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H_∞ FEEDBACK CONTROLS BASED ON DISCRETE-TIME STATE OBSERVATIONS FOR SINGULAR HYBRID SYSTEMS WITH NONHOMOGENEOUS MARKOVIAN JUMP

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Abstract In this paper, the H_∞-control problem for singular Markovian jump systems (SMJSs) with variable transition rates by feedback controls based on discrete-time state observations is studied. The mode-dependent time-varying character of transition rates is supposed to be piecewise-constant. By designing a feedback controller based on discrete-time state observations, employing a stochastic Lyapunov-Krasovskii functional, and combining with the linear matrix inequalities (LMIs) technologies, sufficient conditions under the case of nonhomogeneous transition rates are developed such that the controlled system is regular, impulse free, and stochastically stable. Subsequently, the upper bound on the duration τ between two consecutive state observations and prescribed H_∞ performance γ are derived. Moreover, the achieved results can be easily checked by the Matlab LMI Tool Box. Finally, two numerical examples are presented to show the effectiveness of the proposed methods.

Keywords Discrete-time state observation, stochastically stable, H_∞feedback control, nonhomogeneous Markovian jump, singular system.

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1. Introduction

Markovian jump systems serve as an important kind of stochastic hybrid systems that have been attracting increasing attention. In the past few years, since this class of systems is very appropriate to model some practical systems which are subject to random abrupt changes, such as, unexpect events, uncontrolled configuration and random faults, and so on. Therefore, a considerable research attention has been devoted to the study on Markovian jump systems. The crucial issues in the investigation of systems are being placed on the analysis of stability [19, 20, 26, 36] or the study of bifurcations for nonlinear systems [6, 7, 39]. Specifically, the problem of stability of stochastic differential equation with Markovian jump was addressed [19], and scholar in [20] studied the exponential stability of stochastic delay interval systems with Markovian jump. The issue of exponential stabilization for uncertain linear systems with Markovian jump parameters and mode-dependent input delays

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is considered in [28]. The works for stochastic hybris systems have been provided in [1, 10]. In particular, the literature [21] is the first book in this area of stochastic differential equations with Markovian switching. For more details on the relevant fruits, we can refer to the literatures [11, 22, 23, 25, 31, 37]. In addition, switched systems are a kind of hybrid systems, which have already been studied by numerous scholars [5, 27] due to its powerful potential in some practical applications. Compared with the switched systems, Markovian jump systems not only have the merits of switched systems, but also contain the statistical information of the switching signal. Therefore, the considered system in our work maybe have wider range in practice in some certain.

On the other hand, the researches of singular systems began at the end of 1970s, although they were first mentioned in 1973 in [12]. According to the various area of application, singular systems are known as generalized state space systems, descriptor systems, differential-algebraic or implicit systems. Singular systems have attracted a large number of researchers from the control and the mathematics communities due to the fact that singular systems can describe the behavior of some physical systems better than normal state-space ones can. For example, electrical networks, power system, aerospace engineering, social economic systems, chemical processes, biological systems, network analysis, time-series analysis, and so on. Thus, singular systems should be taken into account in order to simulate more practical systems. The problem of reduced-order H_∞ filter for singular systems has been discussed in [38]. In [13], the H_∞ optimal singular and normal filter design for uncertain singular systems have been analyzed, and several conditions for the solvability of this problem have been attained in terms of LMIs approaches. Because singular system models are natural representation of dynamic systems and able to describe wider range of systems than the normal linear ones. So, many filtering and control problems based on singular systems have been extensively investigated in recent years [2, 8, 24].

Some attention has been focused on LMIs conditions for controller design and stability analysis of singular Markovian jump systems. Many important and fundamental results have been proposed for singular Markovian jump systems in these literatures [3, 4, 16, 17, 29, 30, 34]. For instance, the stabilization problem was addressed for singular Markovian jump systems in [33] and the desired linear H_∞ filter was designed that ensuring the considered system to be regular, impulse free and stochastically stable. What's more, time delays are frequently encountered in many fields of science and engineering, including biology, economy, manufacturing systems and communication network. Therefore, time-delay should be sufficiently considered in order to better describe the real world. During the past two decades, the problems of stochastic stability analysis and control for time-delay systems have been considered, to see literatures [18, 32, 35, 41]. In [33], the problem of H_∞ filtering for singular Markovian jump continuous linear system with time delay has been analyzed, in which by means of the LMIs techniques, bounded real lemma of delay-dependent was given to discuss the problem of stochastically admissible and the prescribed H_∞ performance conditions also obtained. The mean-square stabilization of continuous-time hybrid stochastic differential equations by feedback control based on state observations has been addressed in [23]. Inspired by these works, the problem of stochastically admissible for one class of stochastic hybrid systems with singular Markovian jump by feedback controls based on discrete-time state observations is discussed in this paper.

The aim of this article is to design a feedback controller which is based on discrete-time state observations that guarantees stochastic admissibility of the singular hybrid system with Markovian jump. Compared to traditional feedback controllers, the discrete time state observations feedback controller has the advantages of low cost and easy control. In addition, the involved time delays here are time varying which are different from time-delay in literature [33] where the considered time delays are time invariant. The remainder of this article is organized as follows. Section 2 gives out the preliminaries that contain the notations will be used in this paper and the basic model is established. The main results and the procedure of proof are presented In Section 3. In Section 4, performance evaluations are illustrated and the experiment results have been exhibited. The paper ends with conclusion shown in Section 5.

2. Preliminaries

2.1. Notation

In this section, some basic notations are presented that will be used in the rest of this paper. Throughout this paper, unless otherwise specified, let $\mathbb{R}^n$ represent the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ be the set of all $m \times n$ real matrices. Given $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (it means it is right continuous and $\{\mathcal{F}_0\}$ contains all $\mathbb{P}-$null sets). We defined $\mathcal{E}\{\cdot\}$ be the expectation operator with respect to some probability measure $\mathbb{P}$. If A denotes a matrix, then A^T stands for the transpose of A, here $\|A\| = max\{|Ax| : |x| = 1\}$ and $|A| = \sqrt{trace(A^T A)}$ are referred to as the operator norm and its trace norm, respectively. Further, if the matrix A is a symmetric matrix (i.e. $A^T{=}A$), its largest and smallest eigenvalues are denoted by $\lambda_{max}(A)$ and $\lambda_{min}(A)$ respectively, and the label $'*'$ indicates the term that can be induced by the symmetry and I is an identity matrix. $A < 0$ $(A \leq 0)$ means that A is a negative definite (negative semi-definite) matrix respectively. $\mathbb{C}$ is the set of complex number.

2.2. Problem formulation

Fix a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and the singular hybrid systems with Markovian jump based on discrete-time state observations are considered as follows

$$\begin{cases} E\dot{x}(t){=}A(r(t))x(t){+}F(r(t))u(x(\delta(t)), r(t)){+}B(r(t))\omega(t){+}f(r(t), x(t), \omega(t)), \\ z(t){=}C(r(t))x(t) + \tilde{D}(r(t))x(\delta(t)), \end{cases} \tag{2.1}$$

where $t \geq 0$, with initial value $x(0) = x_0 \in \mathrm{L}^2_{\mathcal{F}_0}$, $x(t) \in \mathbb{R}^n$ is the state variable of the system, $z(t)$ is the measure output, and $\omega(t) = (\omega_1(t), \omega_1(t), \ldots, \omega_m(t))^T$ is the disturbance input that belongs to $L_2[0, \infty)$.

In this paper, we are interested in the design of a stabilizing controller that has the following form

$$u(x(\delta(t)), r(t)) = K(r(t))x(\delta(t)), \tag{2.2}$$

that renders the closed-loop systems (2.1) regular, impulse-free and stochastically stable. Here, $u(x(\delta(t)),\ r(t))$ is the control input based on discrete-time observations of the state $x(t)$ at time $0,\ \tau,\ 2\tau,\ \ldots$. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular and it is assumed that $rankE = r \leq n$. These matrices $A(r(t))$, $B(r(t)) = (B_1(r(t)),\ B_2(r(t)),\ldots,\ B_m(r(t)))$, $C(r(t))$, $\tilde{D}(r(t))$ and $F(r(t))$ are all known real constant matrices with appropriate dimensions for each $r(t) \in S$. The function $f(r(t), x(t), \omega(t)) : \mathbb{S} \times \mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}^n$ is the nonlinear part of the considered system. The parameter $r(t)$ represents a continuous-time Markov chain with right continuous trajectory that taking values in a finite set $S = \{1, 2, \ldots s\}$ with generator $\prod^{(\sigma_{t+h})} = \{\pi_{ij}^{(\sigma_{t+h})}\}$ given by

$$P\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}^{(\sigma_{t+h})} h + o(h), j \neq i, \\ 1 + \pi_{ii}^{(\sigma_{t+h})} h + o(h), j = i, \end{cases} \tag{2.3}$$

where $h > 0$, the limit $\lim\limits_{h\to 0} o(h)/h = 0$, and $\{\pi_{ij}^{(\sigma_{t+h})}\} \geq 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t + h$ and

$$\pi_{ii}^{\sigma_{t+h}} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}^{\sigma_{t+h}}. \tag{2.4}$$

Similarly, the parameter $\sigma_t, t \geq 0$ is also a continuous-time Markov chain with right continuous trajectories and taking values in a finite set $R = \{1, 2, \ldots, l\}$ with transition rate matrix $\Lambda = \{\rho_{mn}\}$ given by

$$P\{\sigma_{t+h} = n | \sigma_t = m\} = \begin{cases} \rho_{mn} h + o(h), n \neq m, \\ 1 + \rho_{mm} h + o(h), n = m, \end{cases} \tag{2.5}$$

where $h > 0$, the limit $\lim\limits_{h\to 0} o(h)/h = 0$, and transition rate $\rho_{mn} \geq 0$, for $n \neq m$, is the transition rate from mode m at time t to mode n at time $t + h$ and

$$\rho_{mm} = -\sum_{n=1, n\neq m}^{l} \rho_{mn}. \tag{2.6}$$

Remark 2.1. If the finite set $S = \{1\}$ or $R = \{1\}$, the nonhomogeneous singular MJLS (2.1) is reduced to a general singular MJLS. In addition, the transition rates play an important role in analysing the stability for the underlying system. However, the most of the existing works assume that the transition rates of Markov chain are time-invariant, the assumption is not realistic in practical engineering problem. So, in order to reasonably describe the real world, it is significant and necessary to research the nonhomogeneous Markov chain with variable transition rates. Subsequently, the relevant works have been reported [9,14]. The piecewise homogeneous transition rates are taken into account in this paper.

The state feedback control gain $K(r(t))$ is a design matrix that is determined for every $r(t) \in S$, and $\tau > 0$

$$\delta(t) = [t/\tau]\tau, \quad \text{for } t \geq 0,$$

in which $[t/\tau]$ is the integer part of t/τ. Indeed, if we define the bounded variable delay $\zeta : [0, \infty) \to [0, \tau)$ by $\zeta(t) = t - \nu\tau$ *for* $\nu\tau \leq t < (\nu + 1)\tau$ and $\nu = 0, 1, 2, \ldots$. Further $x(\delta(t))$ can be written as $x(\delta(t)) = x(t - \zeta(t)) = x(\nu\tau)$. Then $\zeta(t) \in [0, \tau)$ and $\dot{\zeta}(t) = 1$ for $t \neq \nu\tau$. At this moment, the system (2.1) can be rewritten as

$$\begin{cases} E\dot{x}(t) = A(r(t))x(t) + D(r(t))x(t - \zeta(t)) + B(r(t))\omega(t) + f(r(t), x(t), \omega(t)), \\ z(t) = C(r(t))x(t) + \tilde{D}(r(t))x(t - \zeta(t)), \end{cases} \tag{2.7}$$

where $t \in [\nu\tau, (\nu+1)\tau)$ and $D(r(t)) = F(r(t))K(r(t))$.

Assumption 2.1. For every $r(t) \in S$, there exist two positive numbers a and b satisfying the following condition

$$f^T(r(t), x(t), \omega(t)) f(r(t), x(t), \omega(t)) \leq a^2|x(t)|^2 + b^2|\omega(t)|^2. \tag{2.8}$$

Remark 2.2. In the special case when $rankE = n$, the state equation of the system (2.7) is reduced to the following one

$$\begin{aligned}\dot{x}(t) =& E^{-1}[A(r(t))x(t) + D(r(t))x(t-\zeta(t))] + E^{-1}B(r(t))\omega(t) \\ &+ E^{-1}f(r(t), x(t), \omega(t)).\end{aligned}$$

The relevant results of singular systems also can be extended to the case that E is nonsingular, Therefore, the singular systems include the normal systems, which means that the normal system is a special case of the singular system, and the correspond results of singular system are the generalization of the normal one.

For the unforced singular system

$$E\dot{x}(t) = A(r(t))x(t), \tag{2.9}$$

or the pair $(E, A(r(t)))$, its generalized spectral abscissa is defined as follows [17]

$$\alpha(E, A(r(t))) \triangleq \max_{\lambda \in \{s | det(sE - A(r(t)))=0\}} \mathrm{Re}(\lambda).$$

Moreover, for convenience of narrative in the sequel, we write $\alpha(A(r(t))) = \alpha(I, A(r(t)))$ which is the usual spectral abscissa, and denote $A(r(t)) = A_i$. In order to analyze the stability and the prescribed H_∞ performance level for the singular system with Markovian jump in this paper, firstly, let us introduce the following definitions, which would be used to derive the main results of the stability and the given H_∞ performance.

Definition 2.1 ([4,17]). (1) For any given two matrices $E, A_i \in \mathbb{R}^{n\times n}$, the pair (E, A_i) is called to be regular if there exists a constant $s \in \mathbb{C}$ such that $\det(sE - A_i)$ is not identically zero.
(2) The pair (E, A_i) is called to be impulse free if $\deg(\det(sE - A_i)) = \mathrm{rank}(E)$.

Remark 2.3. In existing works, some revelent definitions of singular systems are based on the definitions [4,17,33] that are presented. We have discussed not only the singularity and impulse-free, but also the stochastic characteristic of the considered singular Markovian jump systems in this paper. In order to further describe the corresponding issues of the underlying systems, the following definitions of singular Markovian jump systems are needed, which are extensions of the relevant concepts.

Definition 2.2. (1) The singular Markovian jump system

$$E\dot{x}(t) = [A_i x(t) + D_i x(t - \zeta(t))] + B_i\omega(t) + f(i, x(t), \omega(t)), \tag{2.10}$$

with $\omega(t) = 0$ is said to be regular and impulse free, if the pairs (E, A_i) are regular and impulse free for every $i \in S$.

(2) The singular Markovian jump system (2.7) with $\omega(t) = 0$ is said to be stochastically admissible, if it is regular, impulse free and stochastically stable.

(3) The regular and impulse-free singular Markovian jump system (2.7) with $\omega(t)=0$ is said to be stochastically stable, if there exists a scalar $M(x_0,r_0)$, such that the following inequality holds for any initial conditions (x_0,r_0)

$$\mathcal{E}[\int_0^\infty \|x(t)\|^2 dt|x_0,r_0] \le M(x_0,r_0),$$

and under the assumption of zero initial condition and any nonzero $\omega(t)$, and for a prescribed scalar $\gamma>0$, the controlled output $z(t)$ satisfies

$$\mathcal{E}[\int_0^\infty \|z(t)\|^2 dt] \le \gamma^2 \int_0^\infty \|\omega(t)\|^2 dt,$$

then the system (2.7) is said to be stochastically admissible with H_∞ performance γ, where $\omega(t)\in L_2[0,\infty)$ is non-zero.

The following lemma gives an equivalent condition for regularity, which is important and necessary to obtain our results.

Lemma 2.1 ([4, 17]). *The pair (E,A_i) is regular if and only if there exist two nonsingular matrices M and N which can satisfy*

$$MEN = diag\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \quad MA_iN = diag\begin{bmatrix} A_{i1} & 0 \\ 0 & I_{n_2} \end{bmatrix}, \tag{2.11}$$

where $n_1+n_2=n, A_{i1}\in\mathbb{R}^{n_1\times n_1}, N\in\mathbb{R}^{n_2\times n_2}$ is a nilpotent matrix.

Remark 2.4. From Lemma 2.1, it is worth mentioning that the regularity of pair (E,A_i) guarantees the existence and uniqueness solution of the state equation of system (2.7) for any given initial conditions, because the singular Markovian jump system (2.7) can be transformed into the normal system which is equivalent to the singular system with Markovian jump (2.7) based on Lemma 2.1. Moreover, according to Definition 2.1, it can be deduced that the nonimpulsiveness of the pair (E,A_i) implies that the pair (E,A_i) is regular.

If the regularity of the pair (E,A_i) is unknown, it is always possible to choose two invertible matrices M_1 and N_1 such that matrices E and A_i can be decomposed into the following form

$$M_1EN_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad M_1A_iN_1 = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}. \tag{2.12}$$

Using the singular value decomposition on matrices E and A_i, and based on formula (2.12), it is not difficult to acquire the following results.

Lemma 2.2 ([4, 17]). *The pair (E,A_i) is impulse-free if and only if A_{i4} is non-singular.*

Lemma 2.2 provides a necessary and sufficient condition to prove that singular systems are impulse-free.

Lemma 2.3 ([17]). *Given any real square matrix $\mathcal{X}$ with appropriate dimensions. The matrix measure $\mu(\mathcal{X})$ is defined as*

$$\mu(\mathcal{X}) = \lim_{\theta \to 0^+} \frac{\|I + \theta\mathcal{X}\| - 1}{\theta},$$

which has the following properties:
(1) $-\|\mathcal{X}\| \leq \alpha(\mathcal{X}) \leq \mu(\mathcal{X}) \leq \|\mathcal{X}\|$,
(2) $\mu(\mathcal{X}) = \frac{1}{2}\lambda_{max}(\mathcal{X} + \mathcal{X}^T) = \frac{1}{2}\alpha(\mathcal{X} + \mathcal{X}^T)$.

Based on the above analysis, the considered system has been transformed into an equivalent one. Due to rank$E = r \leq n$, there exist two invertible matrices M and N that can be choose such that

$$\bar{E} = MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \; MA_iN = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}.$$

Lemma 2.4. *For any real matrix $M > 0$, scalars a_1 and a_2 with $a_1 < a_2$, vector function $x(\alpha)$ such that the following integrals are well defined, we have*

$$(a_2 - a_1)\int_{a_1}^{a_2} x(\alpha)^T M x(\alpha) d\alpha \geq (\int_{a_1}^{a_2} x(\alpha) d\alpha)^T M (\int_{a_1}^{a_2} x(\alpha) d\alpha).$$

3. Main results

Theorem 3.1. *The closed-loop system (2.7) is stochastically admissible and has the prescribed H_∞ performance level γ, if there exist symmetric positive-definite matrices P_i, U, U_1, U_2 and matrix S_i such that the following inequalities hold*

$$\begin{bmatrix} \Xi_{1i} & \Xi_{2i} & \Xi_{3i} & C_i^T \\ * & \Xi_{4i} & \Xi_{5i} & \tilde{D}_i^T \\ * & * & \Xi_{6i} & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{3.1}$$

$$\Xi_{7i} = -(\tau - \zeta(t))^T A_i^T U A_i + \tau E^T U E < 0, \tag{3.2}$$

where $R \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column satisfying $E^T R = 0$, and

$$\begin{aligned}
\Xi_{1i} =& 2A_i^T P_{i,\sigma} E - \tau E^T U E + \lambda_{max}(P_{i,\sigma}) a^2 + + E^T P_{i,\sigma} E + \sum_{n \in R} \rho_{mn} E^T P_{in} E \\
&+ \sum_{j \in S} \pi_{ij}^m E^T P_{jm} E + 2S_i R^T A_i + S_i R^T B_i B_i^T R S_i^T + aS_i R^T R S_i^T + aI \\
&+ bS_i R^T R S_i^T + 2(\tau - \zeta(t)) A_i^T U A_i + (\tau - \zeta(t)) 4a^2 \lambda_{max}(U), \\
\Xi_{2i} =& D_i^T P_{i,\sigma} E + S_i R^T D_i + \tau E^T U E + (\tau - \zeta(t)) A_i^T U D_i, \\
\Xi_{3i} =& E^T P_{i,\sigma} B_i + (\tau - \zeta(t)) A_i^T U B_i, \\
\Xi_{4i} =& -2\tau E^T U E + 2(\tau - \zeta(t)) D_i^T U D_i, \\
\Xi_{5i} =& (\tau - \zeta(t)) D_i^T U B_i,
\end{aligned}$$

$$\Xi_{6i} = -\gamma^2 I + \lambda_{max}(P_{i,\sigma})b^2 I + (1+b)I + 2(\tau - \zeta(t))B_i^T U B_i + 4(\tau - \zeta(t))b^2 \lambda_{max}(U)I.$$

Proof. Since rank$E = r \le n$, there exist two invertible matrices M and N such that

$$\bar{E} = MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad R = M^T \begin{bmatrix} 0 \\ I \end{bmatrix} H,$$

where $H \in \mathbb{R}^{(n-r)\times(n-r)}$ is an invertible matrix. Write

$$MA_iN = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}, \quad M^{-T}P_{i,n}M^{-1} = \begin{bmatrix} P_{i,n}^1 & P_{i,n}^2 \\ * & P_{i,n}^3 \end{bmatrix},$$

$$M^{-T}P_{j,m}M^{-1} = \begin{bmatrix} P_{j,m}^1 & P_{j,m}^2 \\ * & P_{j,m}^3 \end{bmatrix}, \quad N^T S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \end{bmatrix},$$

from (3.1) and (3.2), we can obtain $\Xi_{1i} + \Xi_{7i} < 0$ and pre-multiplying and post-multiplying $\Xi_{1i} + \Xi_{7i} < 0$ respectively by N^T and N for every $i \in S$, we have

$$\begin{aligned} N^T(\Xi_{1i} + \Xi_{7i})N =& N^T[2A_i^T P_{i,\sigma}E + \lambda_{max}(P_{i,\sigma})a^2 + +E^T P_{i,\sigma}E + \sum_{n\in R}\rho_{mn}E^T P_{in}E \\ &+ \sum_{j\in S}\pi_{ij}^m E^T P_{jm}E + 2S_iR^TA_i]N + N^T[S_iR^TB_iB_i^TRS_i^T \\ &+ aS_iR^TRS_i^T + aI + bS_iR^TRS_i^T + (\tau-\zeta(t))A_i^TUA_i \\ &+ (\tau - \zeta(t))4a^2\lambda_{max}(U)]N. \end{aligned}$$

Because $P_{i,\sigma}$,U are symmetric positive-definite matrices, a, b are positive numbers and N^T, N are nonsingular matrices, we obtain the following

$$\begin{aligned} & N^T(\Xi_{1i} + \Xi_{7i})N - N^T[\lambda_{max}(P_{i,\sigma})a^2 + E^TP_{i,\sigma}E + S_iR^TB_iB_i^TRS_i^T \\ & + aS_iR^TRS_i^T + aI + bS_iR^TRS_i^T + (\tau - \zeta(t))4a^2\lambda_{max}(U)]N \\ =& \sum_{n\in R}\rho_{mn}N^TE^TP_{in}EN + \sum_{q\in S}\pi_{ij}^m N^TE^TP_{jm}EN + N^TE^TP_{i,\sigma}A_iN \\ & + N^TS_iR^TA_iN + N^TA_i^TRS_i^TN + N^TA_i^TP_{i,\sigma}EN < 0. \end{aligned} \tag{3.3}$$

From the above inequality (3.3), it is not difficult to derive

$$S_{i2}H^TA_{i4} + A_{i4}^THS_{i2}^T < 0. \tag{3.4}$$

According to Lemma 2.3 and the inequality (3.4), it is not difficult to yield

$$\alpha(S_{i2}H^TA_{i4}) \le \mu(S_{i2}H^TA_{i4}) = \frac{1}{2}\lambda_{max}(S_{i2}H^TA_{i4} + A_{i4}^THS_{i2}^T) < 0, \tag{3.5}$$

which is reduced to the following formula

$$\alpha(S_{i2}H^TA_{i4}) = \max_{\lambda\in\{s|\det(sI - S_{i2}H^TA_{i4})=0\}} Re(\lambda) < 0. \tag{3.6}$$

The formula (3.6) means that the real part of eigenvalues of $S_{i2}H^TA_{i4}$ is less than zero. Further, the determinant of $S_{i2}H^TA_{i4}$ is not zero. That is to say, $|S_{i2}H^TA_{i4}| \neq 0$. The result of $|S_{i2}H^TA_{i4}| \neq 0$ implies $|A_{i4}| \neq 0$, which means A_{i4} is nonsingular matrix for each $i \in S$. By Lemma 2.2, we obtain the pair (E, A_i) is regular and impulse-free for every $i \in S$.

According to (E, A_i) regular and impulse free for every $j \in S, n \in R$ and Definition 2.2, we obtain that the unforced one of system (2.7) with $\omega(t) = 0$ has the regularity and the absence of impulse for every $j \in S, n \in R$.

In order to prove that the system (2.7) with $\omega(t) = 0$ is stochastic stable and establish the H_∞ performance index of the system (2.7) with nonzero $\omega(t)$, we choose the Lyapunov-Krasovkii functional $V(x(t), r(t), \sigma(t))$ that has the following form

$$\begin{aligned} &V(x(t), r(t), \sigma(t)) = V_1(x(t), r(t), \sigma(t)) + V_2(x(t), r(t), \sigma(t)), \\ &V_1(x(t), r(t), \sigma(t)) = x(t)^TE^TP(r(t), \sigma(t))Ex(t), \\ &V_2(x(t), r(t), \sigma(t)) = (\tau - \zeta(t)) \int_{t-\zeta(t)}^{t} \dot{x}(s)^TE^TUE\dot{x}(s)ds, \end{aligned} \tag{3.7}$$

where $P_{i,\sigma}$ and $U$are symmetric and positive-definite matrices for every $j \in S, n \in R$. Define the infinitesimal operator $\mathcal{L}$ acting on $V(x(t), r(t), \sigma(t))$ along the trajectory for (2.7) with $\omega(t) = 0$ as

$$\begin{aligned} \mathcal{L}V = \lim_{h\to 0} \frac{1}{h} \{&\mathcal{E}\{V(x(t+h), r(t+h), \sigma(t+h))|x(t), r(t) = i, \sigma(t) = m\} \\ &- V(x(t), r(t) = i, \sigma(t) = m)\}, \end{aligned} \tag{3.8}$$

on the basic of (2.3),(2.5) and (3.7),we can achieve that

$$\mathcal{L}V = \sum_{n\in R} \rho_{mn}V(x_t, i, n) + \sum_{j\in S} \pi_{ij}^m V(x_t, j, m) + \dot{V}(x_t, i, m), \tag{3.9}$$

the derivative of (3.7) along the solution of state equation for systems (2.7) is obtained

$$\begin{aligned} \mathcal{L}V =&2x(t)^TE^TP_{i,m}E\dot{x}(t) + x(t)^TE^T[\sum_{n\in R} \rho_{mn}P_{i,n} + \sum_{j\in S} \pi_{ij}^m P_{j,m}] \\ &- \int_{t-\zeta(t)}^{t} \dot{x}(s)^TE^TUE\dot{x}(s)ds + (\tau - \zeta(t))\dot{x}(t)^TE^TUE\dot{x}(t) \\ &+ (\tau - \zeta(t)) \int_{t-\zeta(t)}^{t} \dot{x}(s)^TE^T[\sum_{n\in R} \rho_{mn}U + \sum_{j\in S} \pi_{ij}^m U]E\dot{x}(s)ds. \end{aligned} \tag{3.10}$$

Multiplying the both sides of the following identity by $2x(t)^TS_iR^T$ from the left side, it gets

$$-E\dot{x}(t) + [A(r(t))x(t) + D(r(t))x(t - \zeta(t))] + B(r(t))\omega(t) + f(r(t), x(t), \omega(t)) = 0,$$

and noting that $S_iR^TE = 0$, one can have

$$\begin{aligned} -2x(t)^TS_iR^TE\dot{x}(t) + 2x(t)^TS_iR^T[A(r(t))x(t) + D(r(t))x(t - \zeta(t))] \\ +2x(t)^TS_iR^TB(r(t))\omega(t) + 2x(t)^TS_iR^Tf(r(t), x(t), \omega(t)) = 0, \end{aligned} \tag{3.11}$$

and

$$\begin{aligned}\dot{x}^T(t)E^TUE\dot{x}(t) =&[A_ix(t)+D_ix(t-\zeta(t))+B_i\omega(t)+f_i(x(t),\omega(t))]^TU[A_ix(t)\\&+D_ix(t-\zeta(t))+B_i\omega(t)+f_i(x(t),\omega(t))]\\=&x(t)^TA_i^TUA_ix(t)+x(t)^TA_i^TUD_ix(t-\zeta(t))+x(t)^TA_i^TUB_i\omega(t)\\&+x(t)^TA_i^TUf_i(x(t),\omega(t))+x(t-\zeta(t))^TD_i^TUA_ix(t)\\&+x(t-\zeta(t))^TD_i^TUD_ix(t-\zeta(t))+x(t-\zeta(t))^TD_i^TUB_i\omega(t)\\&+x(t-\zeta(t))^TD_i^TUf_i(x(t),\omega(t))+\omega(t)^TB_i^TUA_ix(t)\\&+\omega(t)^TB_i^TUD_ix(t-\zeta(t))+\omega(t)^TB_i^TUB_i\omega(t)\\&+\omega(t)^TB_i^TUf_i(x(t),\omega(t))+f_i^T(x(t),\omega(t))UA_ix(t)\\&+f_i^T(x(t),\omega(t))UD_ix(t-\zeta(t))+f_i^T(x(t),\omega(t))UB_i\omega(t)\\&+f_i^T(x(t),\omega(t))Uf_i(x(t),\omega(t)). \qquad (3.12)\end{aligned}$$

By Assumption 2.1, the following inequalities hold

$$\begin{aligned}2x(t)^TS_iR^Tf(r(t),x(t),\omega(t)) \le&2\|x(t)^TS_iR^T\|[\|f(r(t),x(t),\omega(t))\|]\\\le&2\|x(t)^TS_iR^T\|[\|a\|x(t)\|+b\|\omega(t)\|\|]\\\le&ax(t)^TS_iR^TRS_i^Tx(t)+ax(t)^Tx(t)\\&+bx(t)^TS_iR^TRS_i^Tx(t)+b\omega(t)^T\omega(t), \qquad (3.13)\\2x(t)^TA_i^TUf_i(x(t),\omega(t)) \le&x(t)^TA_i^TUA_ix(t)+\lambda_{max}(U)[a^2x(t)^Tx(t)\\&+b^2\omega(t)^T\omega(t)], \qquad (3.14)\\2x(\nu\tau)^TD_i^TUf_i(x(t),\omega(t)) \le&x(\nu\tau)^TD_i^TUD_ix(\nu\tau)+\lambda_{max}(U)[a^2x(t)^Tx(t)\\&+b^2\omega(t)^T\omega(t)], \qquad (3.15)\\2\omega(t)^TB_i^TUf_i(x(t),\omega(t)) \le&\omega(t)^TB_i^TUB_i\omega(t)+\lambda_{max}(U)[a^2x(t)^Tx(t)\\&+b^2\omega(t)^T\omega(t)], \qquad (3.16)\\f_i(x(t),\omega(t))^TUf_i(x(t),\omega(t)) \le&\lambda_{max}(U)[a^2x(t)^Tx(t)+b^2\omega(t)^T\omega(t)]. \qquad (3.17)\end{aligned}$$

It follows from Lemma 2.4 that

$$\begin{aligned}&-\int_{t-\zeta(t)}^t\dot{x}(s)^TE^TUE\dot{x}(s)ds\le-\tau(\int_{t-\zeta(t)}^tE\dot{x}(s)ds)^TU(\int_{t-\zeta(t)}^tE\dot{x}(s)ds)\\&\le-\tau[x(t)^TE^T-x^T(t-\zeta(t))E^T]U[Ex(t)-Ex(t-\zeta(t))]\\&\le-\tau[x(t)^TE^TUEx(t)-x(t)^TE^TUEx(t-\zeta(t))\\&\quad-x^T(t-\zeta(t))E^TUEx(t)+x^T(t-\zeta(t))E^TUEx(t-\zeta(t))], \qquad (3.18)\end{aligned}$$

under the formulaes of (3.11)-(3.18), the $\mathcal{L}V$ can be reduced to the following one when $\omega(t)=0$

$$\begin{aligned}\mathcal{L}V <&2\dot{x}(t)^TE^TP_{i,\sigma}Ex(t)-\tau x(t)^TE^TUEx(t)+2\tau x(t)^TE^TUEx(t-\zeta(t))\\&-2\tau x(t-\zeta(t))^TE^TUEx(t-\zeta(t))+x(t)^T[\sum_{n\in R}\rho_{mn}P_{i,n}+\sum_{j\in S}\pi_{ij}^mP_{j,m}]x(t)\\&+(\tau-\zeta(t))[x(t)^TA_i^TUA_ix(t)+x(t)^TA_i^TUD_ix(t-\zeta(t))\end{aligned}$$

$$
\begin{aligned}
&+ x(t-\zeta(t))^T D_i^T U A_i x(t) + x(t-\zeta(t))^T D_i^T U D_i x(t-\zeta(t))] \\
&+ 2x(t)^T S_i R^T A_i x(t) + 2x(t)^T S_i R^T D_i x(t-\zeta(t)). \qquad (3.19)
\end{aligned}
$$

The following inequality holds

$$
\begin{aligned}
\mathcal{L}V \leq & x(t)^T \Phi_{1i} x(t) + x(t)^T \Phi_{2i} x(t-\zeta(t)) + x(t-\zeta(t))^T \Phi_{2i}^T x(t) \\
& + x(t-\zeta(t))^T \Phi_{4i} x(t-\zeta(t)) \\
\leq & \phi(t)^T \begin{bmatrix} \Phi_{1i} & \Phi_{2i} \\ \Phi_{2i}^T & \Phi_{4i} \end{bmatrix} \phi(t),
\end{aligned} \qquad (3.20)
$$

where $\phi^T(t) = [x^T(t)\ x^T(t-\zeta(t))]$ and $\Phi_{1i} = 2A_i^T P_{i,\sigma} E - \tau E^T U E + \sum_{n\in R} \rho_{mn} E^T P_{in} E + \sum_{j\in S} \pi_{ij}^m E^T P_{jm} E + 2S_i R^T A_i + (\tau - \zeta(t)) A_i^T U A_i$, $\Phi_{2i} = D_i^T P_{i,\sigma} E + S_i R^T D_i + \tau E^T U E + (\tau - \zeta(t)) A_i^T U D_i$, $\Phi_{4i} = -2\tau E^T U E + (\tau - \zeta(t)) D_i^T U D_i$.
According to the condition (3.1), (3.2) and [33, Lemma 3], we can derive

$$
\begin{bmatrix} \Phi_{1i} & \Phi_{2i} \\ \Phi_{2i}^T & \Phi_{4i} \end{bmatrix} < 0. \qquad (3.21)
$$

Thereby, it is not difficult to obtain

$$
\Phi_{1i} + \Phi_{2i} + \Phi_{2i}^T + \Phi_{4i} < 0, \qquad (3.22)
$$

combining (3.20) and (3.22) with $V \geq 0$, we obtain $\mathcal{L}V < 0$, which means the system (2.7) with $\omega(t) = 0$ is stochastically stable. Now let us define a performance index

$$
\mathcal{J}_{z\omega}(T) = \mathcal{E}\{\int_0^T [z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t)]dt\}. \qquad (3.23)
$$

Based on zero initial condition, the following formula naturally holds

$$
\begin{aligned}
\mathcal{J}_{z\omega}(T) = & \mathcal{E}\{\int_0^T [z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) + \mathcal{L}V(x(t), r(t), t)]dt\} \\
& - \mathcal{E}\int_0^T \mathcal{L}V(x(t), r(t), t)dt,
\end{aligned}
$$

where

$$
\begin{aligned}
& z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) \\
= & [x^T(t)C_i^T + x^T(t-\zeta(t))\tilde{D}_i^T][C_i x(t) + \tilde{D}_i x(t-\zeta(t))] \\
& - \gamma^2 \omega^T(t)\omega(t) \\
= & x^T(t) C_i^T C_i x(t) - \gamma^2 \omega^T(t)\omega(t) + x^T(t-\zeta(t))\tilde{D}_i^T C_i x(t) \\
& + x^T(t-\zeta(t))\tilde{D}_i^T \tilde{D}_i x(t-\zeta(t)) + x^T(t) C_i^T \tilde{D}_i x(t-\zeta(t)),
\end{aligned} \qquad (3.24)
$$

and L_i has the following expression

$$
\begin{aligned}
L_i = & (\tau - \zeta(t))\dot{x}^T(t) E^T U E \dot{x}(t) \\
= & (\tau - \zeta(t))[A_i x(t) + D_i x(t-\zeta(t)) + B_i \omega(t) + f_i(x(t), \omega(t))]^T U [A_i x(t)
\end{aligned}
$$

$$
\begin{aligned}
&+ D_i x(t-\zeta(t)) + B_i\omega(t) f_i(x(t),\omega(t))] \\
=& x^T(t)A_i^T U A_i x(t) + x^T(t)A_i^T U D_i x(t-\zeta(t)) + x^T(t)A_i^T U B_i\omega(t) \\
&+ x(t)^T A_i^T U f_i(x(t),\omega(t)) + x^T(t-\zeta(t))D_i^T U A_i x(t) \\
&+ x^T(t-\zeta(t))D_i^T U D_i x(t-\zeta(t)) + x^T(t-\zeta(t))D_i^T U B_i\omega(t) \\
&+ x^T(t-\zeta(t))D_i^T U f_i(x(t),\omega(t)) + \omega^T(t)B_i^T U A_i x(t) + \omega^T(t)B_i^T U D_i x(t-\zeta(t)) \\
&+ \omega^T(t)B_i^T U B_i\omega(t) + \omega(t)^T B_i^T U f_i(x(t),\omega(t)) + f_i^T(x(t),\omega(t))U A_i x(t) \\
&+ f_i^T(x(t),\omega(t))U D_i x(t-\zeta(t)) + f_i^T(x(t),\omega(t))U B_i\omega(t) \\
&+ f_i^T(x(t),\omega(t))U f_i(x(t),\omega(t)). \qquad (3.25)
\end{aligned}
$$

Then, by Dynkin's formula and (3.23)-(3.25) with the condition $V \geq 0$, we have

$$
\begin{aligned}
\mathcal{J}_{z\omega}(T) =& \mathcal{E}\{\int_0^T [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \mathcal{L}V_1(x(t),r(t),t)]dt\} \\
&- \mathcal{E}\int_0^T \mathcal{L}V_1(x(t),r(t),t)dt \\
\leq& \mathcal{E}\{\int_0^T [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \mathcal{L}V_1(x(t),r(t),t)]dt\} \\
=& \mathcal{E}\{\int_0^T \xi^T(t)\Theta_i\xi(t)dt\}, \qquad (3.26)
\end{aligned}
$$

where

$$
\xi^T(t) = [x^T(t),\, x^T(t-\zeta(t)),\, \omega^T(t)], \text{ and } \Theta_i = \begin{bmatrix} \Xi_{1i} + C_i^T C_i & \Xi_{2i} + C_i^T\tilde{D}_i & \Xi_{3i} \\ * & \Xi_{4i} + \tilde{D}_i^T\tilde{D}_i & \Xi_{5i} \\ * & * & \Xi_{6i} \end{bmatrix}.
$$

Therefore, by Schur complement in [3], we get from (3.1) and (3.2) that for all $t > 0$, $\mathcal{J}_{zw}(T) < 0$. Thus, the proof has been accomplished. □

Compared with the continuous time state feedback, the discrete-time state observations feedback control method has the advantages of saving cost and easily controlling. In order to derive the time interval τ during the two adjacent discrete-time state observations, the following corollary will be presented.

Remark 3.1. It is worth noting that the Lyapunov-Krasovkii functional in (3.7) is quite general. In the following, on the basic of the obtained results in Theorem 3.1, a special case will be considered, which can present a constraint of time delay τ in the special one of $P_{i,\sigma} = U = I (i \in S, \sigma \in R)$.

Corollary 3.1. *If there exist matrices D_i, S_i, R $(i \in S)$ such that the inequalities hold as follows*

$$
\begin{aligned}
&\mathcal{X}_{1i} < 0, \\
&\tau \leq \min\{\frac{1}{2}, \frac{-\lambda_{min}(\mathcal{X}_{1i})}{K_1}\},
\end{aligned} \qquad (3.27)
$$

then the system (2.7) is stochastically admissible, where $K_1 = \lambda_{max}(A_i^T A_i) + \lambda_{max}(D_i^T D_i) + 2\|A_i^T D_i\| + \|E^T E\|$, $\mathcal{X}_{1i} = E^T A_i + S_i R^T A_i + A_i^T E + A_i^T R S_i^T + 2E^T E + E^T D_i + S_i R^T D_i + D_i^T E + D_i^T R S_i^T$.

Proof. The prove of the corollary is accomplished under the special case $P_{i,\sigma} = U = I$ $(i \in S, \sigma \in R)$. According to $\Phi_{1i} + \Phi_{2i} + \Phi_{2i}^T + \Phi_{4i} < 0$ in Theorem 3.1, we can derive that the system (2.7) with $\omega(t) = 0$ is stochastically stable. At first,

$$\begin{aligned}\Phi_{1i} + \Phi_{2i} + \Phi_{2i}^T + \Phi_{4i} =& 2A_i^T P_{i,\sigma} E - \tau E^T U E + \sum_{n\in R} E^T P_{in} E + \sum_{j\in S} \pi_{ij}^m E^T P_{jm} E \\ &+ 2S_i R^T A_i + (\tau - \zeta(t)) A_i^T U A_i + 2D_i^T P_{i,\sigma} E + 2S_i R^T D_i \\ &+ 2\tau E^T U E + 2(\tau - \zeta(t)) A_i^T U D_i - 2\tau E^T U E \\ &+ (\tau - \zeta(t)) D_i^T U D_i.\end{aligned}$$

According to

$$\tau \leq \frac{-\lambda_{min}(\mathcal{X}_{1i})}{K_1} \tag{3.28}$$

and the condition $E^T A_i + S_i R^T A_i + A_i^T E + A_i^T R S_i^T + 2E^T E + E^T D_i + S_i R^T D_i + D_i^T E + D_i^T R S_i^T < 0$, we could obtain

$$\begin{aligned} &2A_i^T P_{i,\sigma} E - \tau E^T U E + \sum_{n\in R} E^T P_{in} E + \sum_{j\in S} \pi_{ij}^m E^T P_{jm} E + 2S_i R^T A_i \\ &+ (\tau - \zeta(t)) A_i^T U A_i + 2D_i^T P_{i,\sigma} E + 2S_i R^T D_i + 2\tau E^T U E \\ &+ 2(\tau - \zeta(t)) A_i^T U D_i - 2\tau E^T U E + (\tau - \zeta(t)) D_i^T U D_i \\ \leq& E^T A_i + S_i R^T A_i + A_i^T E + A_i^T R S_i^T + (2-\tau) E^T E + \tau A_i^T A_i + E^T D_i \\ &+ S_i R^T D_i + 2\tau \|A_i^T D_i\| + D_i^T E + D_i^T R S_i^T + \tau D_i^T D_i < 0,\end{aligned} \tag{3.29}$$

moreover,

$$\begin{aligned} &2A_i^T P_{i,\sigma} E - \tau E^T U E + \sum_{n\in R} E^T P_{in} E + \sum_{j\in S} \pi_{ij}^m E^T P_{jm} E + 2S_i R^T A_i \\ &+ (\tau - \zeta(t)) A_i^T U A_i + 2D_i^T P_{i,\sigma} E + 2S_i R^T D_i + 2\tau E^T U E + 2(\tau - \zeta(t)) A_i^T U D_i \\ &- 2\tau E^T U E + (\tau - \zeta(t)) D_i^T U D_i \leq 2A_i^T E + 2S_i R^T A_i + 2ED_i + 2S_i R^T D_i \\ &+ 2E^T E + \tau(\lambda_{max}(A_i^T A_i) + \lambda_{max}(D_i^T D_i) + 2\|A_i^T D_i\| + \|E^T E\|) < 0. \end{aligned} \tag{3.30}$$

Combining with (3.29), (3.30), we get $\Phi_{1i} + \Phi_{2i} + \Phi_{2i}^T + \Phi_{4i} < 0$.

As a consequence, base on $P_{i,\sigma} = U = I$ $(i \in S, \sigma \in R)$, we have proved Corollary 1. That is to say, the system (2.7) is stochastically admissible in the special case of $P_{i,\sigma} = U = I$ $(i \in S, \sigma \in R)$. □

Remark 3.2. By the inequality (3.27), we can find the upper bound of time delay τ, which is different from paper [33]. In [33], there has no the specific inequality constraints of delay. In our work, we find the specific inequality restrictions of upper bound on the duration τ from the theoretical aspect. It should be noted that the conditions of delay are less conservative in our work than ones in papers [29,30,34].

Remark 3.3. On the one hand, it is worth noting that dealing with the stability problem for singular systems with Markovian jump parameters, it is required to consider not only stochastic stability, but also regularity and absence of impulsiveness (for continuous singular Markovian jump systems) or causality (for discrete singular Markovian jump systems) simultaneously, whereas for normal state-space

Markovian jump systems the latter two issues do not arise. As a result, the stability problem of singular Markovian jump systems is much more complicated than that for normal state-space Markovian jump systems. On the other hand, the corresponding results of singular systems also can be applied to the case of $E = I$ that E is nonsingular. Hence, the singular systems are more extensively than the normal systems in the range of application.

4. Numerical example

In this section, two examples are illustrated to verify the effectiveness of the designed approach.

Example 4.1. As we known, singular systems can describe the behavior of some physical systems better than the normal systems. Singular systems arise in some practical systems like power systems, electrical circuits, networks, and so on. Random abrupt changes in singular systems represent a kind of systems which has stochastic behavior and can be appropriately described by the linear time-variant model that is used extensively in the field of control. Some undesired results may be caused by these abrupt changes, for example, the system turns to be unstable or the poor performance emerges. Therefore, the problem of the stability and stabilization analysis of this class of systems is very important and significant. Some stabilization techniques have been considered and most of the results are obtained in terms of LMIs techniques, which make the results easy to be verified by the Matlab LMIs Tools. We can refer the more detail to [10,12,16]. We carry out the simulations on system (2.1) to demonstrate results in Theorem 3.1. In particular, the corresponding system parameters are presented as

In mode 1

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -1.5 & 0.5 \\ -0.45 & -0.832 \end{bmatrix}, B_1 = \begin{bmatrix} 0.15 \\ 0.01 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -0.5 & 0.4 \\ 2 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 0.21 \\ 0.25 \end{bmatrix}, \tilde{D}_1 = \begin{bmatrix} 0.31 \\ 1.5 \end{bmatrix}^T.$$

In mode 2

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.4 \\ -0.36 & 0.79 \end{bmatrix}, B_2 = \begin{bmatrix} -0.1 \\ 1 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \tilde{D}_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}^T.$$

$f(i, x, t) = [e^{-t} \sin(x_1(t) - x_2(t)), e^{-t} \sin(x_1(t))]^T$ for $i = 1, 2$. Here, we assume that $r_t \in S = \{1, 2\}$ and $\sigma_t \in R = \{1, 2, 3\}$. The corresponding transition matrices are given by

$$\Pi^1 = \begin{bmatrix} -0.1 & 0.1 \\ 0.72 & -0.72 \end{bmatrix}, \qquad \Pi^2 = \begin{bmatrix} -0.5 & 0.5 \\ 0.7 & -0.7 \end{bmatrix},$$

$$\Pi^3 = \begin{bmatrix} -0.2 & 0.2 \\ 0.35 & -0.35 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} -0.4 & 0.1 & 0.3 \\ 0.2 & -0.9 & 0.7 \\ 0.2 & 0.3 & -0.5 \end{bmatrix}.$$

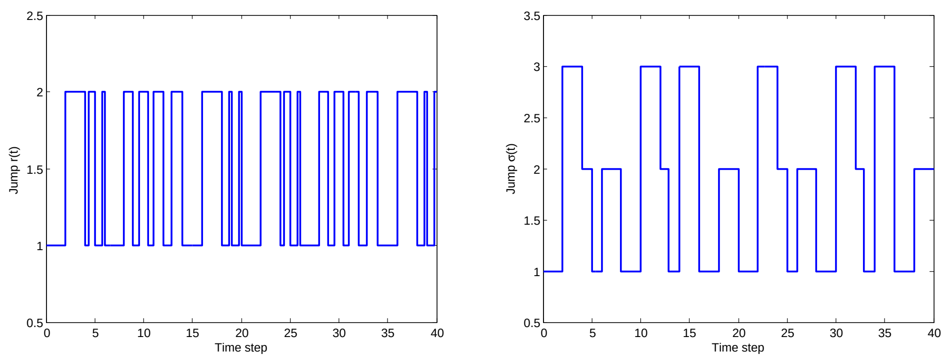

Figure 1. (a) Random jump with two modes (b) Random jump with three modes

For the underlying system with above parameters and time-delay $\tau = 0.2$, performance index $\gamma = 0.1$, we will design a controller of form (2.2) and use LMI toolbox solve inequalities (3.1), (3.2), which can find

$$D_1 = \begin{bmatrix} 0.0031 & -0.0821 \\ 0.0293 & -0.5439 \end{bmatrix}, D_2 = \begin{bmatrix} -1.5391 & -0.1541 \\ -0.5496 & -0.0556 \end{bmatrix}, U = \begin{bmatrix} 2.3098 & -0.4460 \\ -0.4460 & 1.8516 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 4.9278 & -0.0435 \\ -0.0435 & 5.8002 \end{bmatrix}, P_{12} = \begin{bmatrix} 5.0115 & -0.0090 \\ -0.0090 & 5.7998 \end{bmatrix}, P_{13} = \begin{bmatrix} 4.9384 & -0.0323 \\ -0.0323 & 5.800 \end{bmatrix},$$

$$P_{21} = \begin{bmatrix} 5.7598 & 0.7842 \\ 0.7842 & 5.8852 \end{bmatrix}, P_{22} = \begin{bmatrix} 5.7419 & 0.7832 \\ 0.7832 & 5.8850 \end{bmatrix}, P_{23} = \begin{bmatrix} 5.6489 & 0.7794 \\ 0.7794 & 5.8847 \end{bmatrix}.$$

According to the relationship of $D_i = F_i K_i$, and the fact that F_i is given out, further, the control gain parameters can be obtained

$$K_1 = \begin{bmatrix} 0.0066 & -0.1042 \\ 0.0160 & -0.3355 \end{bmatrix}, K_2 = \begin{bmatrix} -0.2984 & -0.0300 \\ 0.0472 & 0.0043 \end{bmatrix}.$$

A possible case for the switching signal is depicted in Figure 1. State response for the open-loop system is plotted in Figure 2(a) under the initial values $x(0) = [0.7551 - 1.0522]^T$. Figure 2(b) shows the state response for the underlying

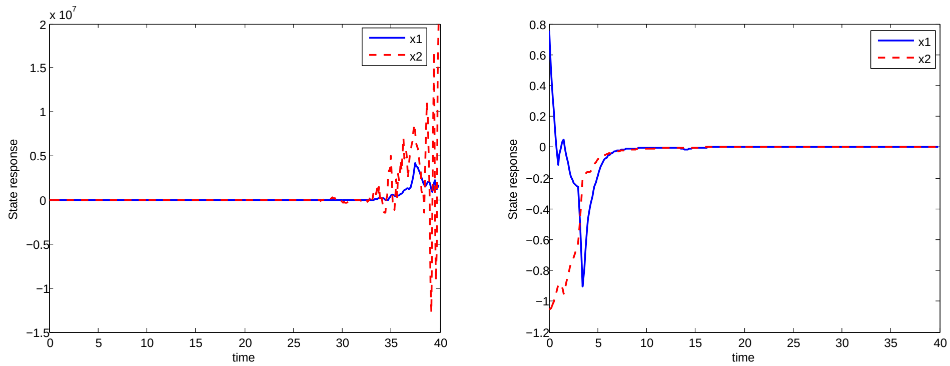

Figure 2. (a) State response for open-loop system (b) State response for closed-loop system

system with the feedback controller based on discrete-time state observations, for illustration, we let controller with $x(0) = [0.7551\ \ -1.0522]^T$ and the exogenous disturbance signal $\omega(t) = \sin(0.01t)e^{-0.001t}$. Based on Figure 2(a) and (b), we can see that the open loop system is unstable, while the closed-loop systems can be stabilized by the designed controller. This example shows our result is effective.

Example 4.2. Based on the inequality (3.27) and let $E = I$. These parameters $A_1, A_2, B_1, B_2, C_1, C_2, \tilde{D}_1, \tilde{D}_2, F_1, F_2$ are the same as in Example 4.1. Let $\gamma = 1$ and make sure that $\tau < 0.5$, the solution of hybrid system (2.7) is stochastically stable. It is noted that it is required for $\tau < 0.0000308$ in [22] and $\tau < 0.0046$ in [23], while in our result the τ only satisfies $\tau \leq 0.5$. Therefore, it shows that our theory has improved the existing result significantly.

5. Conclusion

The issue of stochastically admissible with H_∞ performance for hybrid systems with singular Markovian jump by state feedback based on discrete-time observations has been handled. The designed controller is more practical compared with the continuous-time state feedback strategies, and has the merits of cost less and save resource. By employing Lyapunov-Krasovskii functional and LMIs technologies, in terms of LMIs, our results can be readily testified using numerical software MATLAB. Furthermore, the criteria are established which ensure the resulting systems are regular, impulse free, and stochastically stable. Two numerical examples are supplied to manifest the effectiveness of the designed methods. In future work, in order to well describe the practical systems, the more general systems shall be considered, for instance, the semi-Markov jump systems [40] with sliding mode control scheme [15] maybe an interesting approach.

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References

[1] E. K. Boukas, *Static output feedback control for stochastic hybrid systems: LMI approach*, Automatica, 2006, 42(1), 183–188.

[2] E. K. Boukas, *Communications and control engineering, Control of singular systems with random abrupt changes*, Springer, 2008.

[3] Y. K. Cui, J. Shen and Y. Chen, *Stability analysis for positive singular systems with distributed delays*, Automatica, 2018, 94, 170–177.

[4] L. Y. Dai, *Lectures notes in control and information sciences: Vol. 118. Singular control systems*, Springer, New York, 1989.

[5] J. C. Geromel and P. Colaneri, *Stability and stabilization of continuous-time switched linear systems*, Siam J. Control Optim., 2006, 45(5), 1915–1930.

[6] M. A. Han and L. J. Sheng, *Bifurcation of limit cycles in piecewise smooth systems via Melnikov function*, J. Appl. Anal. Comput., 2015, 5(4), 809–815.

[7] M. A. Han, *On the maximum number of periodic solutions of piecewise smooth periodic equations by average method*, J. Appl. Anal. Comput., 2017, 7(2), 788–794.

[8] Y. He, M. Wu, J. H. She and G. P. Liu, *Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays*, Syst. Control Lett., 2004, 51(1), 57–65.

[9] M. G. Hua, D. D. Zheng and F. Q. Deng, *Partially mode-dependent $l_2 - l_\infty$ filtering for discrete-time nonhomogeneous Markov jump systems with repeated scalar nonlinearities*, Inform. Sciences, 2018, 451-452, 223-239.

[10] I. Y. Kats and A. A. Martynyuk, *Stability and stabilization of nonlinear systems with random structure*, Taylor & Francis, 2002.

[11] Y. G. Kao, J. Xie and C. H. Wang, *An anti-windup design to robust H_∞ control for singular Markovian jump systems with actuator saturation and general unknown transition rates*, J. Franklin Inst., 2015, 352(12), 5708–5734.

[12] N. N. Krasovskii and E. A. Lidskii, *Analysis design of controller in systems with random attributes-Part 2*, Automat. Rem. Contr., 1961, 22, 1141–1146.

[13] C. M. Lee and I. K. Fong, *H_∞ optimal singular and normal filter design for uncertain singular systems*, IET Control Theory A., 2007, 1(1), 119–126.

[14] F. B. Li, P. Shi, C. C. Lim and L. G. Wu, *Fault detection filtering for nonhomogeneous Markovian jump systems via fuzzy approach*, IEEE Trans. Fuzzy Syst., 2018, 26(1), 131–141.

[15] M. Li, M. Liu, Y. C. Zhang and Y. H. Geng, *Fault tolerant sliding mode control for T-S fuzzy stochastic time-delay system via a novel sliding mode observer approach*, Int. J. Syst. Sci., 2018, 49(7), 1353–1367.

[16] Y. K. Li, M. Chen, L. Cai, and Q. X. Wu, *Resilient control based on disturbance observer for nonlinear singular stochastic hybrid system with partly unknown Markovian jump parameters*, J. Franklin Inst., 2018, 355(5), 2243–2265.

[17] M. Z. Luo, S. M. Zhong and J. Cheng, *Finite-time event-triggered control and fault detection for singular Markovian jump mixed delay systems under asynchronous switching*, Adv. Differ. Equ., 2018. DOI: 10.1186/s13662-018-1533-y.

[18] Y. C. Ma, N. N. Ma and L. Chen, *Synchronization criteria for singular complex networks with Markovian jump and time-varying delays via pinning control*, Nonlinear Anal.-Hybri., 2018, 29, 85–99.

[19] X. R. Mao, *Stability of stochastic differential equations with Markovian switching*, Stoch. Proc. Appl., 1999, 79, 45–67.

[20] X. R. Mao *Exponential stability of stochastic delay interval systems with Markovian switching*, IEEE Trans. Automat. Contr., 2002, 47(10), 1604–1612.

[21] X. R. Mao and C. G. Yuan, *Stochastic differential equations with Markovian switching*, Imperial College Press, 2006.

[22] X. R. Mao, *Stabilization of continuous-time hybrid stochastic differential equations by discrete-time feedback control*, Automatics, 2013, 49(12), 3677–3681.

[23] X. R. Mao, W. Liu, L. J, Hu, Q. Luo and J. Q. Lu, *Stabilization of hybrid stochastic differential equations by feedback control based on discrete-time state observations*, Syst. Control Lett., 2014, 73, 88–95.

[24] S. Y. Pan, Z. Y. Ye and J. Zhou, *H_∞ robust control based on event-triggered sampling for hybrid systems with singular Markovian jump*, Math. Mett. Appl. Sci., 2019, 42, 790–805.

[25] B. Pang and Q. L. Zhang, *Sliding mode control for polynomial fuzzy singular systems with time delay*, IET Control Theory A., 2018, 12(10), 1483–1490.

[26] W. H. Qi, G. D. Zong and H. R. Karim, *Observer-based adaptive SMC for nonlinear uncertain singular semi-Markov jump systems with applications to DC motor*, IEEE Trans. Circuits Syst. I. Reg. Papers, 2018, 65(9), 2951–2960.

[27] R. Shorthen, F.Wirth , O. Mason, K. Wulff and C. King, *Stability criteria for switched and hybrid systems*, Siam Rev., 2007, 49(4), 545–592.

[28] M. H. Sun, J. Lam, S. Y. Xu and Y. Zou, *Robust exponential stabilization for Markovian jump systems with mode-dependent input delay*, Automatica, 2007, 43, 1799–1807.

[29] H. J. Wang, A. K. Xue and R. Q. Lu, *New stability criteria for singular systems with time-varying delay and nonlinear perturbations*, Int. J. Syst. Sci., 2014, 45(12), 2576–2589.

[30] G. L. Wang, Q. L. Zhang and C. Y. Yang, *Exponential stability of stochastic singular delay systems with general Markovian switchings*, Int. J. Robust Nonlin., 2014, 25(17), 3478–3494.

[31] J. M. Wang, S. P. Ma, C. H. Zhang and M. Y. Fu, *Finite-time H_∞ filtering for nonlinear singular systems with nonhomogeneous Markov jumps*, IEEE T. Cybernetics, 2018, 49(6), 2133–2143.

[32] Z. G. Wu, H. Y. Su and J. Chu, *Robust exponential stability of uncertain singular Markovian jump time-delay systems*, Acta Automatica Sinica, 2010, 36(4), 558–563.

[33] Z. G. Wu, H. Y. Su and J. Chu, *H_∞ filtering for singular Markovian jump systems with time delay*, Int. J. Robust Nonlin., 2010, 20(8), 939–957.

[34] Y. F. Xie, W. H. Gui, Y. L. Wang and Z. H. Jiang, *Memory state feedback controller design for singular systems with multiple internal constant point delays*, IET Control Theory A., 2009, 3(6), 631–641.

[35] J. Xie, Y. G. Kao, C. H. Zhang and H. R. Karimi, *Quantized control for uncertain singular Markovian jump linear systems with general incomplete transition rates*, Int. J. Control Autom. Syst., 2017, 15(3), 1107–1116.

[36] S. Y. Xing, F. Q. Deng and D. Hou, *Delay-dependent H_∞ filtering for singular Markov jump systems with wiener process and generally uncertain transition rates*, Int. J. Syst. Sci., 2018, 49(8), 1685–1702.

[37] S. Y. Xu, T. Chen and J. Lam, *Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent time delays*, IEEE Trans. Automat. Contr., 2003, 48(5), 900–907.

[38] S. Y. Xu and J. Lam, *Reduced-order H_∞ filter for singular systems*, Syst. Control Lett., 2007, 56(1), 48–57.

[39] P. Yu, M. A. Han and Y. Z. Bai, *Dynamics and bifurcation stidy on an extended lorenz system*, J. nonlinear model. anal., 2019, 1(1), 107–128.

[40] L. X. Zhang, B. Cai and Y. Shi, *Stabilization of hidden semi-Markov jump systems: Emission probability approach*, Automatica, 2019, 101, 87–95.

[41] J. Zuo, G. B. Liu, Y. L. Wei, Z. D. Wei and J. W. Feng, *Mixed filter design for nonlinear singular Markovian jump systems with time-varying delays based on a dissipativity performance index*, T. I. Meas. Control, 2018, 40(9), 2779–2788.