

INCLINED MAGNETIC FIELD AND SORET EFFECTS ON MIXED CONVECTION FLOW BETWEEN VERTICAL PARALLEL PLATES*

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Abstract This present paper investigates the influence of thermal diffusion and inclined magnetic field effects on mixed convection flow through a channel. Spectral Quasilinearization Method (SQLM) is used to solve the dimensionless governing equations, those were obtained by using suitable transformations from the system of governing partial differential equations. The influence of the variation of different parameters like magnetic parameter, Hall parameter, Soret parameter and the intensity of angle of inclination on velocities, temperature and concentration are investigated and presented through plots. According to acquired results, under the influence of magnetic field (in an inclined direction) the velocity profiles were amplified and the temperature profile got diminished, where as there is a reverse tendency under the effect of Hall parameter. Finally the nature of the physical parameters were displayed in table form.

Keywords Inclined magnetic field, mixed convection, Hall effect, Soret effect, SQLM.

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1. Introduction

Combined heat and mass transfer by mixed convection in a porous medium has been created much attention in the last several decades due to its many significant engineering and geophysical applications, like in the design of cooling systems for electronic devices and in the field of solar energy collection, etc.,. Heat exchanger technology involves convective flows in vertical channels. The problem of mixed convection heat and mass transfer and fluid flow between vertical parallel plates have been examined analytically and mostly numerically by several researchers [2,4,8]. In view of applications, Hossain and Floryan [17] studied the nature of mixed convection flow in a periodically heated channel. Barzegar *et al.* [5] investigated the thermal radiation on traditional Jeffery-Hamel flow to stretchable convergent/divergent channels. Mokhtari *et al.* [23] presented the numerical study on mixed convection

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heat transfer of various fin arrangements in a horizontal channel. Recently, Celik *et al.* [7] used $Al_2O_3^-$ Nanoparticles to enhance the heat transfer in partially heated vertical channel under mixed convection. Kaladhar and Komuraiah [18] applied Homotopy method to study the influence of cross diffusions on mixed convection chemical reaction flow in a vertical channel with Navier slip. Most recently, Sheikholeslami *et al.* [30] applied the Neural Network for estimation of heat transfer treatment of $Al_2O_3 - H_2O$ nanofluid through a channel.

Inclined magnetic field is just a magnetic field with a nonzero inclination. Inclination is the angle between the direction of the vector \vec{B} and another preferred direction in the problem. In astrophysics, they usually mean the radial direction. Many researchers advised the magnetic field vertical to the plates. But, in many sensible circumstances such magnetic field may not applicable always. Also, the influence of Hall current becomes most powerful mechanism for the electrical conduction in ionized gases and plasmas when the applied magnetic field is capable [3, 27, 34]. Nandkeolyar and Das [28] have been considered the MHD free convective radiative flow past a flat plate with ramped temperature in the presence of an inclined magnetic field. Gupta *et al.* [11] presented the effect of variable thermal conductivity and the inclined magnetic field on MHD plane poiseuille flow past non-uniform plate temperature. Hariri *et al.* [12] analyzed the numerical investigation of the heat transfer of a ferrofluid inside a tube in the presence of a non-uniform magnetic field. Mokhtari *et al.* [24] presented the effect of non-uniform magnetic field on heat transfer of swirling ferrofluid flow inside tube with twisted tapes. Hayat *et al.* [13] contributed towards the outcome of slip features on the peristaltic flow of a prandtl nanofluid with inclined magnetic field and chemical reaction. Recently, Barzegar *et al.* [9] analyzed the influence of non-uniform magnetic field on heat transfer intensification of ferrofluid inside a T-junction. Most recently, Sheikholeslami *et al.* [31] used the numerical mesoscopic method for transportation of H_2O -based nanofluid through a porous channel considering Lorentz forces.

The mass flux induced by temperature gradients is called a Soret effect. The Soret effect is more important in many physical processes and its effect on the double diffusive convection in porous media has attracted the attention of several researcher in the recent past. The Soret effect encountered in many areas for instance geosciences and chemical engineering, etc., and neglected in frequently related to the transfer of heat and mass due to that it is a smaller order of magnitude than the effects described by Fourier's and Fick's laws. Srinivasacharya and Kaladhar [33] presented the influence of Soret and dufour parameters on mixed convection couple stress fluid with heat and mass fluxes. Hayat *et al.* [14] investigated the Soret effect on convection flow. Pattnaik *et al.* [29] investigated the diffusion-thermo effect with Hall current on unsteady hydromagnetic flow past an infinite vertical porous plate. Hayat *et al.* [16] presented Low-speed peristaltic transport in a vertical channel subject to the Soret and Dufour effects. Most recently, Animasaun *et al.* [1] analyzed the comparative analysis between 36 nm and 47 nm alumina-water nanofluid flows in the presence of Hall effect. Hayat *et al.* [15] studied the physical significance of heat generation/absorption and Soret effects on peristalsis flow of pseudoplastic fluid in an inclined channel.

In this paper, the mixed convection flow through a vertical channel in presence of inclined magnetic field, Hall current and Soret effect has been investigated. The Spectral quasilinearization method is employed to solve the nonlinear problem. The quasilinearization method was proposed by Bellman *et al.* [6] as a generalization of

the Newton-Raphson method. Mandelzweig and his co-workers Krivec *et al.* [20–22] have been extended the application of the quasilinearization method to a wide variety of nonlinear BVP's and established that the method converges quadratically. The accuracy and validity of the Spectral quasilinearization schemes is presented by Motsa *et al.* [25, 26]. Recently, Goqo *et al.* [10] studied the capable of multi-domain bivariate spectral collocation solution for MHD laminar natural convection flow from a vertical permeable flat plate with uniform surface temperature and thermal radiation. Most recently, Srinivasacharya and Himabindu [32] applied for spectral quasilinearization method to study the influence of slip and convective boundary conditions on entropy generation in a porous channel due to micropolar fluid flow. Kaladhar *et al.* [19] applied Spectral quasilinearization method to study the nature of inclined magnetic field, thermal radiation and Hall current effects in a channel. The behavior of emerging flow parameters on the velocity, temperature and concentration is discussed and presented through plots.

2. Mathematical Modelling

Consider a steady fully developed laminar mixed convection flow between two permeable vertical plates with distance $2d$ apart. Choose the coordinate system such that x – axis be taken along vertically upward direction through the central line of the channel, y is perpendicular to the plates and the two plates are infinitely extended in the direction of x . The plate $y = -d$ has given the uniform temperature T_1 and concentration C_1 , while the plate $y = d$ is subjected to a uniform temperature T_2 and concentration C_2 . Since the boundaries in the x direction are of infinite dimensions, without loss of generality, we assume that the physical quantities depend on y only. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. The flow is a mixed convection flow taking place under thermal buoyancy and uniform pressure gradient in the flow direction. The flow configuration and the coordinates system are shown in Figure 1. A uniform external magnetic field B_0 is applied in the direction which makes an angle α with the positive direction of x –axis. The fluid velocity u is assumed to be parallel to the x – axis, so that only the x -component u of the velocity vector does not vanish but the transpiration flow velocity v_0 remains constant, where $v_0 < 0$ is the velocity of suction and $v_0 > 0$ is the velocity of injection.

With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible fluid are

$$v_0 \rho \frac{\partial u}{\partial y} = \rho g^* (\beta_T (T - T_1) + \beta_C (C - C_1)) + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} - \frac{\sigma B_0^2 \cos \alpha}{1 + m^2 \cos^2 \alpha} [u \cos \alpha - v_0 \sin \alpha + mw \cos^2 \alpha], \quad (2.1)$$

$$v_0 \rho \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2 \cos^2 \alpha}{1 + m^2 \cos^2 \alpha} [mu \cos \alpha - mv_0 \sin \alpha - w], \quad (2.2)$$

$$\rho C_P v_0 \frac{\partial T}{\partial y} = K_f \frac{\partial^2 T}{\partial y^2} + 2\mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right], \quad (2.3)$$

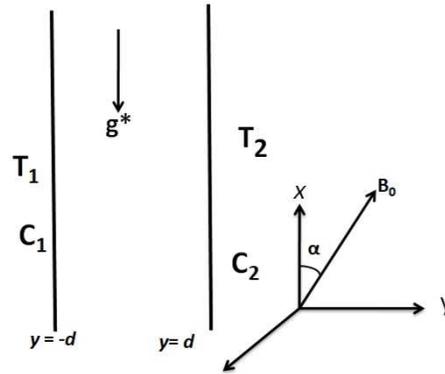


Figure 1. Physical model and coordinate system.

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (2.4)$$

with

$$u(-d) = w(-d) = 0, T(-d) = T_1, C(-d) = C_1, \quad (2.5a)$$

$$u(d) = w(d) = 0, T(d) = T_2, C(d) = C_2, \quad (2.5b)$$

where the velocity components in x, y and z are u, v, w respectively, g^* is the acceleration due to gravity, K_f is the thermal diffusion ratio, ρ is the density, C_p is the specific heat, μ is the coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, D is the mass diffusivity, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio.

Introducing the following similarity transformations

$$y = \eta d, u = u_0 f, w = u_0 g, T - T_1 = (T_2 - T_1) \theta, C - C_1 = (C_2 - C_1) \phi \quad (2.6)$$

in equations (2.1)–(2.4), we obtain the governing dimensionless equations as

$$f'' - Rf' + \frac{Gr_T}{Re} \theta + \frac{Gr_C}{Re} \phi - \frac{Ha^2 \cos \alpha}{1 + m^2 \cos^2 \alpha} [f \cos \alpha - \lambda \sin \alpha + mg \cos^2 \alpha] - A = 0, \quad (2.7)$$

$$g'' - Rg' + \frac{Ha^2 \cos^2 \alpha}{1 + m^2 \cos^2 \alpha} [mf \cos \alpha - g - m\lambda \sin \alpha] = 0, \quad (2.8)$$

$$\theta'' - RePr\theta' + 2Br [(f')^2 + (g')^2] = 0, \quad (2.9)$$

$$\phi'' - ReSc\phi' + ScSr\theta'' = 0 \quad (2.10)$$

with

$$\begin{aligned} f(-1) = g(-1) = \theta(-1) = \phi(-1) &= 0, \\ f(1) = g(1) = 0, \theta(1) = \phi(1) &= 1, \end{aligned} \quad (2.11)$$

where the primes represents differentiation with respect to η , $Re = \frac{\rho u_0 d}{\mu}$ is the Reynolds number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $Gr_T = \frac{g^* \beta_T (T_2 - T_1) d^3}{\nu^2}$ is the thermal Grashof number, $Gr_C = \frac{g^* \beta_C (C_2 - C_1) d^3}{\nu^2}$ is the Solutal Grashof number,

$R = \frac{\rho v_0 d}{\mu}$ is the suction/induction number, $Pr = \frac{\mu C_p}{K_f}$ is the Prandtl number, $Br = \frac{\mu \nu^2}{K_f d^2 (T_2 - T_1)}$ is the Brinkman number, $S_r = \frac{DK_T (T_2 - T_1)}{\nu T_m (C_2 - C_1)}$ is the thermo diffusion parameter, $A = \frac{dP}{dx}$ is the constant pressure gradient, $Ha = dB_0 \sqrt{\frac{\sigma}{\nu}}$ is the magnetic parameter and m is the Hall parameter.

The physical quantities of interest in this problem are the skin friction coefficient, heat and mass transfer rates. The shearing stress, heat, mass fluxes at the vertical plates can be obtained from

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=\pm d}; \quad q_w = -K_f \left. \frac{\partial T}{\partial y} \right|_{y=\pm d}; \quad q_m = -D \left. \frac{\partial C}{\partial y} \right|_{y=\pm d}. \tag{2.12}$$

The non-dimensional shear stress $C_f = \frac{\tau_w}{\rho u_0^2}$, the Nusselt number $Nu = \frac{q_w d}{K_f (T_2 - T_1)}$ and the Sherwood number $Sh = \frac{q_m d}{D (C_2 - C_1)}$ are given by

$$ReC_{f_1} = f'(-1); \quad ReC_{f_2} = f'(1); \quad Nu_{1,2} = -\theta'(\eta)|_{\eta=-1,1}; \quad Sh_{1,2} = -\phi'(\eta)|_{\eta=-1,1}.$$

Effect of the various parameters involved in the investigation on these coefficients is discussed in the following section.

3. THE SPECTRAL QUASI-LINEARISATION (QLM) SOLUTION OF THE PROBLEM

In this section, we introduce the quasi-linearisation (QLM) method (which is a generalized Newton- Raphson Method, that was initially used by Bellman and Kalaba [6]) for solving the governing system of equations (2.7)–(2.10) subject to the boundary conditions (2.11). In this method, the iteration scheme is obtained by linearizing the nonlinear component of a differential equation using the Taylor series expansion.

Let f_r, g_r, θ_r and ϕ_r be an approximate current solution and $f_{r+1}, g_{r+1}, \theta_{r+1}$ and ϕ_{r+1} be an improved solution of the system of eqs (2.7)–(2.10). By taking Taylor series expansion of non-linear terms in eqs (2.7)–(2.10) around the current solution and neglecting the second and higher order derivative terms, we get the linearized equations as:

$$f''_{k+1} - Rf'_{k+1} + \frac{Gr_T}{Re} \theta_{k+1} + \frac{Gr_C}{Re} \phi_{k+1} + a_1 f_{k+1} + a_2 g_{k+1} = a_3, \tag{3.1}$$

$$g''_{k+1} - Rg'_{k+1} + a_4 f_{k+1} + a_5 g_{k+1} = a_6, \tag{3.2}$$

$$\theta''_{k+1} - RePr\theta'_{k+1} + a_7 f'_{k+1} + a_8 g'_{k+1} = a_9, \tag{3.3}$$

$$\phi''_{k+1} - ReSc\phi'_{k+1} + ScSr\theta''_{k+1} = 0, \tag{3.4}$$

where the coefficients a_s ($s = 1, 2 \dots 9$) are known functions (from previous calculations) and are defined as

$$\begin{aligned} a_1 &= -\frac{Ha^2 \cos^2 \alpha}{1 + m^2 \cos^2 \alpha}, & a_2 &= -\frac{mHa^2 \cos^3 \alpha}{1 + m^2 \cos^2 \alpha}, & a_3 &= -\frac{\lambda Ha^2 \cos \alpha \sin \alpha}{1 + m^2 \cos^2 \alpha} + A, \\ a_4 &= \frac{mHa^2 \cos^3 \alpha}{1 + m^2 \cos^2 \alpha}, & a_5 &= -\frac{Ha^2 \cos^2 \alpha}{1 + m^2 \cos^2 \alpha}, & a_6 &= \frac{m\lambda Ha^2 \cos^2 \alpha \sin \alpha}{1 + m^2 \cos^2 \alpha}, \\ a_7 &= 4Brf'_k, & a_8 &= 4Br g'_k, & a_9 &= 2Br(f_k'^2 + g_k'^2). \end{aligned}$$

It must be pointed out that the above system (3.1)–(3.4) constitute a linear system of coupled differential equations with variable coefficients and can be solved iteratively using any numerical method for $r = 1, 2, 3, \dots$. In this work, as will be discussed below, the Chebyshev pseudo-spectral method was used to solve the QLM scheme (3.1)–(3.4). Starting from a given set of initial approximations $f_0, g_0, \theta_0, \phi_0$ the iteration schemes (3.1)–(3.4) can be solved iteratively for $\mathbf{F}_{r+1}(\eta)$, $\mathbf{G}_{r+1}(\eta)$, $\Theta_{r+1}(\eta)$ and $\Phi_{r+1}(\eta)$ when $r = 0, 1, 2, \dots$. To solve equation (3.1)–(3.4) we discretize the equation using the Chebyshev spectral collocation method. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix D which is used to approximate the derivatives of the unknown functions $f(\eta), g(\eta), \theta(\eta)$ and $\phi(\eta)$ at the N_x number of collocation points

$$\eta_j = \cos \frac{j\pi}{N_x}, (j = 0, 1, 2, \dots, N_x) \quad (3.5)$$

leads to the matrix equation:

$$\mathbf{A}_r \mathbf{X}_{r+1} = \mathbf{B}_r \quad (3.6)$$

where \mathbf{A}_r is a $(4N_x + 4) \times (4N_x + 4)$ square matrix and \mathbf{X}_{r+1} and \mathbf{B}_r are $(4N_x + 4) \times 1$ column vectors defined by

$$\mathbf{A}_r = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}, \quad (3.7)$$

$$\mathbf{X}_{r+1} = [\mathbf{F}_{k+1} \quad \mathbf{G}_{k+1} \quad \Theta_{k+1} \quad \Phi_{k+1}]^T, \quad (3.8)$$

$$\mathbf{B}_r = [\mathbf{K}_1 \quad \mathbf{K}_2 \quad \mathbf{K}_3 \quad \mathbf{K}_4]^T, \quad (3.9)$$

where

$$\begin{aligned} \mathbf{F}_{r+1} &= [f_{r+1}(\xi_0), f_{r+1}(\xi_1), \dots, f_{r+1}(\xi_{N_x})]^T, \\ \mathbf{G}_{r+1} &= [g_{r+1}(\xi_0), g_{r+1}(\xi_1), \dots, g_{r+1}(\xi_{N_x})]^T, \\ \Theta_{r+1} &= [\theta_{r+1}(\xi_0), \theta_{r+1}(\xi_1), \dots, \theta_{r+1}(\xi_{N_x})]^T, \\ \Phi_{r+1} &= [\phi_{r+1}(\xi_0), \phi_{r+1}(\xi_1), \dots, \phi_{r+1}(\xi_{N_x})]^T, \\ \mathbf{K}_1 &= [a_3(\xi_0), a_3(\xi_1), \dots, a_3(\xi_{N_x})]^T, \\ \mathbf{K}_2 &= [a_6(\xi_0), a_6(\xi_1), \dots, a_6(\xi_{N_x})]^T, \\ \mathbf{K}_3 &= [a_9(\xi_0), a_9(\xi_1), \dots, a_9(\xi_{N_x})]^T, \\ \mathbf{K}_4 &= O, \\ A_{11} &= D^2 - RD + a_1, A_{12} = a_2, A_{13} = (Gr_T/Re) * I, A_{14} = (Gr_C/Re) * I, \\ A_{21} &= a_4, A_{22} = D^2 - RD + a_5, A_{23} = 0, A_{24} = O, \\ A_{31} &= a_7D, A_{32} = a_8D, A_{33} = D^2 - RePrD, A_{34} = O, \\ A_{41} &= O, A_{42} = O, A_{43} = SrScD^2, A_{44} = D^2 - ReScD. \end{aligned} \quad (3.10)$$

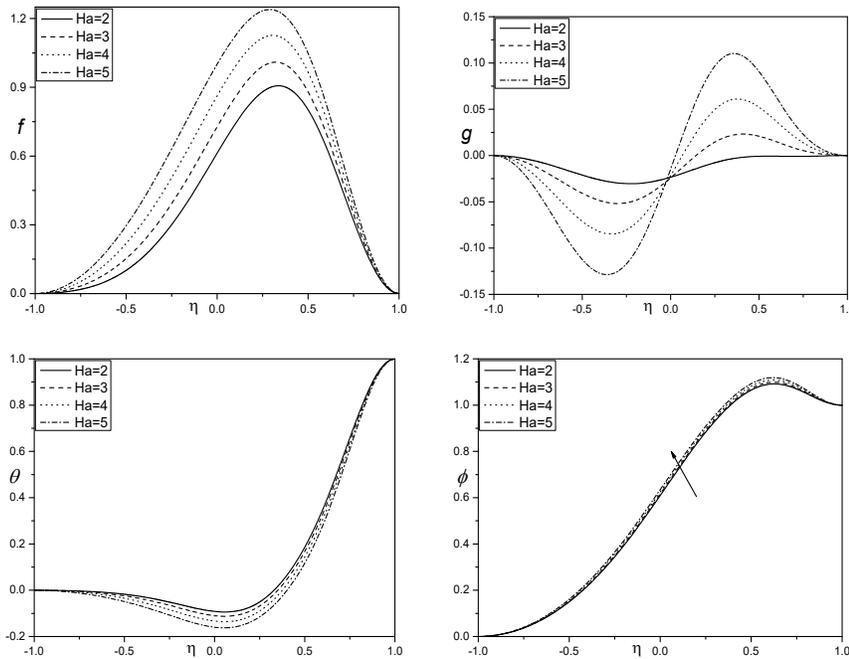


Figure 2. Effect of Ha on (a) x - velocity, (b) cross velocity, (c) temperature, and (d) concentration profiles.

Here I and O represents $(N_x + 1) \times (N_x + 1)$ identity matrix and zero matrix respectively. The approximate solutions for f, g, θ and ϕ are obtained as

$$\mathbf{X}_{r+1} = \mathbf{A}_r^{-1} \mathbf{B}_r. \tag{3.11}$$

4. Discussion of Results

The influence of Ha, m, α, Sr on velocities $(f(\eta), g(\eta))$, temperature $(\theta(\eta))$ and concentration $(\phi(\eta))$ are calculated and are explained Figs. 2 to 5 by fixing Pr, Br, Re, Sc, Gr, N, A at 0.71, 0.5, 2, 0.22, 0.5, 2, 1 respectively.

Figure 2 shows the impact of Hartmann number (Ha) on f, g, θ and ϕ when $\alpha = \pi/3, Sr=2, m=2$. It is seen from Fig. 2(a) that as Ha increases, the flow velocity increases. It is noted that the applied magnetic field has an inclination angle $\alpha > 0$ with that the drag force cannot be generated. It is identified from Fig. 2(b) that the cross flow velocity increases as Ha increases. It can depict from Figs. 2(c)-2(d) that the dimensionless temperature diminishes and concentration enhances with the increase of magnetic parameter. This is due to the fact that magnetic field generates resistive force, which leads to the decrease in the temperature.

The influence of m on f, g, θ and ϕ can be found in Fig. 3 at $Ha=2, \alpha = \pi/3, Sr=2$. It is noticed from Figs. 3(a)-3(b) that the flow velocity and the cross velocity decreases as m magnifies. This is due to the influence of inclined magnetic field. When there is an magnetic field acting with an angle $\alpha = \pi/4$, Hall current will

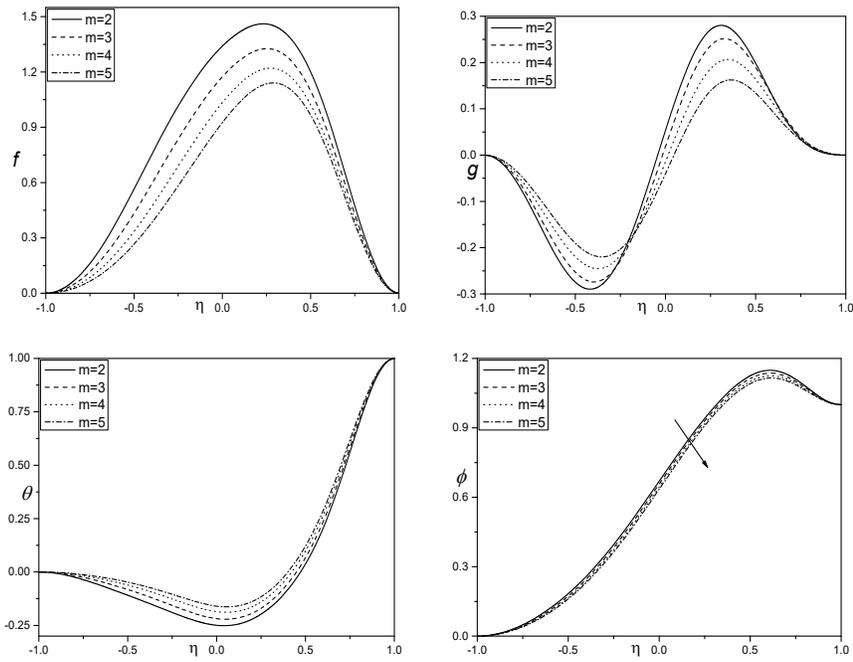


Figure 3. Effect of m on (a) x - velocity, (b) cross velocity, (c) temperature, and (d) concentration profiles.

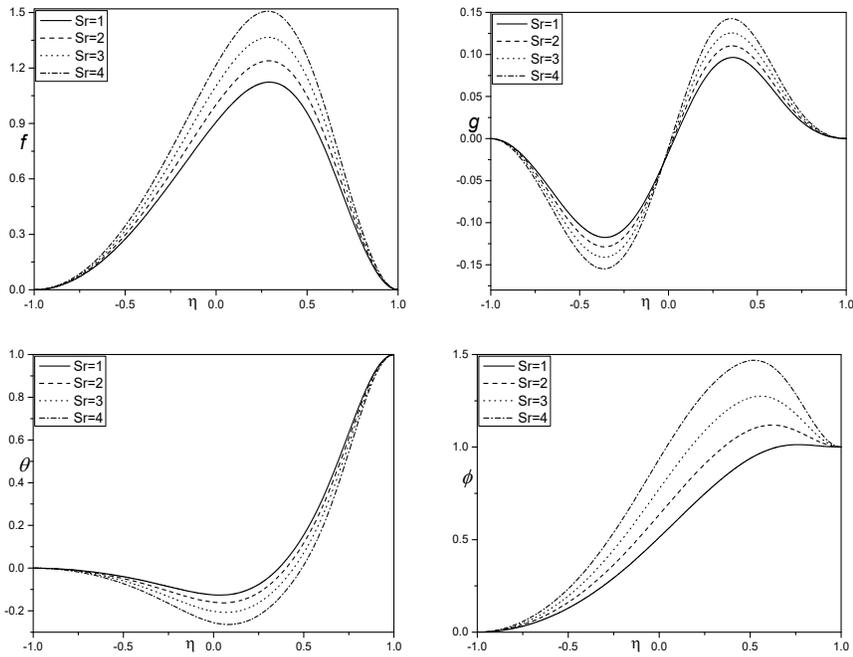


Figure 4. Influence of Sr on (a) x - velocity, (b) cross velocity, (c) temperature, and (d) concentration profiles.

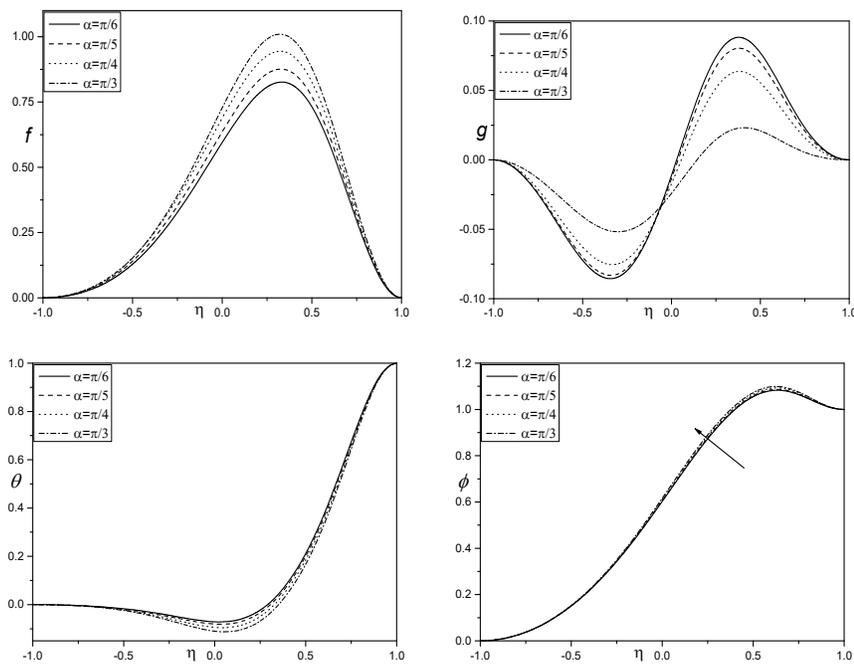


Figure 5. Influence of α on (a) x - velocity, (b) cross velocity, (c) temperature, and (d) concentration profiles.

Table 1. Nature of skin friction coefficient, heat and mass transfer rates for various values of Sr , Ha , m and α when $Re = 2.0$, $Pr = 0.71$, and $Sc = 0.22$

| Ha | Sr | α | m | C_{f_1} | C_{f_2} | Nu_1 | Nu_2 | Sh_1 | Sh_2 |
|------|------|----------|-----|-----------|-----------|---------|----------|----------|---------|
| 1 | 2 | $\pi/3$ | 2 | 0.33273 | -5.35625 | 0.05478 | -5.96633 | -0.47256 | 1.73672 |
| 2 | 2 | $\pi/3$ | 2 | 0.56288 | -5.81880 | 0.08560 | -6.30680 | -0.48303 | 1.88962 |
| 3 | 2 | $\pi/3$ | 2 | 0.91458 | -6.41771 | 0.14527 | -6.76877 | -0.50500 | 2.09717 |
| 2 | 1 | $\pi/3$ | 2 | 0.49458 | -5.23626 | 0.06347 | -5.76648 | -0.39520 | 0.44738 |
| 2 | 2 | $\pi/3$ | 2 | 0.56288 | -5.81880 | 0.08560 | -6.30680 | -0.48303 | 1.88962 |
| 2 | 3 | $\pi/3$ | 2 | 0.63720 | -6.46913 | 0.11376 | -6.99640 | -0.57703 | 3.67567 |
| 2 | 2 | 0 | 2 | 0.21150 | -4.61350 | 0.03447 | -5.38500 | -0.46905 | 1.47551 |
| 2 | 2 | $\pi/4$ | 2 | 0.47483 | -5.46649 | 0.06924 | -5.99907 | -0.47873 | 1.75132 |
| 2 | 2 | $\pi/3$ | 2 | 0.56288 | -5.81880 | 0.08560 | -6.30680 | -0.48303 | 1.88962 |
| 2 | 2 | $\pi/3$ | 1 | 0.73049 | -6.12657 | 0.11087 | -6.53852 | -0.49208 | 1.99365 |
| 2 | 2 | $\pi/3$ | 2 | 0.56288 | -5.81880 | 0.08560 | -6.30680 | -0.48303 | 1.88962 |
| 2 | 2 | $\pi/3$ | 3 | 0.44902 | -5.59808 | 0.07002 | -6.14361 | -0.47764 | 1.81635 |

be generated perpendicular to both the direction and which will act as drag on velocities. This is due to influence of inclined magnetic field. It is noted from Figs. 3(c)-3(d) that the temperature of the fluid increases and the concentration of the fluid decreases as an increase in m . As explained above the Hall current generates extra charge and which leads to increase in temperature of the fluid.

Figure 4(a) presents the influence of Soret parameter on the flow velocity $f(\eta)$ at $Ha=2, m=2, \alpha = \pi/3$. It is noted from Fig. 4(a) that the flow velocity increases with an increase in Sr . The influence of diffusion thermo parameter on azimuthal velocity $g(\eta)$ can be seen in Fig. 4(b). It is clear from this figure that the cross flow velocity increases as Soret parameter increases. This is due to the fact that an increase in Soret parameter leads to increase in temperature gradient, which leads to enhance the velocities. The impact of Soret number on dimensionless temperature can be found in figure. 4(c). Figure 4(d) depicts the effect of Soret parameter on concentration profile. These results clearly disclose that the flow field is appreciably influenced by the Soret parameter.

The effect of angle of inclination parameter α on f, g, θ and ϕ can be noted in Fig. 5 by fixing the other parameters at $Ha=2, m=2, Sr=2$. It is noticed from Fig. 5(a) and 5(b) that the flow velocity increases and cross flow velocity decreases as α increase. It is observed from Figs. 5(c) and 5(d) that the dimensionless temperature decreases and concentration increases as α increases. This is due to the reason that as an inclination angle of applied magnetic field changes (angle of inclination increases) leads to the reduction in drag force will enhance on the net flow.

Variation of Soret parameter (Sr), magnetic parameter (Ha), Hall number (m) together with the inclination angle (α) is presented in Table 1 with fixed values of other parameters. It can be seen from this table that the skin friction coefficient increases at the initial plate and decreases at the terminal plate with an increase in Ha, Sr and α , where as the reverse trend is observed on friction factor with an increase in the Hall parameter m . As the magnetic parameter increases, the resistive force slow downs the friction factor at $\eta = -1$. It is observed from this table that, heat transfer rate increases and mass transfer rate decreases at the left plate but there is a reverse trend at the right plate with an increase of magnetic parameter, Soret parameter and inclination angle but reverse trend is noticed when there is an increase in Hall parameter β_h . The behavior of these parameters are self-evident from the Table 1 and hence are not discussed for brevity. These results are clearly shows that the emerging parameters have remarkable impact on all the profiles.

5. Conclusion

The present study investigates the steady inclined manetohydrodynamic flow of Newtonian fluid in a vertical channel in presence of Hall current and Soret effects. Spectral Quasilinearization Method is used to solve the final dimensionless governing equations. The significant findings are summarized as:

- Magnetic parameter Ha and Soret parameter Sr cause an increase in the velocity profiles.
- An enhancement in the magnetic parameter leads decrease in dimensionless temperature and increase in concentration of the fluid.

- A large Soret parameter Sr causes increase in concentration and decrease in the temperature of the fluid.
- Higher values of Hall parameter m leads to decrease in velocities and concentration but increases in temperature.
- As an angle of inclination increases, the flow velocity and concentration profiles are decreases.
- Inclination angle α causes an increase in cross flow velocity and temperature of the fluid.

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