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STATISTICALLY LOCALIZED SEQUENCES IN METRIC SPACES

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Abstract In this paper we have introduced the statistically localized sequences in metric spaces and investigate basic properties of the statistically localized sequences. Also we have obtained some necessary and sufficient conditions for a localized sequence to be a statistically Cauchy sequence. It is also defined uniformly statistically localized sequences on metric spaces and its relation with statistically Cauchy sequences has been investigated.

Keywords Statistical convergence, a metric space, statistical localor of the sequence.

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1. Introduction and preliminaries

The notation of a localized sequence introduced in [11] can be treated as a generalization of a Cauchy sequence in metric spaces. Introducing the concept of localized sequence and the locator of a sequence, the paper [11] studied the basic properties of localized sequences and obtained some results for the closure operators in a metric space. Let X is a metric space with a metric $d(\cdot,\cdot)$ and (x_n) is a sequence of points in X. If the real number sequence $\alpha_n = d(x_n, x)$ converges for all $x \in M \subset X$ then the sequence (x_n) is called a localized sequence on the subset M. The maximal subset on which (x_n) is a localized sequence is called the locator of the sequence (x_n). If (x_n) is a localized sequence on X then (x_n) is called localized everywhere. If the locator of a sequence (x_n) contains all members of this sequence, expect of a finite number of them, then (x_n) is called localized in itself [11]. It is important to note that, every Cauchy sequence in X is localized everywhere. It is also an interesting fact that if $A : X \to X$ is a mapping with the condition $d(Ax, Ay) \le d(x, y)$ for all $x, y \in X$ then for every $x \in X$ the sequence $(A^n x)$ is localized at every fixed point of the mapping A. This means that fixed points of the mapping A is contained in the locator of the sequence $(A^n x)$. Motivating the above facts and the fact that the locator of a sequence can be extended by changing the usual limit to the statistical limit of a sequence (see [8]), we introduce the concepts of a statistically localized sequence and the statistical locator of a sequence in metric spaces. Recall that if (x_n) is a sequence of points in a metric space X we say that

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(a) (x_n) is statistically convergent to the point $l \in X$ (we write $x_n \overset{st}{\to} l$ or $st\text{-}\lim_{n\to\infty} d(x_n, l) = 0$) if for each $\varepsilon > 0$,

$$\delta\{k : d(x_k, l) \geq \varepsilon\} = 0 \text{ and}$$

(b) (x_n) is statistically Cauchy sequence if for each $\varepsilon > 0$ there is a positive integer $N = N(\varepsilon)$ such that

$$\delta\{\ k : d(x_N, x_k) \geq \varepsilon\} = 0,$$

where

$$\delta\{A\} := \lim_n n^{-1}\{\text{the number of } k \leq n \text{ such that } k \in A\}$$

is a natural density of the set $A \subset \mathbb{N}$ (see [7,8]).

Note that the statistical convergence of a sequence was introduced by Fast [6] and Steinhaus [14]. Later, this concept has been generalized in many directions. More details on statistical convergence and on applications of this concept can be found in Fridy [8], Connor [4], Erdös and Tenenbaum [5], Freedman and Sember [7] and Maio and Kocinac [12], Braha et al. [2], Nuray et al. [13], Yegül and Dündar [15], Baliarsingh et al. [1], Kadak and Mohiuddine [9].

In this paper, we investigate basic properties of statistically localized sequences and obtain some necessary and sufficient conditions for a localized sequence to be a statistically Cauchy sequence. We prove that every statistically bounded sequence has everywhere statistically localized subsequence. It is also defined uniformly s-tatistically localized sequences on metric spaces and its relation with statistically Cauchy sequences has been investigated.

2. Definitions and notations

In this part of the paper we shall introduce some basic definitions and notations. Let X be a metric space with metric $d(x, y)$.

Definition 2.1. (a) A sequence (x_n) in X is called statistically localized in the subset $M \subset X$ if the number sequence $d(x_n, x)$ is statistically converges for all $x \in M$.

(b) the maximal set on which a sequence is statistically localized is called a statistical locator of the sequence. The statistically locator of a sequence (x_n) will be denoted by $loc_{st}(x_n)$.

(c) A sequence (x_n) is called statistically localized everywhere if the statistical locator of (x_n) coincides with X.

(d) A sequence (x_n) is called statistically localized in itself if the statistically locator contains x_n for almost all n, i.e.

$$\delta\{n : x_n \notin loc_{st}(x_n)\} = 0 \text{ or } \delta\{n : x_n \in loc_{st}(x_n)\} = 1.$$

(e) A sequence (x_n) is called statistically localized if $loc_{st}(x_n)$ is not empty.

Definition 2.2. A sequence $(x_n) \in X$ is called a statistically Cauchy sequence if for any $\varepsilon > 0$ there exist $k_\varepsilon \in \mathbb{N}$ such that

$$\delta\{n \in \mathbb{N} : d(x_n, x_{k_\varepsilon}) \geqslant \varepsilon\} = 0.$$

From the above definitions it is clear that every statistically Cauchy sequence in a metric space X is statistically localized everywhere in X. Indeed, since

$$|d(x_n,x)-d(x,x_{k_\varepsilon})|\leqslant d\left(x_n,x_{k_\varepsilon}\right)$$

we have

$$\left\{n\in\mathbb{N}:d\left(x_n,x_{k_\varepsilon}\right)\geqslant\varepsilon\right\}\supset\left\{n\in\mathbb{N}:\left|d(x_n,x)-d\left(x_{k_\varepsilon},x\right)\right|\geqslant\varepsilon\right\}.$$

Therefore the number sequence $d(x_n,x)$ is a statistically Cauchy sequence, then $d(x_n,x)$ is statistically convergent for all $x\in X$. Hence (x_n) is statistically localized everywhere.

3. Basic properties of statistically localized sequences

Proposition 3.1. *Every statistically localized sequence is statistically bounded.*

Proof. Let (x_n) is statistically localized sequence. Then, $d(x_n,x)$ is statistically converges for some $x\in X$. Hence, the sequence $d(x_n,x)$ is a statistically bounded. This implies that $\delta(\{n\in\mathbb{N}:d(x_n,x)>K\})=0$ for some $K>0$. Consequently, the sequence (x_n) is statistically bounded because almost all elements of (x_n) are located in the open ball $B(x,K)$. □

Proposition 3.2. *Let $L=loc_{st}(x_n)$ and the point $z\in X$ be such that for any $\varepsilon>0$ there exists $x\in L$ satisfying*

$$\delta\{n\in\mathbb{N}:|d(x,x_n)-d(z,x_n)|\geqslant\varepsilon\}=0. \tag{3.1}$$

Then $z\in L$.

Proof. It is enough to prove that the number sequence $\alpha_n=d(x_n,z)$ satisfies the statistically Cauchy criteria. Let be $\varepsilon>0$ and $x\in L=loc_{st}(x_n)$ is a point with the property (3.1). Since the sequence $d(x_n,x)$ is statistically Cauchy sequence with the property (3.1), then there exist a subsequence $K=(k_n)$ of $\mathbb{N}$ with $\delta(K)=1$ such that

$$|d(x,x_{k_n})-d(z,x_{k_n})|\to 0 \text{ and}$$

$$|d(x_{k_n},x)-d(x_{k_m},x)|\to 0 \text{ as } m,n\to\infty$$

(see [8]). Obviously, for any $\varepsilon>0$ there exist $n_0\in\mathbb{N}$ such that for all $n\geq n_0$, $m\geq m_0$ we have

$$|d(x,x_{k_n})-d(z,x_{k_n})|<\frac{\varepsilon}{3}, \tag{3.2}$$

$$|d(x,x_{k_n})-d(x,x_{k_m})|<\frac{\varepsilon}{3}. \tag{3.3}$$

Now, combining (3.2) and (3.3) together with the following estimation

$$\begin{aligned}|d(z,x_{k_n})-d(z,x_{k_m})|\leq{}&|d(z,x_{k_n})-d(x,x_{k_n})|+|d(x,x_{k_n})-d(x,x_{k_m})|+\\&+|d(x,x_{k_m})-d(z,x_{k_m})|\end{aligned} \tag{3.4}$$

we find that

$$|d(z,x_{k_n})-d(z,x_{k_m})|<\varepsilon \tag{3.5}$$

for all $n \geq n_0$, $m \geq n_0$. That is

$$|d(z, x_{k_n}) - d(z, x_{k_m})| \to 0 \text{ as } m, n \to \infty$$

for the subset $K = (k_n) \subset N$ with $\delta(K) = 1$. This implies that the number sequence $d(z, x_n)$ is a Cauchy sequence. The proof is completed. □

Proposition 3.3. *Statistically locator of any sequence is a closed subset of the metric space X.*

Proof. Let be $z \in \overline{loc_{st}}(x_n)$. Then, for any $\varepsilon > 0$ the ball $B(z, \varepsilon)$ will contain a point $x \in loc_{st}(x_n)$. Therefore,

$$\delta\{n \in \mathbb{N} : |d(x, x_n) - d(z, x_n)| \geqslant \varepsilon\} = 0$$

for any $\varepsilon > 0$, since for almost all n we have

$$|d(x, x_n) - d(z, x_n)| \leq d(z, x_n) < \varepsilon.$$

Consequently, the hypothesis of Proposition 3.2 is satisfied. Then $z \in loc_{st}(x_n)$, i.e. $loc_{st}(x_n)$ is closed. □

Recall that the point z is a statistical limit point of the sequence $(x_n) \in X$ if there is a set $K = \{k_1 < k_2 < ...\} \subset \mathbb{N}$ such that $\delta(K) \neq 0$ and $\lim_{n\to\infty} d(x_{k_n}, z) = 0$. Similarly, a point ξ is said to be a statistical cluster point if for each $\varepsilon > 0$

$$\delta\{n \in \mathbb{N} : d(x_n, \xi) < \varepsilon\} \neq 0.$$

Since $|d(x_n, y) - d(z, y)| \leq d(x_n, z)$, we have the following proposition.

Proposition 3.4. *If $z \in X$ is a statistical limit point (a statistical cluster point) of a sequence $(x_n) \in X$, then for each $y \in X$ the number $d(z, y)$ is a statistical limit point (a statistical cluster point) of the sequence $\{d(x_n, y)\}$.*

Proposition 3.5. *All statistical limit points (statistical cluster points) of the statistically localized sequence (x_n) have the same distance from each point x of the statistical locator $loc_{st}(x_n)$.*

Proof. Indeed, if z_1 and z_2 are two statistical limit points (statistical cluster points) of the sequence (x_n), then the numbers $d(z_1, x)$ and $d(z_2, x)$ are statistical limit points of the statistically convergent sequence $d(x, x_n)$. Consequently, $d(z_1, x) = d(z_2, x)$. □

Proposition 3.6. *$loc_{st}(x_n)$ does not contain more than one statistical limit (cluster) point of the sequence (x_n). Particularly, everywhere localized sequence has not more than one statistical limit (cluster) point.*

Proof. If $x, y \in loc_{st}(x_n)$ are two statistical limit or cluster points of the sequence (x_n), then by the Proposition 3.5 $d(x, x) = d(x, y)$. But $d(x, x) = 0$. This implies that, $d(x, y) = 0$ for $x \neq y$. This is a contradiction. □

Proposition 3.7. *If the sequence (x_n) has a statistical limit point $z \in loc_{st}(x_n)$, then $x_n \overset{st}{\to} z$.*

Proof. The sequence $\{d(x_n, z)\}$ is statistically convergent and some subsequence of this sequence converges to zero, that is $x_n \overset{st}{\to} z$. □

Definition 3.1. For the given statistically localized sequence (x_n), with the statistically locator $L = loc_{st}(x_n)$, the number

$$\sigma = \inf_{x\in L}\left(st\text{-}\lim_{n\to\infty} d(x,x_n)\right)$$

is called the statistical barrier of (x_n).

Theorem 3.1. *The statistically localized sequence is statistically Cauchy sequence if and only if the statistical barrier is equal to zero.*

Proof. Let (x_n) is a statistically Cauchy sequence in a metric space X. This means there exit a set $K = \{k_1 < k_2 < ... < k_n < ...\} \subset \mathbb{N}$ such that $\delta(K) = 1$ and $\lim_{n,m\to\infty} d(x_{k_n}, x_{k_m}) = 0$. Consequently, for each $\varepsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that

$$d\left(x_{k_n}, x_{k_{n_0}}\right) < \varepsilon \text{ for all } n \geq n_0.$$

Since a statistically Cauchy sequence is statistically localized everywhere, we have $st\text{-}\lim_{n\to\infty} d\left(x_n, x_{k_{n_0}}\right) \leq \varepsilon$, i.e. $\sigma \leq \varepsilon$. Because $\varepsilon > 0$ is arbitrary, we obtain $\sigma = 0$.

Conversely, if $\sigma = 0$ then for each $\varepsilon > 0$ there is $x \in L = loc_{st}(x_n)$ such that $d(x) = st\text{-}\lim_{n\to\infty} d(x,x_n) < \frac{\varepsilon}{2}$. In this case

$$\delta\left\{n \in \mathbb{N} : |d(x) - d(x,x_n)| \geq \frac{\varepsilon}{2} - d(x)\right\} = 0.$$

Then $\delta\left\{n \in \mathbb{N} : d(x,x_n) \geq \frac{\varepsilon}{2}\right\} = 0$, i.e. $st\text{-}\lim_{n\to\infty} d(x,x_n) = 0$. Then (x_n) is a statistically Cauchy sequence. □

If for some subsequence (x_{k_n}) of the sequence (x_n) we have $\delta(K) = 0$, where $K = \{k_1 < k_2 < ... < k_n\} \subset \mathbb{N}$, then $\left(x_{k_n}\right)$ is called a thin subsequence of (x_n). Otherwise, i.e. if $\delta(K) \neq 0$, $\left(x_{k_n}\right)$ is called a nonthin subsequence (see [8]).

Theorem 3.2. *If (x_n) is statistically localized in itself and (x_n) contains a nonthin Cauchy subsequence, then (x_n) is a statistically Cauchy sequence itself.*

Proof. Let (x'_n) is a nonthin Cauchy subsequence of (x_n). Not losing of generality, we can suppose that all members of (x'_n) belong to $loc_{st}(x_n)$. Since (x'_n) is a Cauchy sequence, by Theorem 3.1 $\inf_{x'_n} \lim_{m\to\infty} d(x'_m, x'_n) = 0$. On the other hand, since (x_n) is statistically localized in itself then,

$$st - \lim_{m\to\infty} d(x_m, x'_n) = st - \lim_{m\to\infty} d(x'_m, x'_n) = 0.$$

This implies that,

$$\sigma = \inf_{x\in L}\left(st\text{-}\lim_{m\to\infty} d(x_m, x)\right) = 0,$$

i.e. (x_n) is a statistically Cauchy sequence itself. □

Let be $a \in X$ and $r > 0$. Recall that the sequence (x_n) in a metric space X is called statistically bounded if there is a subset $K = \{k_1 < k_2 < ... < k_n \subset ...\}$ of $\mathbb{N}$ such that $\delta(K) = 1$ and $\left(x_{k_n}\right) \subset B(a,r)$, where $B(a,r)$ is the ball with center at the point a and with radius r. It is clear that, $\left(x_{k_n}\right)$ is a bounded sequence in X and it has a localized in itself subsequence (see [11]). Consequently, the following assertion also becames true.

Theorem 3.3. *Every statistically bounded sequence in a metric space has a statistically localized in itself subsequence.*

Definition 3.2. We say an infinite subset $M \subset X$ is thick relatively to a nonempty subset $Y \subset X$ if for each $\varepsilon > 0$ there is the a point $y \in Y$ such that the ball $B(y, \varepsilon)$ has infinitely many points of M. In particular, if the set M is thick relatively to its subset $Y \subset M$ then M is called thick in itself.

Proposition 3.8 (see [11]). *If the set M is thick relatively to some set Y, then the set M is thick in itself.*

Using the notation thick set the separability of metric spaces can be characterized.

Proposition 3.9 (see [11]). *The metric space X is separable if and only if each noncountable subset of X is thick in itself.*

Theorem 3.4. *The following statements are equivalent:*

(i) Every statistically localized in itself sequence in X is a statistically Cauchy sequence.
(ii) Every bounded subset of X is totally bounded.
(iii) Every bounded infinite set of X is thick in itself.

Proof. Let (i) is satisfied, but (ii) is not true. Then there is a subset $M \subset X$ such that M is not totally bounded. This means that there exists $\varepsilon > 0$ and a sequence $(x_n) \subset M$ such that $d(x_n, x_m) > \varepsilon$ for any $n \neq m$.

Since (x_n) is statistically bounded by Theorem 3.3 it has a statistically localized in itself sequence (x'_n). Since $d(x'_n, x'_m) > \varepsilon$ for any $n \neq m$ the subsequence is not a statistically Cauchy sequence. This contradicts (i). Hence, (i) implies (ii).

It is elementary to obtain that (ii) implies (iii) (see [11]).

Now let show that (iii) implies (i). Let $(x_n) \subset X$ is a statistically localized in itself. Then (x_n) is statistically bounded sequence in X. Then here is an infinite set M of points of (x_n) such that M is a bounded subset of X.

By the assumption the set M is thick in itself. Then for every $\varepsilon > 0$ we can choose $x_k \in M$ such that the ball $B(x_k, \varepsilon)$ contains infinitely many points of X, say $x'_1, ..., x'_n, ...$ For the sequence (x'_n) the sequence $d(x'_n, x_k)_{n=1}^{\infty}$ is statistically converges and $st\text{-}\lim_{n\to\infty} d(x'_n, x_K) \leq \varepsilon$. Hence, the statistically barrier of (x_n) is equal to zero. That is (x_n) is a Cauchy sequence. Proof is completed. □

From Theorem 3.2 and 3.3 we obtain the property (i) is equivalent to

(iv) every statistically bounded sequence has a statistically Cauchy subsequence.

In separable metric spaces the property (iv) can be weakened as

(v) every statistically bounded sequence has everywhere statistically localized sequence.

There is nonseparable metric spaces on which the property (v) is also satisfied.

Example 3.1. Let $\ell^p(x)$ is the set of real valued functions defined on a set X with

$$\|f\|^p = \sup_{Y \subset X} \sum_{y \in Y} |f(y)|^p,$$

where Y is an arbitrary finite subset of X, $p \geqslant 1$ fixed real number. Then $\ell^p(x)$ is a vector space, and the map $f \to \|f\|$ is a norm on it.

Let $\{f_n\}$ is a statistically bounded with respect to this norm sequence in $\ell^p(x)$.

Obviously, there is finite or accountable set $(x_n) \subset X$ such that at most one of the functions f_n has nonzero values on the sequence (x_n). Let $f_{x_m}(x) =$

$\begin{cases} 1, \, x = x_m \\ 0, \, x \neq x_m \end{cases}$ and $\mathcal{H} = \overline{span}\,\{f_{x_n}\}$. Then $(f_n) \subset H$ and therefore $\mathcal{H}$ is separable, i.e. $\mathcal{H}$ is isomorphic to ordinary sequence space ℓ^p or its finite subspace. Consequently, the sequence $\{f_n\}$ has a subsequence $\{f'_n\}$ with statistically locator containing in $\mathcal{H}$.

It is true that the subsequence $\{f'_n\}$ is statistically localized everywhere in $\ell^p\,(x)$. If $f \in l^p\,(x)$, then $f = f' + f''$, $f' \in \mathcal{H}$, f'' satisfies

$$\delta\,\{n \in \mathbb{N} : f''\,(x_n = 0)\} = 0.$$

Since $\|f'_n - f\|^p = \|f'_n - f'\|^p + \|f''\|^p$ and $st\text{-}\lim\limits_{n\to\infty} \|f'_n - f'\|$ exists we have $st\text{-}\lim\limits_{n\to\infty} \|f'_n - f\|$ is also exist, i.e. (f_n) is statistically localized everywhere.

Note that in the space ℓ^{st}_{∞}, of all statistically bounded number sequence the property (v) is not valid. For example, the sequence (e_n) having unit e at n th coordinate for the element e_n and zero other coordinates, doesn't contain statistically localized everywhere subsequence.

Definition 3.3. A sequence (x_n) in metric space X is called uniformly statistically localized on the subset M of X if the sequence $\{d\,(x, x_n)\}$ uniformly statistically converges for all $x \in M$.

Proposition 3.10. *Let (x_n) is uniformly statistically localized on the set $M \subset X$ and $z \in Y$ is such that for every $\varepsilon > 0$, there is $y \in M$ with the property*

$$\delta\,\{n \in \mathbb{N} : |d\,(z, x_n) - d\,(y, x_n)| \geqslant \varepsilon\} = 0.$$

Then, $z \in loc_{st}\,(x_n)$ and (x_n) is uniformly statistically localized on a set containing such points z.

The proof of the Proposition 3.10 is analogously to the Proposition 3.2.

Remark 3.1. The function $d_M\,(x, z) = \sup\limits_{y\in M} |d\,(x, y) - d\,(y, z)|\,,\ x, z \in Y$ is called a pseudo metric on the set Y.

The following result is proved by the standard manner (see [3, 10]).

Proposition 3.11. *(x_n) is uniformly statistically localized on the set $M \subset X$ if and only if the sequence (x_n) is statistically Cauchy on the pseudo metric d_M.*

Theorem 3.5. *Every uniformly statistically localized in itself sequence is a statistically Cauchy sequence.*

Proof. Let $\delta\,\{n \in \mathbb{N} : x_n \notin M\} = 0$ for the uniformly statistically localized in itself sequence (x_n). From the definition of d_M we get that if at most one of the points x, z belongs to M, then $d_M\,(x, z) = d\,(x, z)$. In particular, there is $n_0 \in \mathbb{N}$ such that $\delta\,\{n \in \mathbb{N} : d_M\,(x_n, x_{n_0}) \neq d\,(x_n, x_{n_0})\} = 0$. Now the assertion is obtained from the Proposition 3.11. □

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