381–2388
Website:http://jaac.ijournal.cn
DOI:10.11948/20190112

EXPLICIT AND EXACT NON-TRAVELING WAVE SOLUTIONS OF (3+1)-DIMENSIONAL GENERALIZED SHALLOW WATER EQUATION

Jianguo Liu[1,†], Wenhui Zhu[2], Li Zhou[1] and Yan He[1]

Abstract In this paper, a (3+1)-dimensional generalized shallow water equation is considered. New exact solutions in forms of the hyperbolic functions and the trigonometric functions are obtained based on an extended (G'/G)-expansion method and the variable separation method, which contain traveling wave solutions and non-traveling wave solutions. The particular localized excitations and the interactions between two solitary waves for these obtained exact solutions are shown in some three-dimensional graphics.

Keywords Extended (G'/G)-expansion method, (3+1)-dimensional generalized shallow water equation, non-traveling wave solutions.

MSC(2010) 35C08, 68M07, 33F10.

1. Introduction

Nonlinear evolution equations (NLEEs) have been used to represent various nonlinear phenomenas in fluid dynamics, plasma physics, nonlinear optics, solid state physics, biological molecules and so on [5, 7, 9, 12, 13, 17–19, 30, 36, 37]. To understand these physical phenomenas, searching for exact solutions of NLEEs is of great important. Many methods to have been proposed [1–4, 6, 8, 10, 14, 15, 20–26, 31, 35].

Shallow water equations have applications in weather simulations, tidal waves, river and irrigation flows, tsunami prediction and so on [27]. In this work, based on the (G'/G)-expansion method and symbolic computation, we will consider the following (3+1)-dimensional generalized shallow water equation [32]

$$u_{yt} - u_{xz} - 3u_x\, u_{xy} - 3\, u_y\, u_{xx} + u_{xxxy} = 0, \tag{1.1}$$

where $u = u(x, y, z, t)$. Eq. (1.1) describes the propagation of long water waves in oceans, estuaries, and impoundments. Tian [29] obtained the soliton-type solutions to Eq. (1.1) by using the generalized *tanh* algorithm method. Zayed [33] constructed the traveling wave solutions of Eq. (1.1) by utilizing the (G'/G)-expansion method. Tang [28] derived the Grammian and Pfaffian solutions of Eq. (1.1) by the Hirota's bilinear form. Multiple-soliton solutions were derived by Zeng [34]. Liu [11]

†the corresponding author. Email address:20101059@jxutcm.edu.cn (J. Liu)
[1]College of Computer, Jiangxi University of Traditional Chinese Medicine, Jiangxi 330004, China
[2]Institute of artificial intelligence, Nanchang Institute of Science and Technology, Jiangxi 330108, China

presented new periodic solitary wave solutions. Meng [16] atudied the rational solutions of Eq. (1.1). We will discuss the non-traveling wave exact solutions by using an extended (G'/G)-expansion method, which are different from those presented in Refs. [28, 29, 33].

The organization of this paper is as follows. Section 2 proposes an extended (G'/G)-expansion method and obtains new exact solutions for the (3+1)-dimensional generalized shallow water equation. Some special soliton-structure excitations are shown by some three-dimensional graphics. Section 3 lists the discussion and summary.

2. The extended (G'/G)-expansion method and exact non-traveling wave solutions

Considering the following NLEE

$$F(u, u_x, u_y, u_z, u_t, u_{xy}, u_{xz}, u_{xt}, u_{yt}, u_{xx}, u_{tt}, \ldots) = 0. \tag{2.1}$$

For finding the exact solutions of Eq. (2.1), we suppose

$$u = \sum_{i=-m}^{m} a_i \, (\frac{G'}{G})^i, \tag{2.2}$$

where $G = G(\vartheta)$, $\vartheta = \vartheta(x, y, z, t)$, $a_i (i = -m, \cdots, m)$ is unknown constant. Eq. (2.2) contains more arbitrary parameters than previous work [28, 29, 33]. The G satisfies the second-order linear ordinary differential equation

$$G'' + \lambda \, G' + \rho \, G = 0. \tag{2.3}$$

The general solutions of Eq. (2.3) are presented as follows

$$\frac{G'}{G} = \left\{ \begin{array}{l} -\frac{\varrho}{2} + \zeta_1 \frac{C_1 \cosh(\zeta_1 \vartheta) + C_2 \sinh(\zeta_1 \vartheta)}{C_1 \sinh(\zeta_1 \vartheta) + C_2 \cosh(\zeta_1 \vartheta)}, \varrho^2 - 4\rho > 0, \zeta_1 = \frac{\sqrt{\varrho^2 - 4\rho}}{2}, \\ -\frac{\varrho}{2} + \zeta_2 \frac{-C_1 \sin(\zeta_2 \vartheta) + C_2 \cos(\zeta_2 \vartheta)}{C_1 \cos(\zeta_2 \vartheta) + C_2 \sin(\zeta_2 \vartheta)}, \varrho^2 - 4\rho < 0, \zeta_2 = \frac{\sqrt{-\varrho^2 + 4\rho}}{2}, \\ -\frac{\varrho}{2} + \frac{C_2}{C_1 + C_2 \vartheta}, \varrho^2 - 4\rho = 0, \end{array} \right\} \tag{2.4}$$

where ϱ, ρ, C_1 and C_2 are arbitrary constants.

Using the homogenous balance principle, we have $m = 1$. Thus Eq. (2.2) can be changed into

$$u = a_{-1} \, (\frac{G}{G'}) + a_0 + a_1 \, (\frac{G'}{G}). \tag{2.5}$$

Supposing $\vartheta(x, y, z, t) = f(y + c\,z) + a\,x + h(t)$, substituting Eq. (2.5) into Eq. (1.1) and setting the coefficients of all terms with the same powers of $(G'/G)^k (k = -5, \cdots, -2, -1, 1, 2, \cdots, 5)$ to zero, we have

Case 1:

$$a_1 = 0, a_{-1} = 2a\rho, h(t) = At + B,$$

$$a_0 = cy + \frac{[(\varrho^2 - 4\rho)\, a^3 - 4ca + A]f}{3a^2} + W(z,t), \tag{2.6}$$

where $W(z,t)$ and $f = f(y+cz)$ are arbitrary functions, a, c, A and B are arbitrary constants.

Substituting Eq. (2.6) and the general solutions of Eq. (2.3) into Eq. (2.5), we can present new non-traveling wave solutions for Eq. (1.1).

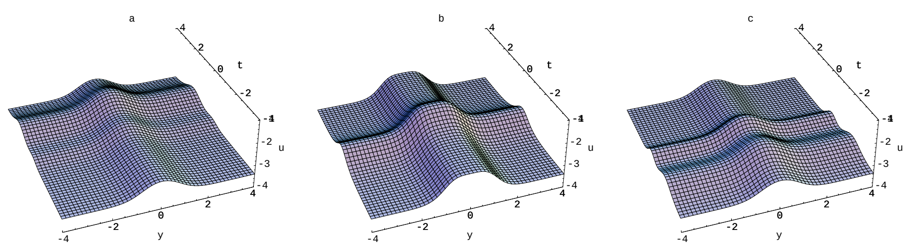
(1): When $\varrho^2 - 4\rho > 0$, the first non-traveling wave solutions of Eq. (1.1) can be written as

$$\begin{aligned} u_1 =& cx + \frac{f[(\varrho^2 - 4\rho)\, a^3 - 4ca + A]}{3a^2} + W(z,t) + [4a\rho C_1 \sinh[\zeta_1(B + ax + At + f)] \\ &+ 4a\rho C_2 \cosh[\zeta_1(B+ax+At+f)]]/[(2\zeta_1 C_1 - \varrho C_2)\cosh[\zeta_1(B+ax+At+f)] \\ &+ (2\zeta_1 C_2 - \varrho C_1)\sinh[\zeta_1(B + ax + At + f)]]. \end{aligned} \tag{2.7}$$

The physical structures of the solution (2.7) are shown in Figs.1-4.

(2): When $\varrho^2 - 4\rho < 0$, we have the second non-traveling wave solutions of Eq. (1.1)

$$\begin{aligned} u_2 =& cx + \frac{f[(\varrho^2 - 4\rho)\, a^3 - 4ca + A]}{3a^2} + W(z,t) + 4a\rho[C_1 \cos((B + f + At + ax)\zeta_2) \\ &+ C_2 \sin((B + f + At + ax)\zeta_2)]/[(2C_2\zeta_2 - \varrho C_1)\cos[(B + f + At + ax)\zeta_2] \\ &- (\varrho C_2 + 2C_1\zeta_2)\sin[(B + f + At + ax)\zeta_2]]. \end{aligned} \tag{2.8}$$



(a) $z = -1$ (b) $z = 0$ (c) $z = 1$

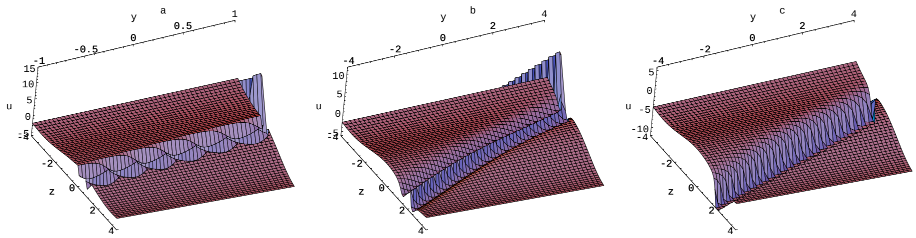
Figure 1. Solution (2.7) at $f = 1 + sech^2(y + cz)$, $W(z,t) = sech(z^2 + t^2)$, $C_1 = 1$, $C_2 = 2$, $\varrho = 3$, $\rho = 0.1$, $c = 2$, $x = 1$, $a = A = 1$, $B = 0$.

Case 2:

$$\begin{aligned} &a_1 = -2a, a_{-1} = 2a\rho, h(t) = At + B, \varrho = 0, \\ &a_0 = cx + \frac{[(\varrho^2 - 16\rho)\, a^3 - 4ca + A]f}{3a^2} + W(z,t), \end{aligned} \tag{2.9}$$

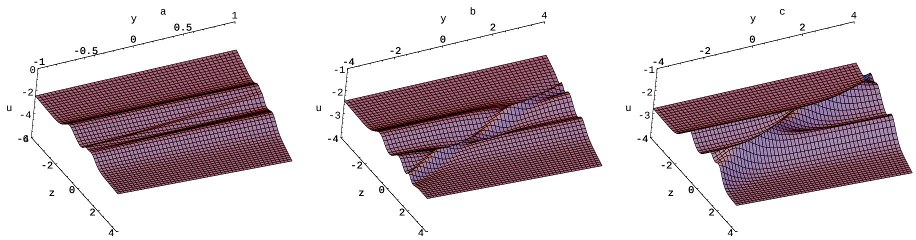
where $W(z,t)$ and $f = f(y+cz)$ are arbitrary functions, a, c, A and B are arbitrary constants.

Substituting Eq. (2.9) and the general solutions of Eq. (2.3) into Eq. (2.5), we can present another new non-traveling wave solutions for Eq. (1.1) as follows



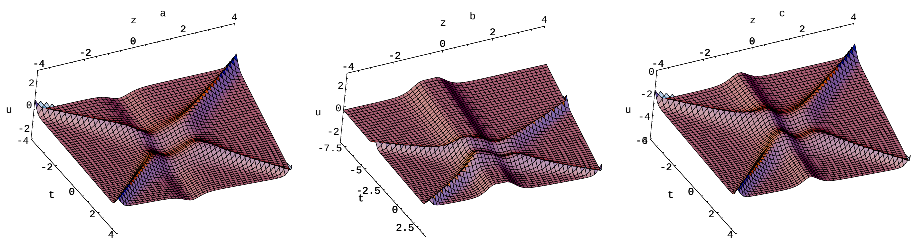
(a) $x = -1$ (b) $x = 0$ (c) $x = 1$

Figure 2. Solution (2.7) at $W(z,t) = \cos\sqrt{z^2 - t^2}$, $B = 0$, $C_1 = 1$, $C_2 = 2$, $f = \coth^2(y + cz) + csch^2(y + cz)$, $\varrho = 3$, $\rho = 0.1$, $c = 2$, $t = 1$, $a = A = 1$.



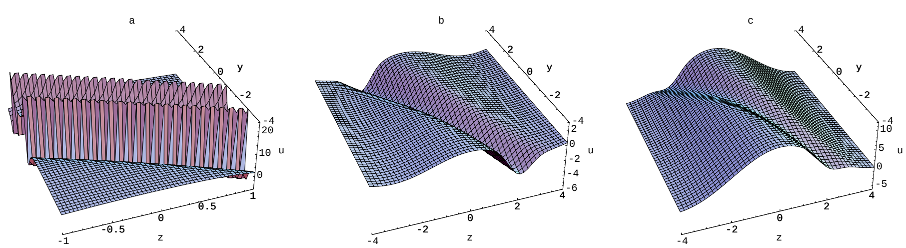
(a) $t = -1$ (b) $t = 0$ (c) $t = 1$

Figure 3. Solution (2.7) at $c = 2$, $f = \frac{1}{1+0.2sech^2(y+cz)+0.2\tanh^2(y+cz)} - sech^2(y+cz)$, $\varrho = 3$, $\rho = 0.1$, $C_2 = 2$, $a = A = 1$, $B = 0$, $W(z,t) = \tanh(z^2 + t^2) + sech(t^2 - z^2)$, $x = 1$.



(a) $x = -1$ (b) $x = 0$ (c) $x = 1$

Figure 4. Solution (2.7) at $c = 2$, $f = \frac{1}{1+0.2sech^2(y+cz)+0.2tanh^2(y+cz)} - sech^2(y+cz)$, $\varrho = 3$, $\rho = 0.1$, $C_2 = 2$, $a = A = 1$, $B = 0$, $W(z,t) = \tanh(z^2 + t^2) + sech(t^2 - z^2)$, $y = 1$.



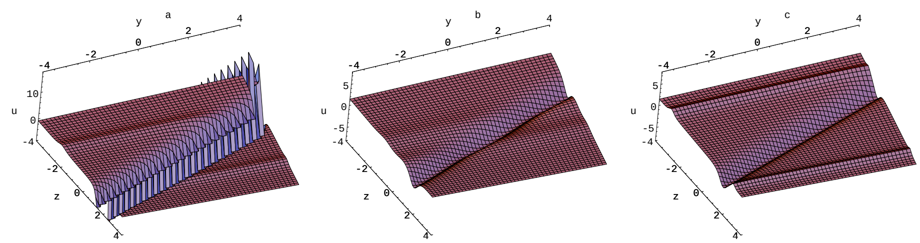
(a) $t = -2$ (b) $t = 0$ (c) $t = 3$

Figure 5. Ssolution (2.10) at $W(z,t) = \cos\sqrt{z^2 - t^2}$, $C_1 = 1$, $\varrho = 3$, $\rho = -1$, $c = 2$, $f = \frac{1}{\coth^2(y+cz)+csch^2(y+cz)}$, $C_2 = 2$, $x = 1$, $B = 0$, $a = A = 1$.

(1): When $\rho < 0$, the third non-traveling wave solutions of Eq. (1.1) can be obtained as

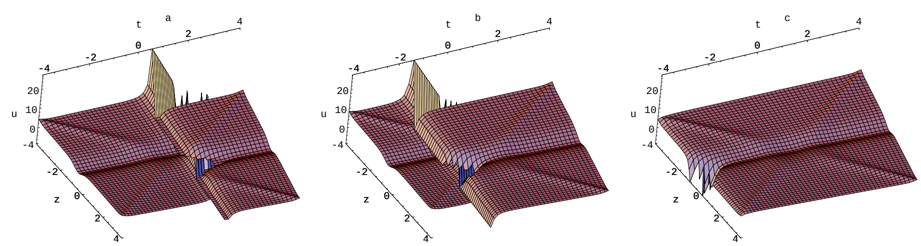
$$\begin{aligned}
u_3 =& cx + \frac{f[A - 4a\left(4\rho a^2 + c\right)]}{3a^2} + W(z,t) \\
& - 2a\sqrt{-\rho}[C_1 \sinh[\sqrt{-\rho}(B + At + ax + f)] + C_2 \cosh[\sqrt{-\rho}(B + At + ax + f)]] \\
& /[C_1 \cosh[\sqrt{-\rho}(B + At + ax + f)] + C_2 \sinh[\sqrt{-\rho}(B + At + ax + f)]] \\
& - 2a\sqrt{-\rho}[C_1 \cosh[(B + f + At + ax)\sqrt{-\rho}] + C_2 \sinh[(B + f + At + ax)\sqrt{-\rho}]] \\
& /[C_1 \sinh[(B + f + At + ax)\sqrt{-\rho}] + C_2 \cosh[(B + f + At + ax)\sqrt{-\rho}]]. \qquad (2.10)
\end{aligned}$$

The physical structures of the solution (2.10) are shown in Figs. 5-7.



(a) $t = -2$ (b) $t = 0$ (c) $t = 3$

Figure 6. Solution (2.10) at $\varrho = 3$, $f = \frac{1}{1+0.2sech^2(y+cz)+0.2\tanh^2(y+cz)} - sech^2(y + cz)$,$\rho = -1$, $c = 2$, $C_2 = 2$, $a = A = 1$, $B = 0$, $W(z,t) = \tanh(z^2 + t^2) + sech(t^2 - z^2)$, $x = 1$.



(a) $x = -2$ (b) $x = 0$ (c) $x = 3$

Figure 7. Solution (2.10) at $\varrho = 3$, $f = \frac{1}{1+0.2sech^2(y+cz)+0.2\tanh^2(y+cz)} - sech^2(y + cz)$, $\rho = 0.1$, $c = 2$, $C_2 = 2$, $a = A = 1$, $B = 0$, $W(z,t) = \tanh(z^2 + t^2) + sech(t^2 - z^2)$, $y = 1$.

(2): When $\rho > 0$, the fourth non-traveling wave solutions of Eq. (1.1) can be expressed as

$$\begin{aligned}
u_4 =& cx + \frac{f[A - 4a\left(4\rho a^2 + c\right)]}{3a^2} + W(z,t) \\
& + 2a\sqrt{\rho}[C_1 \sin[\sqrt{\rho}(B + At + ax + f)] - C_2 \cos[\sqrt{\rho}(B + At + ax + f)]] \\
& /[C_1 \cos[\sqrt{\rho}(B + At + ax + f)] + C_2 \sin[\sqrt{\rho}(B + At + ax + f)]] \\
& + 2a\sqrt{\rho}[C_1 \cos[(B + f + At + ax)\sqrt{-\rho}] + C_2 \sin[(B + f + At + ax)\sqrt{\rho}]] \\
& /[-C_1 \sin[(B + f + At + ax)\sqrt{\rho}] + C_2 \cos[(B + f + At + ax)\sqrt{\rho}]]. \qquad (2.11)
\end{aligned}$$

In Figs.1-7, we research the excitation process of a special dromion soliton structure of solutions (2.7) and (2.10) for Eq. (1.1). The variation of solutions (2.7) and (2.10) with time and the interactions between two solitary waves are also described. It is obviously that other choices of $f(y+cz)$ and $W(z,t)$ in solutions (2.7) and (2.10) may form rich localized soliton structures. In other words, solutions (2.8) and (2.11) may also be employed to excite abundant soliton structures.

3. Discussion and summary

In this paper, we have obtained some new exact non-traveling wave solutions for Eq. (1.1) by using an extended (G'/G)-expansion method. Furthermore, by selecting the different values for $\vartheta(x,y,z,t)$ in the solutions (2.7) and (2.10), we can see various interesting localized soliton excitations. Also, the models proposed in this article describe important applications in physics and engineering.

Acknowledgements. We would like to thank the Editor and the Referee for their timely and valuable comments.

References

[1] A. Biswas, A. Sonmezoglu, M. Ekici et al., *Optical soliton perturbation with fractional temporal evolution by extended* (G'/G)*-expansion method*, Optik., 2018, 161, 301–320.

[2] S. T. Chen, W. X. Ma, *Lump solutions of a generalized Calogero-Bogoyavlenskii-Schiff equation*, Comput. Math. Appl., 2018, 76(7), 1680–1685.

[3] Y. M. Chen, S. H. Ma, Z. Y. Ma, *New exact solutions of (3+1)-dimensional Jimbo-Miwa system*, Chin. Phys. B., 2013, 22(5), 050510.

[4] C. Q. Dai, X. F. Zhang, Y. Fan et al., *Localized modes of the (n+1)-dimensional schrödinger equation with power-law nonlinearities in PT-symmetric potentials*, Commun. Nonlinear. Sci., 2017, 43, 239–250.

[5] J. Gao, L. J. Han, Y. H. Huang, *Solitary Waves for the Generalized Nonautonomous Dual-power Nonlinear Schrödinger Equations with Variable Coefficients*, Journal of Nonlinear Modeling and Analysis, 2019, 1, 251–260.

[6] B. Ghanbari, M. S. Osman, D. Baleanu, *Generalized exponential rational function method for extended Zakharov-Kuzetsov equation with conformable derivative*, Mod. Phys. Lett. A, 2019. DOI: 10.1142/S0217732319501554.

[7] Y. Kong, L. Xin, Q. Qiu et al., *Exact periodic wave solutions for the modified Zakharov equations with a quantum correction*, Appl. Math. Lett., 2019, 94, 140–148.

[8] Z. Z. Lan, W. Q. Hu, B. L. Guo, *General propagation lattice Boltzmann model for a variable-coefficient compound KdV-Burgers equation*, Appl. Math. Model., 2019. DOI: 10.1016/j.apm.2019.04.013.

[9] F. H. Lin, S. T. Chen, Q. X. Qu et al., *Resonant multiple wave solutions to a new (3+1)-dimensional generalized Kadomtsev-Petviashvili equation: Linear superposition principle*, Appl. Math. Lett., 2018, 78, 112–117.

[10] Y. Z. Li, J. G. Liu, *Multiple periodic-soliton solutions of the (3+1)-dimensional generalised shallow water equation*, Pramana, 2018, 90, 71.

[11] J. G. Liu, Y. He, *New periodic solitary wave solutions for the (3+1)-dimensional generalized shallow water equation*, Nonlinear Dyn., 2017, 90, 363–369.

[12] W. X. Ma, X. L. Yong, H. Q. Zhang, *Diversity of interaction solutions to the (2+1)-dimensional Ito equation*, Comput. Math. Appl., 2018, 75(1), 289–295.

[13] W. X. Ma, *Riemann-Hilbert problems of a six-component fourth-order AKNS system and its soliton solutions*, Comput. Appl. Math., 2018, 37, 6359–6375.

[14] W. X. Ma, *Lumps and their interaction solutions of (3+1)-dimensional linear PDEs*, J. Geom. Phys., 2018, 133, 10–16.

[15] W. X. Ma, J. Li, C. M. Khalique, *A Study on Lump Solutions to a Generalized Hirota-Satsuma-Ito Equation in (2+1)-Dimensions*, Complexity, 2018, 11, 9059858.

[16] X. H. Meng, *Rational solutions in Grammian form for the (3+1)-dimensional generalized shallow water wave equation*, Comput. Math. Appl., 2018, 75, 4534–4539.

[17] M. Mirzazadeh, M. Eslami, A. Biswas, *Soliton solutions of the generalized Klein-Gordon equation by using G'/G-expansion method*, Comput. Appl. Math., 2014, 33(3), 831–839.

[18] M. S. Osman, *Multiwave solutions of time-fractional (2+1)-dimensional Nizhnik-Novikov-Veselov equations*, pramana, 2017, 88, 67.

[19] M. S. Osman, *On complex wave solutions governed by the 2d ginzburg-landau equation with variable coefficients*, Optik., 2018, 156, 169–174.

[20] M. S. Osman, D. C. Lu, M. A. Khater, *A study of optical wave propagation in the nonautonomous Schrödinger-Hirota equation with power-law nonlinearity*, Results Phys., 2019, 13, 102157.

[21] M. S. Osman, J.A.T. Machado, *New nonautonomous combined multi-wave solutions for (2+1)-dimensional variable coefficients KdV equation*, Nonlinear Dyn., 2018, 93, 733–740.

[22] M. S. Osman, *One-soliton shaping and inelastic collision between double solitons in the fifth-order variable-coefficient Sawada-Kotera equation*, Nonlinear Dyn., 2019, 96, 1491–1496.

[23] M. S. Osman, B. Ghanbari, J. A. T. Machado, *New complex waves in nonlinear optics based on the complex Ginzburg-Landau equation with Kerr law nonlinearity*, Eur. Phys. J. Plus., 2019, 134, 20.

[24] M. S. Osman, B. Ghanbari, *New optical solitary wave solutions of Fokas-Lenells equation in presence of perturbation terms by a novel approach*, Optik., 2018, 175, 328–333.

[25] M. S. Osman, J. A. T. Machado, *The dynamical behavior of mixed-type soliton solutions described by (2+1)-dimensional Bogoyavlensky-Konopelchenko equation with variable coefficients*, J. Electromagnet. Wave., 2018, 32, 1457–1464.

[26] M. S. Osman, H. I. Abdel-Gawad, M. A. El Mahdy, *Two-layer-atmospheric blocking in a medium with high nonlinearity and lateral dispersion*, Results Phys., 2018, 8, 1054–1060.

[27] K. U. Tariq, M. Younis, R. Hadi et al., *Optical solutions to the space-time fractional nonlinear Schrödinger equation with quadratic-cubic nonlinearity*, Mod. Phys. Lett. B, 2018, 32(26), 1850317.

[28] Y. N. Tang, W. X. Ma, W. Xu, *Grammian and Pfaffian solutions as well as Pfaffianization for a (3+1)-dimensional generalized shallow water equation*, Chin. Phys. B., 2012, 21(7), 070212.

[29] B. Tian, Y. T. Gao, *Beyond travelling waves: a new algorithm for solving nonlinear evolution equations*, Comput. Phys. Commun., 2996, 95, 139–142.

[30] H. Wang, *Lump and interaction solutions to the (2+1)-dimensional Burgers equation*, Appl. Math. Lett., 2018, 85, 27–34.

[31] E. M. E. Zayed, *New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized (G'/G)-expansion method*, J. Phys. A: Math. Theor., 2019, 42(19), 195202–195214.

[32] E. M. E. Zayed, K. A. Gepreel, *The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics*, J. Math. Phys., 2009, 50, 013502.

[33] E. M. E. Zayed, *Traveling wave solutions for higher dimensional nonlinear evolution equations using the (G'/G)-expansion method*, J. Appl. Math. Inform., 2010, 28, 383–395.

[34] Z. F. Zeng, J. G. Liu, B. Nie, *Multiple-soliton solutions, soliton-type solutions and rational solutions for the (3+1)-dimensional generalized shallow water equation in oceans, estuaries and impoundments*, Nonlinear Dyn., 2016, 86, 667–675.

[35] S. Zhang, J. Tong, W. Wang, *A generalized (G'/G)-expansion method for the mKdV equation with variable coefficients*, Phys. Lett. A., 2008, 372, 2254–2257.

[36] Y. Zhou, S. Manukure, W. X. Ma, *Lump and lump-soliton solutions to the Hirota-Satsuma-Ito equation*, Commun. Nonlinear Sci. Numer. Simulat., 2019, 68, 56–62.

[37] D. W. Zuo, Y .T. Gao, G. Q. Meng et al., *Multi-soliton solutions for the three-coupled kdv equations engendered by the neumann system*, Nonlinear Dyn., 2014, 75(4), 1–8.