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EXTINCTION IN A NONAUTONOMOUS COMPETITIVE SYSTEM WITH TOXIC SUBSTANCE AND FEEDBACK CONTROL*

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Abstract This paper deals with a nonautonomous competitive system with infinite delays and feedback control. Sufficient conditions for the permanence of the system are first obtained. By constructing a suitable Lyapunov function, we obtain the sufficient conditions which guarantee that one of the components is driven to extinction. Our result shows that feedback control have an influence on the extinction of the system. Examples together with their numerical simulations illustrate the feasibility of our main results.

Keywords Competitive system, permanence, extinction, feedback control, toxic substance.

MSC(2010) 34C25, 92D25, 34D20, 34D40.

1. Introduction

In the real ecosystem, competition is everywhere. Mathematically, the dynamic behaviors of competitive systems have been attracting much attention, see for example [1, 4, 8, 10, 14, 15, 19] and references therein.

In [14], Montes de Oca and Zeeman considered the following nonautonomous Lotka-Volterra competitive system

$$\dot{x}_i(t) = x_i(t)\left\{b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)\right\}, \ i = 1, 2, \ldots, n, \tag{1.1}$$

where $x_i(t)$ is the population density of the $i-$th species at time t, respectly. They show that all but one of the species is driven to extinction if exhibiting simple algebraic criteria on the parameters.

Ecologically, the impact of toxic substances on ecological communities is an important problem [10, 12, 15–17]. In addition to competition between two species, each species produces a toxic substance to the other, but only when the other is present. Chattopadhyay [3] studied the following two species autonomous compet-

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itive system with toxic substances

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\big\{r_1 - a_{11}x_1(t) - a_{12}x_2(t) - b_1x_1(t)x_2(t)\big\},\\ \dot{x}_2(t) &= x_2(t)\big\{r_2 - a_{21}x_1(t) - a_{22}x_2(t) - b_2x_1(t)x_2(t)\big\},\end{aligned}\tag{1.2}$$

where $x_1(t)$ and $x_2(t)$ denote the population densities of two competitive species at time t for a common pool of resources, $r_1, r_2, a_{11}, a_{12}, a_{21}, a_{22}, b_1$ and b_2 are positive constants. The first three terms in the right side of (1.2) represent competitive growth, and the last term denotes the effect of toxic substances. Chattopadhyay [3] showed that toxic substances play an important role in stabilizing the system.

Many biological or environmental parameters do subject to fluctuate with time, so considering the nonautonomous parameters, Li and Chen [11] studied the following two species nonautonomous competitive system with toxic substances

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\big\{r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - b_1(t)x_1(t)x_2(t)\big\},\\ \dot{x}_2(t) &= x_2(t)\big\{r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - b_2(t)x_1(t)x_2(t)\big\}.\end{aligned}\tag{1.3}$$

They showed that if the coefficients of the system satisfy a series of conditions, $x_2(t)$ will be driven to extinction while $x_1(t)$ will stabilize at a certain solution of a logistic equation.

On the one hand, in the real world, the movement of most species is not affected by the current state, but related to the past state. Therefore, in order to describe the state of species more precisely or the operation of the system better, it is necessary to introduce the part describing the influence of the past state on the system in the differential system, namely delay, see some works [7, 18]. However, when the time lag is quite large, we usually consider the infinite delay, see also some works [2, 5, 9, 13, 20]. On the other hand, it is well known that the population in an ecosystem is often affected by various factors from outside, which leads to changes in various parameters of the ecosystem. The experiment shows that the feedback controls have the ideal effect of eliminating the external interference. Therefore, feedback control is of great significance for the protection of the species diversity and the maintenance of the sustainable development of the ecological environment.

Chen et al. [2] and Hu et al. [9] studied the following two-species nonautonomous Lotka-Volterra competitive system with infinite delays and feedback controls

$$\begin{aligned}\dot{x}_1(t) &= x_1(t)\bigg\{r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)\int_0^{+\infty} k_1(s)x_2(t-s)ds\\ &\qquad -c_1(t)\int_0^{+\infty} k_2(s)u_1(t-s)ds\bigg\},\\ \dot{x}_2(t) &= x_2(t)\bigg\{r_2(t) - a_{21}(t)\int_0^{+\infty} k_3(s)x_1(t-s)ds - a_{22}(t)x_2(t)\\ &\qquad +c_2(t)\int_0^{+\infty} k_4(s)u_2(t-s)ds\bigg\},\\ \dot{u}_1(t) &= -d_1(t)u_1(t) + e_1(t)\int_0^{+\infty} k_5(s)x_1(t-s)ds,\\ \dot{u}_2(t) &= f(t) - d_2(t)u_2(t) - e_2(t)\int_0^{+\infty} k_6(s)x_2(t-s)ds,\end{aligned}\tag{1.4}$$

Chen et al. [2] obtained sufficient conditions for the permanence of system (1.4) and showed that controls can avoid the extinction of the species. Then Hu et al. [9] improved and extended sufficient conditions given in [2].

Stimulated by the works of [2,11], we propose the following system

$$
\begin{aligned}
\dot{x}_1(t) &= x_1(t)\Big\{ r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)\int_0^{+\infty} k_1(s)x_2(t-s)ds \\
&\qquad -b_1(t)x_1(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds - c_1(t)\int_0^{+\infty} k_3(s)u_1(t-s)ds\Big\}, \\
\dot{x}_2(t) &= x_2(t)\Big\{ r_2(t) - a_{21}(t)\int_0^{+\infty} k_4(s)x_1(t-s)ds - a_{22}(t)x_2(t) \\
&\qquad -b_2(t)x_2(t)\int_0^{+\infty} k_5(s)x_1(t-s)ds - c_2(t)\int_0^{+\infty} k_6(s)u_2(t-s)ds\Big\}, \\
\dot{u}_1(t) &= -d_1(t)u_1(t) + e_1(t)\int_0^{+\infty} k_7(s)x_1(t-s)ds, \\
\dot{u}_2(t) &= -d_2(t)u_2(t) + e_2(t)\int_0^{+\infty} k_8(s)x_2(t-s)ds,
\end{aligned}
\tag{1.5}
$$

together with initial conditions

$$
\begin{aligned}
x_i(t) &= \phi_i(t) \in BC^+,\ t\in(-\infty,0]; \\
u_i(t) &= \varphi_i(t) \in BC^+,\ t\in(-\infty,0];
\end{aligned}
\tag{1.6}
$$

where $\phi_i, \psi_i \in BC^+$ and

$$BC^+ = \{\phi \in C((-\infty,0],[0,+\infty)) : \phi(0) > 0 \text{ and } \phi \text{ is bounded}\},\ \ i = 1,2.$$

It is well known that by the fundamental theory of functional differential equations [6], problem (1.5) (1.6) admits a unique solution $(x_1(t), x_2(t), u_1(t), u_2(t))$. Moreover, $x_i(t) > 0$ and $u_i(t) > 0$ for all $i = 1,2$ in maximal interval of existence of the solution.

The organization of the this paper is as follows. Section 2 deals with several basic assumptions for problem (1.5) and four useful lemmas are given. Section 3 is devoted to the sufficient conditions for the permanence of the system and section 4 is for the extinction of species x_2. Examples are presented in section 5 to show the feasibility of our main results.

2. Preliminaries

For convenience, we define $f^u = \max\limits_{t\in R} f(t)$ and $f^l = \inf\limits_{t\in R} f(t)$. Before we state the main result of this paper, we first introduce some lemmas.

Lemma 2.1. (see [13]) *Let $x : R \to R$ be a bounded nonnegative continuous function, and let $k : [0,+\infty) \to (0,+\infty)$ be a continuous kernel such that $\int_0^{+\infty} k(s)ds = 1$. Then*

$$
\begin{aligned}
\liminf_{t\to+\infty} x(t) &\le \liminf_{t\to+\infty} \int_0^{+\infty} k(s)x(t-s)ds \\
&\le \limsup_{t\to+\infty} \int_0^{+\infty} k(s)x(t-s)ds \le \limsup_{t\to+\infty} x(t).
\end{aligned}
$$

We consider the following nonautonomous logistic equation

$$\dot{x}(t) = x(t)(a(t) - b(t)x(t)), \tag{2.1}$$

where function $a(t)$ and $b(t)$ are bounded continuous defined on R_+ and $b(t) \geq 0$ for all $t \geq 0$. The following result is known.

Lemma 2.2. (see [21]) *Suppose there are constants $\omega > 0$ and $\lambda > 0$ such that*

$$\liminf_{t\to\infty} \int_t^{t+\omega} a(s)ds > 0 \text{ and } \liminf_{t\to\infty} \int_t^{t+\lambda} b(s)ds > 0.$$

Then, there exist constants $M \geq m > 0$ such that

$$m \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq M$$

for any positive solution $x(t)$ of Eq. (2.1).

Further, considering the following nonautonomous linear equation

$$\dot{x}(t) = a(t) - b(t)x(t), \tag{2.2}$$

where functions $a(t)$ and $b(t)$ are bounded continuous defined on R_+. We have the following results.

Lemma 2.3. (see [9]) *Suppose $a(t) \geq 0$ for all $t \geq 0$ and there are constants $\omega > 0$ and $\lambda > 0$ such that*

$$\liminf_{t\to\infty} \int_t^{t+\omega} a(s)ds > 0 \text{ and } \liminf_{t\to\infty} \int_t^{t+\lambda} b(s)ds > 0.$$

Then, there exist constants $M \geq m > 0$ such that

$$m \leq \liminf_{t\to+\infty} x(t) \leq \limsup_{t\to+\infty} x(t) \leq M$$

for any positive solution $x(t)$ of Eq. (2.2).

Lemma 2.4. (see [8]) *Suppose that there exists a constant $\omega > 0$ such that*

$$\liminf_{t\to\infty} \int_t^{t+\omega} b(s)ds > 0.$$

Then, for any constants $\varepsilon > 0$ and $M > 0$ there exist constants $\delta = \delta(\varepsilon) > 0$ and $T_0 = T_0(M) > 0$ such that for any $t_0 \in R_+, x_0 \in R$ and $|x_0| \leq M$, when $|a(t)| < \delta$ for all $t \geq t_0$, one has

$$|x(t, t_0, x_0)| < \varepsilon \text{ for all } t \geq t_0 + T_0,$$

where $x(t, t_0, x_0)$ is the solution of Eq. (2.2) with initial condition $x(t_0) = x_0$.

3. Permanence

In this section, we adopt the following assumptions.

(H_1) $r_i(t)$, $a_{ij}(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e_i(t)(i, j = 1, 2)$ are bounded and continuously defined on $[0, +\infty)$. Furthermore, $a_{ij}(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e_i(t)$ are nonnegative on $[0, +\infty)$.

(H_2) $k_i : [0, +\infty) \to [0, +\infty)(i = 1, 2, \ldots, 8)$ are piecewise continuous and satisfy

$$\int_0^{+\infty} k_i(s)ds = 1 \text{ and } \sigma_i = \int_0^{+\infty} sk_i(s)ds < \infty.$$

(H_3) There exists a positive constant ω such that for each $i = 1, 2$,

$$\liminf_{t\to\infty} \int_t^{t+\omega} r_i(s)ds > 0.$$

(H_4) There exists a positive constant λ_i such that for each $i = 1, 2$,

$$\liminf_{t\to\infty} \int_t^{t+\lambda_i} a_{ii}(s)ds > 0.$$

(H_5) There exists a positive constant β_i such that for each $i = 1, 2$,

$$\liminf_{t\to\infty} \int_t^{t+\beta_i} e_i(s)ds > 0.$$

(H_6) There exists a positive constant γ_i such that for each $i = 1, 2$,

$$\liminf_{t\to\infty} \int_t^{t+\gamma_i} d_i(s)ds > 0.$$

(H_7) There exists a positive constant δ such that

$$\liminf_{t\to\infty} \int_t^{t+\delta} (r_1(s) - \overline{x}_2 a_{12}(s))ds > 0.$$

(H_8) There exists a positive constant η such that

$$\liminf_{t\to\infty} \int_t^{t+\eta} (r_2(s) - \overline{x}_1 a_{21}(s))ds > 0,$$

where $\overline{x}_1$ and $\overline{x}_2$ are defined in Theorem 3.1 (also, one can see Remark 3.1 for more details).

Theorem 3.1. *Suppose that assumptions $(H_1)-(H_6)$ hold, for any positive solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ of system (1.5) (1.6), we have*
(i) There exist positive constants $\overline{x}_i$ and $\overline{u}_i$ such that

$$\limsup_{t\to+\infty} x_i(t) \le \overline{x}_i \text{ and } \limsup_{t\to+\infty} u_i(t) \le \overline{u}_i,\ i = 1, 2.$$

(ii) If (H_7) holds, there exists a positive constant $\underline{x}_1$ such that

$$\liminf_{t\to+\infty} x_1(t) \ge \underline{x}_1.$$

(iii) If (H_8) holds, there exists a positive constant $\underline{x}_2$ such that

$$\liminf_{t\to+\infty} x_2(t) \ge \underline{x}_2.$$

Proof. The proof of Theorem 3.1 is similar to that of Theorems 3.1 and 3.2 of Hu et al. [9], so we omit the detail here. □

Remark 3.1. If in system (1.5) all parameters $r_i(t)$, $a_{ij}(t)$, $b_i(t)$, $c_i(t)$, $d_i(t)$, $e_i(t)(i,j=1,2)$ have the positive lower and upper bound on R_+, then it is not hard to prove that in Theorem 3.1(i) we can choose

$$\overline{x}_i = \frac{r_i^u}{a_{ii}^l} \text{ and } \overline{u}_i = \frac{e_i^u r_i^u}{d_i^l a_{ii}^l},\ i=1,2.$$

Theorem 3.2. *Assume further that $(H_1)-(H_8)$ hold, then the system (1.5) (1.6) is permanent. That is, for every solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ of system (1.5) with initial condition (1.6), for every $i=1,2$, there exist positive constants $\overline{x}_i, \underline{x}_i, \overline{u}_i$ and $\underline{u}_i$ such that*

$$\underline{x}_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le \overline{x}_i,$$

$$\underline{u}_i \le \liminf_{t\to+\infty} u_i(t) \le \limsup_{t\to+\infty} u_i(t) \le \overline{u}_i.$$

Proof. From Theorems 3.1, we only need prove that there exist constants $\underline{u}_i > 0$ such that

$$\liminf_{t\to+\infty} u_i(t) \ge \underline{u}_i,\ i=1,2. \tag{3.1}$$

From Lemma 2.1, Theorems 3.1(ii) and (iii), we can choose positive constants ε and T_0 such that for all $t \ge T_0$

$$\int_0^{+\infty} k_7(s)x_1(t-s)ds \ge \underline{x}_1 - \varepsilon \text{ and } \int_0^{+\infty} k_8(s)x_2(t-s)ds \ge \underline{x}_2 - \varepsilon. \tag{3.2}$$

From the third and fourth equation of system (1.5) and (3.2), for all $t \ge T_0$, we have

$$\dot{u_i}(t) \ge -d_i(t)u_i(t) + e_i(t)(\underline{x}_i - \varepsilon),\ i=1,2. \tag{3.3}$$

Consider the following auxiliary problem

$$\begin{aligned} &\dot{v_i}(t) = -d_i(t)v_i(t) + e_i(t)(\underline{x}_i - \varepsilon),\ i=1,2,\ t > T_0,\\ &v_i(T_0) = u_i(T_0). \end{aligned} \tag{3.4}$$

By assumptions (H_5) and (H_6) and Lemma 2.3, we obtain that there is a positive constant $\underline{u}_i$ such that

$$\liminf_{t\to+\infty} v_i(t) \ge \underline{u}_i,\ i=1,2.$$

It follows from the comparison theorem that

$$u_i(t) \ge v_i(t) \text{ for all } t \ge T_0,\ \ i=1,2.$$

Thus, we finally obtain

$$\liminf_{t\to+\infty} u_i(t) \ge \underline{u}_i,\ i=1,2.$$

This completes the proof. □

4. Extinction

In this section, we discuss the extinction of species x_2 of problem (1.5) (1.6). Let us define the functions

$$A_{12}(t) = \int_0^{+\infty} k_1(s)a_{12}(t+s)ds, \quad A_{21}(t) = \int_0^{+\infty} k_4(s)a_{21}(t+s)ds,$$

$$C_1(t) = \int_0^{+\infty} k_3(s)c_1(t+s)ds, \quad C_2(t) = \int_0^{+\infty} k_6(s)c_2(t+s)ds,$$

$$E_1(t) = \int_0^{+\infty} k_7(s)e_1(t+s)ds, \quad E_2(t) = \int_0^{+\infty} k_8(s)e_2(t+s)ds.$$

Theorem 4.1. *Suppose that assumptions $(H_1)-(H_4)$ hold, assume further that*

$$\limsup_{t\to\infty} \frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} < \liminf_{t\to\infty} \frac{b_2(t)}{b_1(t)}, \tag{H_9}$$

$$\limsup_{t\to\infty} \frac{C_1(t)}{d_1(t)} < \liminf_{t\to\infty} \left(\frac{A_{21}(t)}{E_1(t)} \liminf_{t\to\infty} \frac{\displaystyle\int_t^{t+\omega} r_1(s)ds}{\displaystyle\int_t^{t+\omega} r_2(s)ds} - \frac{a_{11}(t)}{E_1(t)} \right), \tag{H_{10}}$$

$$\liminf_{t\to\infty} \frac{C_2(t)}{d_2(t)} > \limsup_{t\to\infty} \left(\frac{A_{12}(t)}{E_2(t)} \limsup_{t\to\infty} \frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} - \frac{a_{22}(t)}{E_2(t)} \right). \tag{H_{11}}$$

Then, we have

$$\lim_{t\to\infty} x_2(t) = 0, \ \lim_{t\to\infty} u_2(t) = 0, \ \int_0^{+\infty} x_2(t)dt < \infty,$$

for any positive solution $(x_1(t), x_2(t), u_1(t), u_2(t))$ of problem (1.5) (1.6).

Proof. From assumption (H_3), we have that there exist positive constants η_0 and T_0 such that

$$\int_t^{t+\omega} r_i(s)ds \geq \eta_0 \quad \text{for all } t \geq T_0, \ \ i = 1, 2.$$

From conditions $(H_9)-(H_{11})$, we can find positive constants $\alpha, \beta, \varepsilon, \gamma, \delta$ and $T_1 \geq T_0$ such that for all $t \geq T_1$,

$$\frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} < \frac{\alpha}{\beta} - \varepsilon < \frac{\alpha}{\beta} < \frac{b_2(t)}{b_1(t)}, \tag{4.1}$$

$$\frac{C_1(t)}{d_1(t)} < \frac{\gamma}{\alpha} < \frac{\beta A_{21}(t) - \alpha a_{11}(t)}{\alpha E_1(t)}, \tag{4.2}$$

and

$$\frac{C_2(t)}{d_2(t)} > \frac{\delta}{\beta} > \frac{\alpha A_{12}(t) - \beta a_{22}(t)}{\beta E_2(t)}. \tag{4.3}$$

Therefore, we have

$$\int_t^{t+\omega} \beta r_2(s) - \alpha r_1(s) ds < -\varepsilon\beta \int_t^{t+\omega} r_1(s) ds < -\varepsilon\beta\eta_0, \tag{4.4}$$

$$\alpha a_{11}(t) - \beta A_{21}(t) + \gamma E_1(t) < 0, \tag{4.5}$$

$$\alpha A_{12}(t) - \beta a_{22}(t) - \delta E_2(t) < 0, \tag{4.6}$$

$$\alpha b_1(t) - \beta b_2(t) < 0, \tag{4.7}$$

$$-\gamma d_1(t) + \alpha C_1(t) < 0, \tag{4.8}$$

and

$$\delta d_2(t) - \beta C_2(t) < 0 \tag{4.9}$$

for all $t \geq T_1$. Consider the following Lyapunov function

$$\begin{aligned}
V(t) = x_1^{-\alpha}(t) x_2^{\beta}(t) \exp\Big\{ & \gamma u_1(t) - \delta u_2(t) + \alpha \int_0^{+\infty} \int_{t-s}^{t} k_1(s) a_{12}(\theta + s) x_2(\theta) d\theta ds \\
& + \alpha \int_0^{+\infty} \int_{t-s}^{0} k_2(s) b_1(\theta + s) x_1(\theta + s) x_2(\theta) d\theta ds \\
& + \alpha \int_0^{+\infty} \int_0^{t} k_2(s) b_1(\theta) x_1(\theta) x_2(\theta) d\theta ds \\
& + \alpha \int_0^{+\infty} \int_{t-s}^{t} k_3(s) c_1(\theta + s) u_1(\theta) d\theta ds - \beta \int_0^{+\infty} \int_{t-s}^{t} k_4(s) a_{21}(\theta + s) x_1(\theta) d\theta ds \\
& - \beta \int_0^{+\infty} \int_{t-s}^{0} k_5(s) b_2(\theta + s) x_1(\theta) x_2(\theta + s) d\theta ds \\
& - \beta \int_0^{+\infty} \int_0^{t} k_5(s) b_2(\theta) x_1(\theta) x_2(\theta) d\theta ds \\
& - \beta \int_0^{+\infty} \int_{t-s}^{t} k_6(s) c_2(\theta + s) u_2(\theta) d\theta ds + \gamma \int_0^{+\infty} \int_{t-s}^{t} k_7(s) e_1(\theta + s) x_1(\theta) d\theta ds \\
& - \delta \int_0^{+\infty} \int_{t-s}^{t} k_8(s) e_2(\theta + s) x_2(\theta) d\theta ds \Big\}.
\end{aligned}$$

Calculating the derivative of $V(t)$ with respect to t yields

$$\begin{aligned}
\dot{V}(t) = V(t) \Big\{ & -\alpha r_1(t) + \alpha a_{11}(t) x_1(t) + \alpha a_{12}(t) \int_0^{+\infty} k_1(s) x_2(t-s) ds \\
& + \alpha b_1(t) x_1(t) \int_0^{+\infty} k_2(s) x_2(t-s) ds + \alpha c_1(t) \int_0^{+\infty} k_3(s) u_1(t-s) ds \\
& + \beta r_2(t) - \beta a_{22}(t) x_2(t) - \beta a_{21}(t) \int_0^{+\infty} k_4(s) x_1(t-s) ds \\
& - \beta b_2(t) x_2(t) \int_0^{+\infty} k_5(s) x_1(t-s) ds - \beta c_2(t) \int_0^{+\infty} k_6(s) u_2(t-s) ds
\end{aligned}$$

$$
\begin{aligned}
&- \gamma d_1(t)u_1(t) + \gamma e_1(t)\int_0^{+\infty} k_7(s)x_1(t-s)ds \\
&+ \delta d_2(t)u_2(t) - \delta e_2(t)\int_0^{+\infty} k_8(s)x_2(t-s)ds \\
&+ \alpha x_2(t)\int_0^{+\infty} k_1(s)a_{12}(t+s)ds - \alpha a_{12}(t)\int_0^{+\infty} k_1(s)x_2(t-s)ds \\
&- \alpha b_1(t)x_1(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds + \alpha b_1(t)x_1(t)x_2(t) \\
&+ \alpha u_1(t)\int_0^{+\infty} k_3(s)c_1(t+s)ds - \alpha c_1(t)\int_0^{+\infty} k_3(s)u_1(t-s)ds \\
&- \beta x_1(t)\int_0^{+\infty} k_4(s)a_{21}(t+s)ds + \beta a_{21}(t)\int_0^{+\infty} k_4(s)x_1(t-s)ds \\
&+ \beta b_2(t)x_2(t)\int_0^{+\infty} k_5(s)x_1(t-s)ds - \beta b_2(t)x_1(t)x_2(t) \\
&- \beta u_2(t)\int_0^{+\infty} k_6(s)c_2(t+s)ds + \beta c_2(t)\int_0^{+\infty} k_6(s)u_2(t-s)ds \\
&+ \gamma x_1(t)\int_0^{+\infty} k_7(s)e_1(t+s)ds - \gamma e_1(t)\int_0^{+\infty} k_7(s)x_1(t-s)ds \\
&- \delta x_2(t)\int_0^{+\infty} k_8(s)e_2(t+s)ds + \delta e_2(t)\int_0^{+\infty} k_8(s)x_2(t-s)ds\Big\} \\
=&V(t)\Big\{\big(-\alpha r_1(t) + \beta r_2(t)\big) + \big(\alpha a_{11}(t) - \beta A_{21}(t) + \gamma E_1(t)\big)x_1(t) \\
&+ \big(-\beta a_{22}(t) + \alpha A_{12}(t) - \delta E_2(t)\big)x_2(t) + \big(-\gamma d_1(t) + \alpha C_1(t)\big)u_1(t) \\
&+ \big(\delta d_2(t) - \beta C_2(t)\big)u_2(t) + \big(\alpha b_1(t) - \beta b_2(t)\big)x_1(t)x_2(t)\Big\}.
\end{aligned}
$$

From inequalities (4.5)-(4.9), we can obtain

$$\dot{V}(t) < V(t)\big(-\alpha r_1(t) + \beta r_2(t)\big), \quad t \geq T_2. \tag{4.10}$$

For any $t \geq T_2$, we choose an integer $n \geq 0$ such that $t \in \big[T_2 + n\omega, T_2 + (n+1)\omega\big)$. Integrating (4.10) from T_2 to t and using (4.4) give

$$
\begin{aligned}
V(t) &\leq V(T_2)\exp\Big\{\int_{T_2}^{t}(-\alpha r_1(s) + \beta r_2(s))ds\Big\} \\
&= V(T_2)\exp\Big(\int_{T_2}^{T_2+n\omega} + \int_{T_2+n\omega}^{t}\Big)(-\alpha r_1(s) + \beta r_2(s))ds \\
&\leq V(T_2)\exp\Big\{-\varepsilon\beta\eta_0 n + M_1\Big\} < V(T_2)\exp\Big\{-\varepsilon\beta\eta_0\Big(\frac{t-T_2}{\omega} - 1\Big) + M_1\Big\} \\
&= V(T_2)\exp\Big\{-\frac{\varepsilon\beta\eta_0 t}{\omega} + \frac{\varepsilon\beta\eta_0 T_2}{\omega} + \varepsilon\beta\eta_0 + M_1\Big\} = V(T_2)\exp(-\lambda t + M_1^*),
\end{aligned} \tag{4.11}
$$

where $\lambda = \dfrac{\varepsilon\beta\eta_0}{\omega}$, $M_1^* = \dfrac{\varepsilon\beta\eta_0 T_2}{\omega} + \varepsilon\beta\eta_0 + M_1$ and $M_1 = \sup\limits_{t\geq 0}\big|\beta r_2(t) - \alpha r_1(t)\big|\omega$. On the other hand, from assumptions (H_1) and (H_2), for all $t \geq 0$, we have

$$\begin{aligned} V(t) \geq & x_1^{-\alpha}(t)x_2^{\beta}(t)\exp\Big\{ -\delta u_2(t) - \beta\int_0^{+\infty}\int_{t-s}^{t} k_4(s)a_{21}(\theta+s)x_1(\theta)d\theta ds \\ & - \beta\int_0^{+\infty}\int_{t-s}^{0} k_5(s)b_2(\theta+s)x_1(\theta)x_2(\theta+s)d\theta ds \\ & - \beta\int_0^{+\infty}\int_0^{t} k_5(s)b_2(\theta)x_1(\theta)x_2(\theta)d\theta ds \\ & - \beta\int_0^{+\infty}\int_{t-s}^{t} k_6(s)c_2(\theta+s)u_2(\theta)d\theta ds - \delta\int_0^{+\infty}\int_{t-s}^{t} k_8(s)e_2(\theta+s)x_2(\theta)d\theta ds\Big\} \\ \geq & x_1^{-\alpha}(t)x_2^{\beta}(t)\exp\Big\{ -\delta\sup_{t\geq 0}u_2(t) - \beta\sup_{t\geq 0}a_{21}(t)\sup_{t\in R}x_1(t)\int_0^{+\infty} sk_4(s)ds \\ & - \beta\sup_{t\geq 0}b_2(t)\sup_{t\geq 0}x_1(t)\sup_{t\geq 0}x_2(t)\int_0^{+\infty} sk_5(s)ds \\ & - \beta\sup_{t\geq 0}c_2(t)\sup_{t\in R}u_2(t)\int_0^{+\infty} sk_6(s)ds - \delta\sup_{t\geq 0}e_2(t)\sup_{t\in R}x_2(t)\int_0^{+\infty} sk_8(s)ds\Big\}. \end{aligned}$$

Therefore, we obtain that there exists a positive constant $W > 0$ such that for all $t \geq 0$

$$V(t) \geq W x_1^{-\alpha}(t)x_2^{\beta}(t). \tag{4.12}$$

By (4.11), (4.12), for all $t \geq T_2$, we have

$$x_2(t) \leq \big(W^{-1}x_1^{\alpha}(t)V(T_2)\exp(-\lambda t + M_1^*)\big)^{1/\beta} \leq W^*\exp(-\lambda^* t), \tag{4.13}$$

where $W^* = \Big(W^{-1}\sup_{t\geq 0}x_1^{\alpha}(t)V(T_2)\exp M_1^*\Big)^{1/\beta}$ and $\lambda^* = \dfrac{\lambda}{\beta}$. Hence, we finally obtain that $x_2(t) \to 0$ and $\int_0^{+\infty} k_8(s)x_2(t-s)ds \to 0$ as $t \to \infty$.

Further, it follows from the forth equation of system (1.5) and Lemma 2.4 that $u_2(t) \to 0$ as $t \to \infty$. □

Considering the following system without feedback control

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\Big\{ r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)\int_0^{+\infty} k_1(s)x_2(t-s)ds \\ &\qquad -b_1(t)x_1(t)\int_0^{+\infty} k_2(s)x_2(t-s)ds\Big\}, \\ \dot{x}_2(t) &= x_2(t)\Big\{ r_2(t) - a_{21}(t)\int_0^{+\infty} k_4(s)x_1(t-s)ds - a_{22}(t)x_2(t) \\ &\qquad -b_2(t)x_2(t)\int_0^{+\infty} k_5(s)x_1(t-s)ds\Big\}, \end{aligned} \tag{4.14}$$

together with initial conditions $x_i(t) = \phi_i(t) \in BC^+, t \in (-\infty, 0]$, we have the following corollary.

Corollary 4.1. *Suppose that assumptions $(H_1)-(H_4)$ hold, assume further that*

$$\limsup_{t\to\infty}\frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} < \min\left\{\liminf_{t\to\infty}\frac{A_{21}(t)}{a_{11}(t)},\ \liminf_{t\to\infty}\frac{a_{22}(t)}{A_{12}(t)}\right\}. \tag{H_{12}}$$

Then, we have

$$\lim_{t\to\infty} x_2(t) = 0$$

for any positive solutions $(x_1(t), x_2(t))$ of system (4.14).

5. Examples and discussions

In this section, we present some numerical to illustrate our theoretical analysis. First, we present the permanence of system (1.5).

Example 5.1. We consider the following system

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\bigg\{(2.9+0.1\sin t) - 3x_1(t) - 4\int_0^{+\infty} e^{-s}x_2(t-s)ds \\
&\qquad -4x_1(t)\int_0^{+\infty} e^{-s}x_2(t-s)ds - (6+0.2\cos t)\int_0^{+\infty} e^{-s}u_1(t-s)ds\bigg\},\\
\dot{x}_2(t) &= x_2(t)\bigg\{1.5 - \int_0^{+\infty} e^{-s}x_1(t-s)ds - 3x_2(t) \\
&\qquad -2x_2(t)\int_0^{+\infty} e^{-s}x_1(t-s)ds - (1+0.5\cos t)\int_0^{+\infty} e^{-s}u_2(t-s)ds\bigg\},\\
\dot{u}_1(t) &= -2u_1(t) + \int_0^{+\infty} e^{-s}x_1(t-s)ds,\\
\dot{u}_2(t) &= -2u_2(t) + 4.5\int_0^{+\infty} e^{-s}x_2(t-s)ds.
\end{aligned} \tag{5.1}$$

Here, corresponding to system (1.5), we set

$$\begin{aligned}
&r_1(t) = 2.9 + 0.1\sin t, a_{11}(t) = 3, a_{12}(t) = 4, b_1(t) = 4, c_1(t) = 6 + 0.2\cos t,\\
&r_2(t) = 1.5, a_{21}(t) = 1, a_{22}(t) = 3, b_2(t) = 2, c_2(t) = 1 + 0.5\cos t,\\
&d_1(t) = 2, e_1(t) = 1, d_2(t) = 2, e_2(t) = 4.5.
\end{aligned} \tag{5.2}$$

By simple computation, one could see that

$$\overline{x}_1 = \frac{r_1^u}{a_{11}^l} = 1,\quad \overline{x}_2 = \frac{r_2^u}{a_{22}^l} = \frac{1}{2},$$

$$\liminf_{t\to\infty}\int_t^{t+2\pi}(r_1(s) - \overline{x}_2 a_{12}(s))ds \approx 5.65 > 0. \tag{5.3}$$

$$\liminf_{t\to\infty}\int_t^{t+2\pi}(r_2(s) - \overline{x}_1 a_{21}(s))ds \approx 3.14 > 0, \tag{5.4}$$

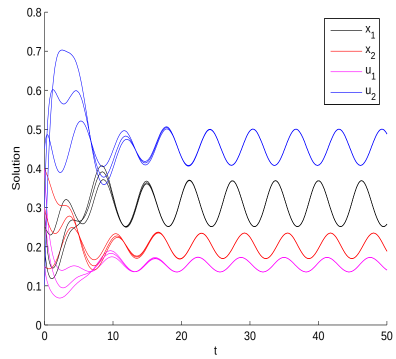

Figure 1. Dynamic behaviors of system (5.1). Here, we take the initial functions $(x_1(\theta),\ x_2(\theta),\ u_1(\theta),\ u_2(\theta))$= $(0.2, 0.4, 0.15, 0.1)$, $(0.25, 0.15, 0.4, 0.45)$ and $(0.3, 0.3, 0.3, 0.3)$ for all $\theta \in (-\infty, 0]$.

which means that the conditions $(H_1)-(H_8)$ hold. Therefore, by Theorem 3.2, the system (5.1) is permanent, see Figure 1.

Next, an example is given to illustrate the feasibility of Theorem 4.1.

Example 5.2. Consider the following nonautonomous Lotka-Volterra competitive system with infinite delays and feedback controls

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\Big\{2+\sin(t)-3x_1(t)-4\int_0^{+\infty} e^{-s}x_2(t-s)ds \\
&\qquad -x_1(t)\int_0^{+\infty} e^{-s}x_2(t-s)ds-\frac{1}{2}\int_0^{+\infty} e^{-s}u_1(t-s)ds\Big\}, \\
\dot{x}_2(t) &= x_2(t)\Big\{1+\cos(t)-2\int_0^{+\infty} e^{-s}x_1(t-s)ds-3x_2(t) \\
&\qquad -2x_2(t)\int_0^{+\infty} e^{-s}x_1(t-s)ds-\int_0^{+\infty} e^{-s}u_2(t-s)ds\Big\}, \\
\dot{u}_1(t) &= -(4+\sin(t))u_1(t)+(2+\sin(t))\int_0^{+\infty} e^{-s}x_1(t-s)ds, \\
\dot{u}_2(t) &= -(1+2\sin(t))u_2(t)+(2+\sin(t))\int_0^{+\infty} e^{-s}x_2(t-s)ds,
\end{aligned} \tag{5.5}$$

where

$$\begin{aligned}
& r_1(t)=2+\sin(t), a_{11}(t)=3, a_{12}(t)=4, b_1(t)=1, c_1(t)=\frac{1}{2}, \\
& r_2(t)=1+\cos(t), a_{21}(t)=2, a_{22}(t)=3, b_2(t)=2, c_2(t)=1, \\
& d_1(t)=4+\sin(t), e_1(t)=2+\sin(t), \\
& d_2(t)=1+2\sin(t), e_2(t)=2+\sin(t).
\end{aligned}$$

Obviously, we have that the period of system (5.5) is $\omega = 2\pi$. Simple computations show that

$$\limsup_{t\to\infty}\frac{\int_t^{t+\omega} r_2(s)ds}{\int_t^{t+\omega} r_1(s)ds}=\frac{1}{2}<\liminf_{t\to\infty}\frac{b_2(t)}{b_1(t)}=2,$$

$$\limsup_{t\to\infty}\frac{C_1(t)}{d_1(t)}=\frac{1}{6}<\liminf_{t\to\infty}\left(\frac{A_{21}(t)}{E_1(t)}\liminf_{t\to\infty}\frac{\int_t^{t+\omega} r_1(s)ds}{\int_t^{t+\omega} r_2(s)ds}-\frac{a_{11}(t)}{E_1(t)}\right)=\frac{1}{3},$$

$$\liminf_{t\to\infty}\frac{C_2(t)}{d_2(t)}=\frac{1}{3}>\limsup_{t\to\infty}\left(\frac{A_{12}(t)}{E_2(t)}\limsup_{t\to\infty}\frac{\int_t^{t+\omega} r_2(s)ds}{\int_t^{t+\omega} r_1(s)ds}-\frac{a_{22}(t)}{E_2(t)}\right)=-1.$$

Therefore, the conditions $(H_9)-(H_{11})$ hold. It follows from Theorem 4.1 that species x_2 will be driven to extinction, see Figure 2.

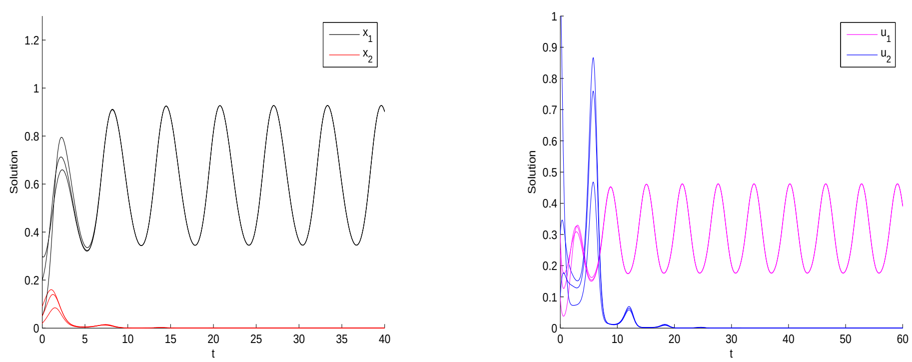

Figure 2. Dynamic behaviors of system (5.5). Here, we take the initial functions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta))= (0.2, 0.05, 0.25, 0.1)$, $(0.05, 0.02, 0.1, 1.19)$ and $(0.3, 0.09, 0.3, 0.3)$ for all $\theta \in (-\infty, 0]$.

Example 5.3. First, we consider the following system

$$\begin{aligned}\dot{x}_1(t)&=x_1(t)\Big\{2+0.2\sin(t)-3x_1(t)-4\int_0^{+\infty}e^{-s}x_2(t-s)ds\\&\quad -4x_1(t)\int_0^{+\infty}e^{-s}x_2(t-s)ds\Big\},\\ \dot{x}_2(t)&=x_2(t)\Big\{1+0.1\sin(t)-2\int_0^{+\infty}e^{-s}x_1(t-s)ds-3x_2(t)\\&\quad -3x_2(t)\int_0^{+\infty}e^{-s}x_1(t-s)ds\Big\},\end{aligned}\tag{5.6}$$

where

$$r_1(t)=2+0.2\sin(t),\ a_{11}(t)=3,\ a_{12}(t)=4,\ b_1(t)=4,$$

$$r_2(t)=1+0.1\sin(t),\ a_{21}(t)=2,\ a_{22}(t)=3,\ b_2(t)=3.$$

Obviously, we have that the period of system (5.6) is $\omega = 2\pi$. Simple computations show that

$$\limsup_{t\to\infty} \frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} = \frac{1}{2} < \liminf_{t\to\infty} \frac{A_{21}(t)}{a_{11}(t)} = \frac{2}{3},$$

$$\limsup_{t\to\infty} \frac{\displaystyle\int_t^{t+\omega} r_2(s)ds}{\displaystyle\int_t^{t+\omega} r_1(s)ds} = \frac{1}{2} < \liminf_{t\to\infty} \frac{a_{22}(t)}{A_{12}(t)} = \frac{3}{4}.$$

It is easy to check that species x_2 is extinct, see Figure 3.

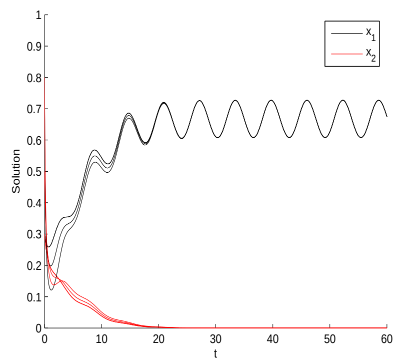

Figure 3. Dynamic behaviors of system (5.6). Here, we take the initial functions ($x_1(\theta)$, $x_2(\theta)$, $u_1(\theta)$, $u_2(\theta)$)= (0.3, 0.3), (0.5, 0.5) and (0.8, 0.8) for all $\theta \in (-\infty, 0]$.

Now we consider the following system

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)\Big\{2 + 0.2\sin(t) - 3x_1(t) - 4\int_0^{+\infty} e^{-s}x_2(t-s)ds \\
&\qquad -4x_1(t)\int_0^{+\infty} e^{-s}x_2(t-s)ds - c_1(t)\int_0^{+\infty} e^{-s}u_1(t-s)ds\Big\}, \\
\dot{x}_2(t) &= x_2(t)\Big\{1 + 0.1\sin(t) - 2\int_0^{+\infty} e^{-s}x_1(t-s)ds - 3x_2(t) \\
&\qquad -3x_2(t)\int_0^{+\infty} e^{-s}x_1(t-s)ds - c_2(t)\int_0^{+\infty} e^{-s}u_2(t-s)ds\Big\}, \\
\dot{u}_1(t) &= -u_1(t) + \int_0^{+\infty} e^{-s}x_1(t-s)ds, \\
\dot{u}_2(t) &= -u_2(t) + \int_0^{+\infty} e^{-s}x_2(t-s)ds.
\end{aligned} \tag{5.7}$$

For the initial functions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta)) = (0.3, 0.3, 0.3, 0.3)$, $(0.5, 0.5, 0.5, 0.5)$ and $(0.8, 0.8, 0.8, 0.8)$ for all $\theta \in (-\infty, 0]$, let the feedback control variable coefficients $c_1(t) = 8$ and $c_2(t) = 1$, from the numerical simulation, species x_1 and species x_2 in system (5.7) become permanent (see Figure 4).

We now change the feedback control variable coefficients to $c_1(t) = 4$ and $c_2(t) = 3$ and in system (5.7), for the same initial conditions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta)) = (0.3, 0.3, 0.3, 0.3)$, $(0.5, 0.5, 0.5, 0.5)$ and $(0.8, 0.8, 0.8, 0.8)$ for all $\theta \in (-\infty, 0]$, the population of species x_1, however, is larger than that of species x_2. Numerical simulation also confirms our results (see Figure 5).

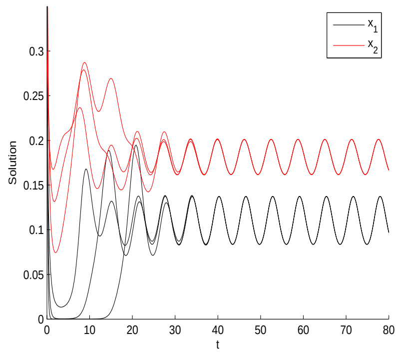

Figure 4. Dynamic behaviors of system (5.7). Here, we take the initial functions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta)) = (0.3, 0.3, 0.3, 0.3)$, $(0.5, 0.5, 0.5, 0.5)$ and $(0.8, 0.8, 0.8, 0.8)$ for all $\theta \in (-\infty, 0]$ and the feedback control variable coefficients $c_1(t) = 8$ and $c_2(t) = 1$.

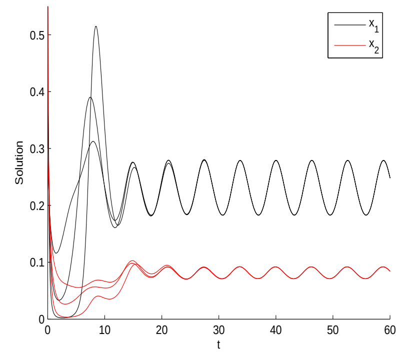

Figure 5. Dynamic behaviors of system (5.7). Here, we take the initial functions $(x_1(\theta), x_2(\theta), u_1(\theta), u_2(\theta)) = (0.3, 0.3, 0.3, 0.3)$, $(0.5, 0.5, 0.5, 0.5)$ and $(0.8, 0.8, 0.8, 0.8)$ for all $\theta \in (-\infty, 0]$ and the feedback control variable coefficients $c_1(t) = 4$ and $c_2(t) = 3$.

Dynamic of the competitive systems is an interesting topic which has attracted a lot of attention. Most existing results were focused on the impact of model parameters and delay on the long time behaviors of the solutions. In this paper, a nonautonomous competitive systems with infinite delays and feedback control is considered. We find that feedback control admits an influence on the extinction of the original system. Example 5.3 shows that, if one of the original system is driven to extinction, the system with feedback control is permanent and therefore feedback control leads to changes in results. We believe that our results can be extended to the reaction diffusion systems modelling two or more species models.

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