Journal of Applied Analysis and Computation
Volume 9, Number 1, February 2019, 231–244
Website:http://jaac-online.com/
DOI:10.11948/2019.231

SOME NEW SEQUENCE SPACES DERIVED BY THE COMPOSITION OF BINOMIAL MATRIX AND DOUBLE BAND MATRIX

Abdulcabbar Sönmez[1,†]

Abstract In this paper, we construct three new sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$ and mention some inclusion relations, where G is generalized difference matrix. Moreover, we give Schauder basis of the spaces $b_0^{r,s}(G)$ and $b_c^{r,s}(G)$. Afterward, we determine $\alpha-$, $\beta-$ and $\gamma-$duals of those spaces. Finally, we characterize some matrix classes related to the space $b_c^{r,s}(G)$.

Keywords Matrix transformations, matrix domain, schauder basis, $\alpha-$, $\beta-$ and $\gamma-$duals, matrix classes.

MSC(2010) 40C05, 40H05, 46B45.

1. Rudiments And Notations

The set of all real or complex valued sequences is symbolized with w. w is a vector space under point-wise addition and scalar multiplication. A sequence space is an arbitrary vector subspace of w. ℓ_∞, c_0, c and ℓ_p are symbolic of all bounded, null, convergent and absolutely p-summable sequence spaces, respectively, where $1 \leq p < \infty$.

A K-space is a sequence space X provided each of the maps $p_n : X \to \mathbb{C}$ defined by $p_n(x) = x_n$ is continuous for all $n \in \mathbb{N}$. A BK-space is a Banach space X which has the property of K-space [11].

The sequence spaces ℓ_∞, c_0 and c are $BK-$spaces according to their usual *sup-norm* defined by $\|x\|_\infty = \sup_{k\in\mathbb{N}} |x_k|$ and ℓ_p is a $BK-$ space with its *p-norm* defined by

$$\|x\|_{\ell_p} = \Big(\sum_{k=0}^{\infty} |x_k|^p \Big)^{\frac{1}{p}}$$

where $1 \leq p < \infty$.

Let $A = (a_{nk})$ be an infinite matrix with complex entries, X and Y be two sequence spaces ,and $x = (x_k) \in w$. Then, the A- transform of x is defined by

$$(Ax)_n = \sum_{k=0}^{\infty} a_{nk} x_k \tag{1.1}$$

†the corresponding author. Email somez@erciyes.edu.tr(A. Sönmez)
1Department Of Mathematics, Erciyes University, Talas Street, Melikgazi, 38039 Kayseri, Turkey

and is assumed to be convergent for all $n \in \mathbb{N}$, the domain of A is defined by

$$X_A = \Big\{x = (x_k) \in w : Ax \in X\Big\} \tag{1.2}$$

which is also a sequence space, and the class of all infinite matrices A is defined by

$$(X : Y) = \big\{A = (a_{nk}) : Ax \in Y \text{ for all } x \in X\big\}$$

[23]. An infinite matrix $A = (a_{nk})$ is called a triangle provided the entries $a_{nk} = 0$ for $k > n$ and $a_{nn} \neq 0$ for all $n, k \in \mathbb{N}$.

The spaces of all bounded and convergent series are defined by the matrix domain of the summation matrix $S = (s_{nk})$ as follows:

$$bs = (\ell_\infty)_S \text{ and } cs = c_S$$

respectively, where $S = (s_{nk})$ is defined by

$$s_{nk} = \begin{cases} 1 \ , \ 0 \leq k \leq n \\ 0 \ , \ k > n \end{cases}$$

for all $n, k \in \mathbb{N}$. Here and in what follows, unless stated otherwise,any term with negative subscript is assumed to be zero and the summation without limits runs from 0 to ∞.

The theory of matrix transformation has a great importance in the theory of summability which was obtained by Cesàro, Norlund, Borel,.... As a consequence of this, lots of authors have constructed new sequence spaces by taking advantage of the matrix domains of infinite matrices. For example: $(\ell_\infty)_{N_q}$ and c_{N_q} in [22], X_p and X_∞ in [19], $c_0(\Delta), c(\Delta)$ and $\ell_\infty(\Delta)$ in [15], $c_0(\Delta^2), c(\Delta^2)$ and $\ell_\infty(\Delta^2)$ in [12], e_0^r, e_c^r in [1], e_p^r and e_∞^rin [2] and [18], $e_0^r(\Delta)$ and $e_c^r(\Delta)$ and $e_\infty^r(\Delta)$ in [3],$e_0^r(\Delta^m)$, $e_c^r(\Delta^m)$ and $e_\infty^r(\Delta^m)$ in [20], $e_0^r(B^{(m)})$, $e_c^r(B^{(m)})$ and $e_\infty^r(B^{(m)})$ in [13], $e_0^r(\Delta, p)$, $e_c^r(\Delta, p)$ and $e_\infty(\Delta, p)$ in [14], $c_0^\lambda(G^m)$ and $c^\lambda(G^m)$ in [5], $\ell_p^\lambda(G^m)$ and $\ell_\infty^\lambda(G^m)$ in [6].

In this paper, we construct three new sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$ and mention some inclusion relations, where G is generalized difference matrix. Moreover, we give Schauder basis of the spaces $b_0^{r,s}(G)$ and $b_c^{r,s}(G)$. Afterward, we determine $\alpha-$, $\beta-$ and $\gamma-$duals of those spaces. Finally, we characterize some matrix classes related to the space $b_c^{r,s}(G)$.

2. Some New Sequence Spaces

In this part, we give some informations concerning previous studies of Binomial matrix and Euler matrix, and construct three new sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$. Furthermore, we show that the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$ are linearly isomorphic to the spaces c_0, c and ℓ_∞, respectively and mention some inclusion relations.

To define sequence spaces, the Euler matrix was first used by Altay, Başar and Mursaleen in [1] and [2]. They defined the Euler sequence spaces e_0^r and e_c^r and e_∞^r as follows:

$$e_0^r = \left\{x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k}(1-r)^{n-k} r^k x_k = 0\right\},$$

$$e_c^r = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k \ exists \right\}$$

and

$$e_\infty^r = \left\{ x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k x_k \right| < \infty \right\}.$$

Afterward, Altay and Polat defined the sequence spaces $e_0^r(\Delta)$ and $e_c^r(\Delta)$ and $e_\infty^r(\Delta)$ in [3] and improved Altay, Başar and Mursaleen's work as follows:

$$e_0^r(\Delta) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) = 0 \right\},$$

$$e_c^r(\Delta) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) \ exists \right\}$$

and

$$e_\infty^r(\Delta) = \left\{ x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \sum_{k=0}^{n} \binom{n}{k} (1-r)^{n-k} r^k (x_k - x_{k-1}) \right| < \infty \right\},$$

where Δ is difference matrix.

Recently, Bişgin has defined the Binomial sequence spaces $b_0^{r,s}$, $b_c^{r,s}$ and $b_\infty^{r,s}$ in [7], [8], [9] and [10], and has generalized Altay, Başar and Mursaleen's work as follows:

$$b_0^{r,s} = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k x_k = 0 \right\},$$

$$b_c^{r,s} = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k x_k \ exists \right\}$$

and

$$b_\infty^{r,s} = \left\{ x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k x_k \right| < \infty \right\},$$

where the Binomial matrix $B^{r,s} = (b_{nk}^{r,s})$ is defined by

$$b_{nk}^{r,s} = \begin{cases} \frac{1}{(s+r)^n} \binom{n}{k} s^{n-k} r^k, & 0 \le k \le n \\ 0, & k > n \end{cases}$$

for all $n, k \in \mathbb{N}$,$r, s \in \mathbb{R}$ and $s.r > 0$. Unless stated otherwise, we henceforth suppose that $s.r > 0$.

Here, we would like to touch on a point, if we take $s + r = 1$, we obtain the Euler sequence spaces e_0^r , e_c^r, and e_∞^r.

Afterward, Meng and Song defined the Binomial difference sequence spaces $b_0^{r,s}(\Delta)$, $b_c^{r,s}(\Delta)$ and $b_\infty^{r,s}(\Delta)$ in [17](in case of $m = 1$) and improved Bişgin's work as follows:

$$b_0^{r,s}(\Delta) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (x_k - x_{k-1}) = 0 \right\},$$

$$b_c^{r,s}(\Delta) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (x_k - x_{k-1}) \ exists \right\}$$

and

$$b_\infty^{r,s}(\Delta) = \left\{ x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (x_k - x_{k-1}) \right| < \infty \right\}.$$

Now, we define the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$ by

$$b_0^{r,s}(G) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (ux_k + vx_{k-1}) = 0 \right\},$$

$$b_c^{r,s}(G) = \left\{ x = (x_k) \in w : \lim_{n\to\infty} \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (ux_k + vx_{k-1}) \ exists \right\}$$

and

$$b_\infty^{r,s}(G) = \left\{ x = (x_k) \in w : \sup_{n\in\mathbb{N}} \left| \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (ux_k + vx_{k-1}) \right| < \infty \right\},$$

where $G = (g_{nk})$ is generalized difference matrix and is defined by

$$g_{nk} = \begin{cases} u \ , \ k = n \\ v \ , \ k = n-1 \\ 0 \ , \ otherwise \end{cases}$$

for all $n, k \in \mathbb{N}$ and $u, v \in \mathbb{R} \setminus \{0\}$. Here, if we take $u = 1$ and $v = -1$, we obtain the difference matrix Δ.

By considering the notation of (1.2) we can redefine the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$, by the matrix domain of the generalized difference matrix G as follows:

$$b_0^{r,s}(G) = (b_0^{r,s})_G, \ b_c^{r,s}(G) = (b_c^{r,s})_G \text{ and } b_\infty^{r,s}(G) = (b_\infty^{r,s})_G. \tag{2.1}$$

Moreover, by defining a triangle matrix $H^{r,s,u,v} = (h_{nk}^{r,s,u,v}) = B^{r,s}G$ such that

$$h_{nk}^{r,s,u,v} = \begin{cases} \frac{s^{n-k-1} r^k}{(r+s)^n} \left[us\binom{n}{k} + vr\binom{n}{k+1} \right] & , \ 0 \le k \le n \\ 0 & , \ k > n \end{cases}$$

for all $n, k \in \mathbb{N}$, the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$, and $b_\infty^{r,s}(G)$ can be rearranged by means of the $H^{r,s,u,v} = (h_{nk}^{r,s,u,v})$ matrix as follows:

$$b_0^{r,s}(G) = (c_0)_{H^{r,s,u,v}} \ , \ b_c^{r,s}(G) = c_{H^{r,s,u,v}} \text{ and } b_\infty^{r,s}(G) = (\ell_\infty)_{H^{r,s,u,v}}. \tag{2.2}$$

In this way ,for a given arbitrary sequence $x = (x_k)$, the $H^{r,s,u,v}-$transform of x is defined by

$$y_k = (H^{r,s,u,v}x)_k = \frac{1}{(r+s)^k} \sum_{i=0}^{k} \binom{k}{i} s^{k-i} r^i (ux_i + vx_{i-1}) \tag{2.3}$$

for all $k \in \mathbb{N}$, or, by considering another representation , the sequence $y = (y_k)$ can rewritten as follows:

$$y_k = (H^{r,s,u,v}x)_k = \frac{1}{(r+s)^k} \sum_{i=0}^{k} \left[us\binom{k}{i} + vr\binom{k}{i+1} \right] s^{k-i-1} r^i x_i \tag{2.4}$$

for all $k \in \mathbb{N}$.

Theorem 2.1. *The sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$, and $b_\infty^{r,s}(G)$ are $BK-$spaces in accordance with their norms defined by*

$$\|x\|_{b_0^{r,s}(G)} = \|x\|_{b_c^{r,s}(G)} = \|x\|_{b_\infty^{r,s}(G)} = \|(H^{r,s,u,v}x)_k\|_\infty = \sup_{k\in\mathbb{N}} |(H^{r,s,u,v}x)_k|.$$

Proof. we know already that the spaces c_0 , c and ℓ_∞ are BK-spaces with the norm $\|x\|_\infty = \sup_{k\in\mathbb{N}} |x_k|$, $H^{r,s,u,v} = (h_{nk}^{r,s,u,v})$ is a triangle matrix and the state (2.2) holds. If we connect these results with Theorem 4.3.12 of Wilansky [23], we obtain that the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$, and $b_\infty^{r,s}(G)$ are $BK-$spaces. This completes the proof of the theorem. □

Theorem 2.2. *The sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$, and $b_\infty^{r,s}(G)$ are linearly isomorphic to the sequence spaces c_0, c and ℓ_∞ , respectively, namely, $b_0^{r,s}(G) \cong c_0$, $b_c^{r,s}(G) \cong c$ and $b_\infty^{r,s}(G) \cong \ell_\infty$.*

Proof. To keep away from the usage of similar statements, the proof of theorem is given for only the sequence space $b_0^{r,s}(G)$. For this purpose, we should show the existence of a linear bijection between the spaces $b_0^{r,s}(G)$ and c_0 .Consider the transformation L defined by $L : b_0^{r,s}(G) \longrightarrow c_0$, $L(x) = H^{r,s,u,v}x$. Then, according to definition of the transformation L , it is obvious that $L(x) = H^{r,s,u,v}x \in c_0$ for all $x \in b_0^{r,s}(G)$. Moreover, it is trivial that L is linear and $x = 0$ whenever $L(x) = 0$. Therefore, L is injective.

For a given arbitrary sequence $y = (y_k) \in c_0$, we define the sequence $x = (x_n)$ by

$$x_n = \frac{1}{u} \sum_{k=0}^{n} \left[\sum_{i=k}^{n} \binom{i}{k} \Big(-\frac{v}{u} \Big)^{n-i} (-s)^{i-k} (r+s)^k r^{-i} \right] y_k$$

for all $n \in \mathbb{N}$. Then, we get

$$\begin{aligned} (H^{r,s,u,v}x)_n &= \frac{1}{(r+s)^n} \sum_{k=0}^{n} \tbinom{n}{k} s^{n-k} r^k (ux_k + vx_{k-1}) \\ &= \frac{1}{(r+s)^n} \sum_{k=0}^{n} \tbinom{n}{k} s^{n-k} r^k \sum_{j=0}^{k} \binom{k}{j} (-s)^{k-j} (r+s)^j r^{-k} y_j \\ &= y_n \end{aligned}$$

for all $n \in \mathbb{N}$, that is

$$\lim_{n\to\infty} (H^{r,s,u,v}x)_n = \lim_{n\to\infty} y_n = 0.$$

Therefore, we obtain that $x = (x_k) \in b_0^{r,s}(G)$ and $L(x) = y$, namely L is surjective. Furthermore, we have for every $x \in b_0^{r,s}(G)$ that

$$\|L(x)\|_\infty = \|H^{r,s,u,v}x\|_\infty = \|x\|_{b_0^{r,s}(G)}.$$

So, L is norm preserving. Consequently, L is a linear bijection. This fact shows us that the sequence spaces $b_0^{r,s}(G)$ and c_0 are linearly isomorphic. This completes the proof. $\square$

Theorem 2.3. *The inclusions $\hat{c_0} \subset b_0^{r,s}(G)$, $\hat{c} \subset b_c^{r,s}(G)$ and $\hat{\ell_\infty} \subset b_\infty^{r,s}(G)$ are strict, where $\hat{c_0}$, $\hat{c}$ and $\hat{\ell_\infty}$ are defined in [16].*

Proof. To avoid the repetition of similar expression, we give the proof of theorem for only the inclusion $\hat{\ell_\infty} \subset b_\infty^{r,s}(G)$.

For a given arbitrary sequence $x = (x_k) \in \hat{\ell_\infty}$, we have that

$$\begin{aligned}
\|x\|_{b_\infty^{r,s}(G)} &= \|H^{r,s,u,v}x\|_\infty \\
&= \sup_{n\in\mathbb{N}} \left| \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k (ux_k + vx_{k-1}) \right| \\
&\leq \sup_{n\in\mathbb{N}} |ux_n + vx_{n-1}| . \sup_{n\in\mathbb{N}} \left| \frac{1}{(r+s)^n} \sum_{k=0}^{n} \binom{n}{k} s^{n-k} r^k \right| \\
&= \|x\|_{\hat{\ell_\infty}}.
\end{aligned}$$

This means that $x = (x_k) \in b_\infty^{r,s}(G)$, namely the inclusion $\hat{\ell_\infty} \subset b_\infty^{r,s}(G)$ holds. Now we define a sequence $x = (x_k)$ such that $x_k = \frac{1}{u} \sum\limits_{i=0}^{k} \big(-\frac{v}{u}\big)^{k-i} \big(-\frac{s+r}{r}\big)^i$ for all $k \in \mathbb{N}$. Then $Gx = ((-\dfrac{s+r}{r})^k) \notin \ell_\infty$ but $H^{r,s,u,v}x = ((-\dfrac{r}{r+s})^k) \in \ell_\infty$. As a consequence, $x = (x_k) \in b_\infty^{r,s}(G) \backslash \hat{\ell_\infty}$. This shows that the inclusion $\hat{\ell_\infty} \subset b_\infty^{r,s}(G)$ is strict. This completes the proof. $\square$

Theorem 2.4. *The inclusions $b_0^{r,s}(G) \subset b_c^{r,s}(G) \subset b_\infty^{r,s}(G)$ strictly hold.*

Proof. It is well known that every null sequence is also convergent and every convergent sequence is also bounded.So, the inclusions $b_0^{r,s}(G) \subset b_c^{r,s}(G) \subset b_\infty^{r,s}(G)$ hold. Now we define two sequences $x = (x_k)$ and $y = (y_k)$ such that $x_k = \frac{1-\left(-\frac{v}{u}\right)^{k+1}}{u+v}$ and $y_k = \frac{1}{u} \sum\limits_{i=0}^{k} \big(-\frac{v}{u}\big)^{k-i} \big(-\frac{r+2s}{r}\big)^i$ for all $k \in \mathbb{N}$. Then we can observe that $H^{r,s,u,v}x = e \in c \backslash c_0$ and $H^{r,s,u,v}y = \Big((-1)^k\Big) \in \ell_\infty \backslash c$, namely $x = (x_k) \in b_c^{r,s}(G) \backslash b_0^{r,s}(G)$and $y = (y_k) \in b_\infty^{r,s}(G) \backslash b_c^{r,s}(G)$. These two facts show that the inclusions $b_0^{r,s}(G) \subset b_c^{r,s}(G) \subset b_\infty^{r,s}(G)$ are strict. This completes the proof. $\square$

Theorem 2.5. *$c \subset b_0^{r,s}(G)$ strictly holds, whenever $u + v = 0$.*

Proof. It is obvious that $Gx \in c_0$ whenever $x \in c$. Also, the Binomial matrix is regular when $r.s > 0$. If we combine these two facts, we obtain that $B^{r,s}Gx \in c_0$ whenever $x \in c$, namely $x \in b_0^{r,s}(G)$ whenever $x \in c$. So , the inclusion $c \subset b_0^{r,s}(G)$ holds. Now we define a sequence $x = (x_k)$ such that $x_k = (-1)^k \left[\frac{1-\left(\frac{v}{u}\right)^{k+1}}{u-v}\right]$ for all$k \in \mathbb{N}$. Then, we can see that $x = (x_k) \notin c$ but $H^{r,s,u,v}x = ((\dfrac{s-r}{s+r})^k) \in c_0$, that is $x \in b_0^{r,s}(G)$. This result shows that the inclusion $c \subset b_0^{r,s}(G)$ is strict. This completes the proof. $\square$

3. The Schauder Basis And $\alpha-$, $\beta-$ and $\gamma-$Duals

In this part, we give the Schauder basis of the Binomial difference sequence spaces $b_0^{r,s}(G)$ and $b_c^{r,s}(G)$. Moreover we determine $\alpha-$, $\beta-$ and $\gamma-$ duals of the sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$.

A sequence $u = (u_k)$ in the sequence space X is called a Schauder basis for a normed space $(X, \|.\|_X)$ if, for every $x = (x_k) \in X$ there exists a unique sequence (λ_k) of scalars such that $x = \sum_k \lambda_k u_k$; i.e. such that

$$\lim_{n\to\infty} \left\| x - \sum_{k=0}^{n} \lambda_k u_k \right\|_X \longrightarrow 0.$$

Theorem 3.1. *Let $\xi_k = (H^{r,s,u,v}x)_k$ for all $k \in \mathbb{N}$. For all fixed $k \in \mathbb{N}$, consider the sequences $d = (d_k)$ defined by $d_k = \frac{1-\left(-\frac{v}{u}\right)^{k+1}}{u+v}$ and $d^{(k)}(r,s,u,v) = \left\{ d_n^{(k)}(r,s,u,v) \right\}_{n\in\mathbb{N}}$ defined by*

$$d_n^{(k)}(r,s,u,v) = \begin{cases} 0, & 0 \le n < k, \\ \frac{1}{u} \sum\limits_{i=k}^{n} \binom{i}{k} \left(-\frac{v}{u}\right)^{n-i} (-s)^{i-k} (r+s)^k r^{-i}, & k \le n. \end{cases}$$

Then the following hold:
(a) The Schauder basis of the sequence space $b_0^{r,s}(G)$ is the sequence $\{d^{(k)}(r,s,u,v)\}_{k\in\mathbb{N}}$ and all $x = (x_k) \in b_0^{r,s}(G)$ can be uniquely written

$$x = \sum_k \xi_k d^{(k)}(r,s,u,v).$$

(b) The Schauder basis of the sequence space $b_c^{r,s}(G)$ is the set $\left\{ d, d^{(0)}(r,s,u,v), d^{(1)}(r,s,u,v), \dots \right\}$ and all $x = (x_k) \in b_c^{r,s}(G)$ can be uniquely written

$$x = ld + \sum_k [\xi_k - l] d^{(k)}(r,s,u,v),$$

where $l = \lim_{k\to\infty} (H^{r,s,u,v}x)_k$.

Proof. One can easily see that $H^{r,s,u,v} d^{(k)}(r,s,u,v) = e^{(k)} \in c_0$ for all $k \in \mathbb{N}$, where $e^{(k)}$ is a sequence with 1 in the k *th* place and zeros elsewhere. Then we conclude that the inclusion $\{d^{(k)}(r,s,u,v)\} \subset b_0^{r,s}(G)$ holds .

Let $x = (x_k) \in b_0^{r,s}(G)$. We write

$$x^{[m]} = \sum_{k=0}^{m} \xi_k d^{(k)}(r,s,u,v)$$

for all $m \in \mathbb{N}$. Then, by applying the matrix $H^{r,s,u,v} = (h_{nk}^{r,s,u,v})$ to $x^{[m]}$, we get

$$H^{r,s,u,v} x^{[m]} = \sum_{k=0}^{m} \xi_k H^{r,s,u,v} d^{(k)}(r,s,u,v) = \sum_{k=0}^{m} (H^{r,s,u,v}x)_k e^{(k)}$$

and

$$\{H^{r,s,u,v}(x-x^{[m]})\}_n = \begin{cases} 0 & , 0 \le n \le m \\ (H^{r,s,u,v}x)_n & , n > m \end{cases}$$

for all $n, m \in \mathbb{N}$. For every $\epsilon > 0$ there exist $m_0 = m_0^{(\epsilon)} \in \mathbb{N}$ such that

$$|(H^{r,s,u,v}x)_m| < \frac{\epsilon}{2}$$

for all $m_0 \le m$. On account of this

$$\|x - x^{[m]}\|_{b_0^{r,s}(G)} = \sup_{m \le n} \left|(H^{r,s,u,v}x)_n\right| \le \sup_{m_0 \le n} \left|(H^{r,s,u,v}x)_n\right| \le \frac{\epsilon}{2} < \epsilon$$

for all $m_0 \le m$. This gives us that

$$x = \sum_k \xi_k d^{(k)}(r,s,u,v).$$

Now, we should show the uniqueness of this representation. We suppose that there exist an another representation of $x = (x_k)$ such that

$$x = \sum_k \mu_k d^{(k)}(r,s,u,v).$$

Then, by the continuity of the transformation, L defined in the proof of theorem 2.2 , we have

$$(H^{r,s,u,v}x)_n = \sum_k \mu_k \left[H^{r,s,u,v} d^{(k)}(r,s,u,v)\right]_n = \sum_k \mu_k e_n^{(k)} = \mu_n$$

for all $n \in \mathbb{N}$. This equality is in contradiction with the fact that $(H^{r,s,u,v}x)_n = \xi_n$ for all $n \in \mathbb{N}$. Therefore, all $x = (x_k) \in b_0^{r,s}(G)$ has a unique representation.

(b) From the part (a) we know that $\left\{d^{(k)}(r,s,u,v)\right\} \subset b_0^{r,s}(G)$ and also $H^{r,s,u,v}d = e \in c$. Thus, the inclusion $\{d, d^{(k)}(r,s,u,v)\} \subset b_c^{r,s}(G)$ clearly holds. Given an arbitrary $x = (x_k) \in b_c^{r,s}(G)$, we constract a sequence $y = (y_k)$ such that $y = x - ld$, where $l = \lim_{k\to\infty} \xi_k$. Then it is clear that $y = (y_k) \in b_0^{r,s}(G)$ and by the part (a) $y = (y_k)$ has a unique representation. This leads us to $x = (x_k)$ has a unique representation of the form

$$x = ld + \sum_k [\xi_k - l] d^{(k)}(r,s,u,v).$$

This completes the proof of the theorem. □

If we combine Theorem 2.1 and Theorem 3.1, we can give the next corollary.

Corollary 3.1. *The sequence spaces $b_0^{r,s}(G)$ and $b_c^{r,s}(G)$ are separable.*

A set defined by

$$M(X,Y) = \left\{a = (a_k) \in w : ax = (a_k x_k) \in Y \, for \ all \ x = (x_k) \in X\right\}$$

is called the multiplier space of the sequence spaces X and Y. Then, the $\alpha-$, $\beta-$ and $\gamma-$duals of the sequence space X are defined by the aid of the notion of multiplier space such that

$$X^{\alpha} = M(X, \ell_1), \quad X^{\beta} = M(X, cs) \ \text{ and } \ X^{\gamma} = M(X, bs),$$

respectively.

Now, we continue with to quote lemma from Stieglitz and Tietz [21] which are needed in the next.

$$\sup_{K\in\mathcal{F}} \sum_n \Big| \sum_{k\in K} a_{nk} \Big| < \infty, \tag{3.1}$$

$$\sup_{n\in\mathbb{N}} \sum_k |a_{nk}| < \infty, \tag{3.2}$$

$$\lim_{n\to\infty} \sum_k |a_{nk}| = \sum_k |\lim_{n\to\infty} a_{nk}|, \tag{3.3}$$

$$\lim_{n\to\infty} a_{nk} = \mu_k \ \text{ for all } \ k \in \mathbb{N}, \tag{3.4}$$

$$\lim_{n\to\infty} \sum_k a_{nk} = \mu, \tag{3.5}$$

where $\mathcal{F}$ represents the set of all finite subsets of $\mathbb{N}$.

Lemma 3.1 ([21]). *Let $A = (a_{nk})$ be an infinite matrix. Then the following statements hold:*

(i) $A = (a_{nk}) \in (c_0 : \ell_1) = (c : \ell_1) = (\ell_\infty : \ell_1) \Leftrightarrow (3.1)$ *holds*

(ii) $A = (a_{nk}) \in (c_0 : \ell_\infty) = (c : \ell_\infty) = (\ell_\infty : \ell_\infty) \Leftrightarrow (3.2)$ *holds*

(iii) $A = (a_{nk}) \in (c_0 : c) \Leftrightarrow (3.2)$ *and* (3.4) *hold*

(iv) $A = (a_{nk}) \in (c : c) \Leftrightarrow (3.2)$, (3.4) *and* (3.5) *hold*

(v) $A = (a_{nk}) \in (\ell_\infty : c) \Leftrightarrow (3.3)$ *and* (3.4) *hold*

(vi) $A = (a_{nk}) \in (c : c_0) \Leftrightarrow (3.2)$, (3.4) *and* (3.5) *hold with* $\mu_k = 0, \forall k \in \mathbb{N}$ *and* $\mu = 0$

Theorem 3.2. *The α- dual of the Binomial sequence spaces $b_0^{r,s}(G)$, $b_c^{r,s}(G)$ and $b_\infty^{r,s}(G)$ is the set*

$$d_1^{r,s,u,v} = \left\{ a = (a_k) \in w : \sup_{K\in\mathcal{F}} \sum_n \Big| \sum_{k\in K} \frac{1}{u} \sum_{i=k}^{n} \binom{i}{k} \left(-\frac{v}{u}\right)^{n-i} (-s)^{i-k} (r+s)^k r^{-i} a_n \Big| < \infty \right\}.$$

Proof. For given $a = (a_n) \in w$, by bearing in mind the sequence that is defined in the proof of Theorem 2.2, we can write

$$a_n x_n = \sum_{k=0}^{n} \left[\frac{1}{u} \sum_{i=k}^{n} \binom{i}{k} \left(-\frac{v}{u}\right)^{n-i} (-s)^{i-k} (r+s)^k r^{-i} a_n \right] y_k = \sum_{k=0}^{n} u_{nk}^{r,s,u,v} y_k = (U^{r,s,u,v} y)_n$$

for all $n \in \mathbb{N}$. Then, $ax = (a_n x_n) \in \ell_1$ whenever $x = (x_k) \in b_0^{r,s}(G)$, $b_c^{r,s}(G)$ or $b_\infty^{r,s}(G)$ if and only if $U^{r,s,u,v} y \in \ell_1$ whenever $y = (y_k) \in c_0$, c or ℓ_∞. This shows us that $a = (a_n) \in \left\{ b_0^{r,s}(G) \right\}^\alpha = \left\{ b_c^{r,s}(G) \right\}^\alpha = \left\{ b_\infty^{r,s}(G) \right\}^\alpha$ if and only if

$U^{r,s,u,v} \in (c_0 : \ell_1) = (c : \ell_1) = (\ell_\infty : \ell_1)$. By combining this result and Lemma 3.1 (i), we deduce that

$$a=(a_n)\in\Big\{b_0^{r,s}(G)\Big\}^\alpha \Leftrightarrow \sup_{K\in\mathcal{F}}\sum_n\Big|\sum_{k\in K}\frac{1}{u}\sum_{i=k}^n\binom{i}{k}\Big(-\frac{v}{u}\Big)^{n-i}(-s)^{i-k}(r+s)^k r^{-i}a_n\Big|<\infty.$$

This means that $\Big\{b_0^{r,s}(G)\Big\}^\alpha = \Big\{b_c^{r,s}(G)\Big\}^\alpha = \Big\{b_\infty^{r,s}(G)\Big\}^\alpha = d_1^{r,s,u,v}$. This completes the proof of theorem. $\square$

Theorem 3.3. *Let four sets $d_2^{r,s,u,v}, d_3^{r,s,u,v}, d_4^{r,s,u,v}$ and $d_5^{r,s,u,v}$ be given as follows:*

$$d_2^{r,s,u,v} = \left\{a = (a_k) \in w : \sup_{n\in\mathbb{N}}\sum_k |v_{nk}^{r,s,u,v}| < \infty\right\},$$

$$d_3^{r,s,u,v} = \left\{a = (a_k) \in w : \lim_{n\to\infty} v_{nk}^{r,s,u,v} \ \textit{exists for all} \ \ k \in \mathbb{N}\right\},$$

$$d_4^{r,s,u,v} = \left\{a = (a_k) \in w : \lim_{n\to\infty}\sum_k |v_{nk}^{r,s,u,v}| = \sum_k |\lim_{n\to\infty} v_{nk}^{r,s,u,v}|\right\}$$

and

$$d_5^{r,s,u,v} = \left\{a = (a_k) \in w : \lim_{n\to\infty}\sum_k v_{nk}^{r,s,u,v} \ \textit{exists}\right\},$$

where the matrix $V^{r,s,u,v} = (v_{nk}^{r,s,u,v})$ is defined by means of the sequence $a = (a_n)$ by

$$v_{nk}^{r,s,u,v} = \begin{cases} \frac{1}{u}\sum\limits_{i=k}^n\sum\limits_{j=k}^i\binom{j}{k}\Big(-\frac{v}{u}\Big)^{i-j}(-s)^{j-k}(r+s)^k r^{-j}a_i &, 0 \le k \le n \\ 0 &, k > n \end{cases}$$

for all $n, k \in \mathbb{N}$. Then, the following hold:

(i) $\Big\{b_0^{r,s}(G)\Big\}^\beta = d_2^{r,s,u,v} \cap d_3^{r,s,u,v}$;

(ii) $\Big\{b_c^{r,s}(G)\Big\}^\beta = d_2^{r,s,u,v} \cap d_3^{r,s,u,v} \cap d_5^{r,s,u,v}$;

(iii) $\Big\{b_\infty^{r,s}(G)\Big\}^\beta = d_3^{r,s,u,v} \cap d_4^{r,s,u,v}$;

(iv) $\Big\{b_0^{r,s}(G)\Big\}^\gamma = \Big\{b_c^{r,s}(G)\Big\}^\gamma = \Big\{b_\infty^{r,s}(G)\Big\}^\gamma = d_2^{r,s,u,v}$.

Proof. Because of the parts (ii), (iii) and (iv) of theorem can be proved by using a similar way, we give the proof of theorem for only the part (i). Let $a = (a_n) \in w$ be given. Then by taking into account the sequence $x = (x_k)$ defined in the proof of Theorem 2.2 , we obtain

$$\begin{aligned}\sum_{k=0}^n a_k x_k &= \sum_{k=0}^n\Big[\frac{1}{u}\sum_{i=0}^k\sum_{j=i}^k\binom{j}{i}\Big(-\frac{v}{u}\Big)^{k-j}(-s)^{j-i}(r+s)^i r^{-j}y_i\Big]a_k \\ &= \sum_{k=0}^n\Big[\frac{1}{u}\sum_{i=k}^n\sum_{j=k}^i\binom{j}{k}\Big(-\frac{v}{u}\Big)^{i-j}(-s)^{j-k}(r+s)^k r^{-j}a_i\Big]y_k = (V^{r,s,u,v}y)_n\end{aligned}$$

for all $n, k \in \mathbb{N}$. Then, $ax = (a_n x_n) \in cs$ whenever $x = (x_k) \in b_0^{r,s}(G)$ if and only if $V^{r,s,u,v} y \in c$ whenever $y \in c_0$. This result show us that $a = (a_k) \in \left\{b_0^{r,s}(G)\right\}^{\beta}$ if and only if $V^{r,s,u,v} \in (c_0 : c)$. By combining this result and Lemma 3.1 (iii), we deduce that $a = (a_k) \in \left\{b_0^{r,s}(G)\right\}^{\beta}$ if and only if

$$\sup_{n\in\mathbb{N}} \sum_k |v_{nk}^{r,s,u,v}| < \infty$$

and

$$\lim_{n\to\infty} v_{nk}^{r,s,u,v} \text{ exists, } \quad \text{for all} \quad k \in \mathbb{N}$$

namely, $\left\{b_0^{r,s}(G)\right\}^{\beta} = d_2^{r,s,u,v} \cap d_3^{r,s,u,v}$. This completes the proof of theorem. □

4. The Matrix Transformations

In this part, we characterize some matrix classes related to the Binomial difference sequence space $b_c^{r,s}(G)$.

Now we give a lemma which is needed in the next corollaries.

Lemma 4.1 ([4]). *Let X, Y be any two sequence spaces, A be an infinite matrix and E be a triangle matrix. Then, $A \in (X : Y_E) \Leftrightarrow EA \in (X : Y)$.*

For simplicity of notation, we use the equalities below throughout the section 4.

$$d_{nk}^{r,s,u,v} = \frac{1}{u} \sum_{i=k}^{\infty} \sum_{j=k}^{i} \binom{j}{k} \Big(- \frac{v}{u} \Big)^{i-j} (-s)^{j-k} (r+s)^k r^{-j} a_{ni}$$

for all $n, k \in \mathbb{N}$.

Theorem 4.1. *$A \in (b_c^{r,s}(G) : \ell_\infty)$ if and only if*

$$\sup_{n\in\mathbb{N}} \sum_k \left| d_{nk}^{r,s,u,v} \right| < \infty, \tag{4.1}$$

$$d_{nk}^{r,s,u,v} \ exist\ for\ all\ n, k \in \mathbb{N}, \tag{4.2}$$

$$\sup_{m\in\mathbb{N}} \sum_k \Big| \frac{1}{u} \sum_{i=k}^{m} \sum_{j=k}^{i} \binom{j}{k} \Big(- \frac{v}{u} \Big)^{i-j} (-s)^{j-k} (r+s)^k r^{-j} a_{ni} \Big| < \infty \ (\ m \in \mathbb{N}), \tag{4.3}$$

$$\lim_{m\to\infty} \frac{1}{u} \sum_{i=k}^{m} \sum_{j=k}^{i} \binom{j}{k} \Big(- \frac{v}{u} \Big)^{i-j} (-s)^{j-k} (r+s)^k r^{-j} a_{ni} \ exist\ for\ all\ m \in \mathbb{N}. \tag{4.4}$$

Proof. Assume that $A \in (b_c^{r,s}(G) : \ell_\infty)$. Then, it is clear that Ax exists and belongs to ℓ_∞ for every $x = (x_k) \in b_c^{r,s}(G)$.This leads us to $\{a_{nk}\}_{k\in\mathbb{N}} \in \left\{b_c^{r,s}(G)\right\}^{\beta}$ for all $n \in \mathbb{N}$. By combining this fact and Theorem 3.3 (ii), we conclude that the conditions (4.2), (4.3) and (4.4) hold. If we consider the fact that $x = \left(\frac{1 - \left(-\frac{v}{u}\right)^{k+1}}{u+v} \right) \in b_c^{r,s}(G)$ and $Ax \in \ell_\infty$ for all $x \in b_c^{r,s}(G)$, one can see that the condition (4.1) holds.

On the contrary assume that the conditions (4.1)-(4.4) hold.Let us take an arbitrary $x = (x_k) \in b_c^{r,s}(G)$ and take into account the equality

$$\sum_{k=0}^{m} a_{nk}x_k = \sum_{k=0}^{m} \Big[\frac{1}{u}\sum_{i=0}^{k}\sum_{j=i}^{k}\binom{j}{i}\Big(-\frac{v}{u}\Big)^{k-j}(-s)^{j-i}(r+s)^i r^{-j} y_i\Big] a_{nk},$$

$$\sum_{k=0}^{m} a_{nk}x_k = \frac{1}{u}\sum_{k=0}^{m}\sum_{i=k}^{m}\Big[\sum_{j=k}^{i}\binom{j}{k}\Big(-\frac{v}{u}\Big)^{i-j}(-s)^{j-k}(r+s)^k r^{-j}\Big] a_{ni}y_k \tag{4.5}$$

for all $m, n \in \mathbb{N}$. Under our assumption if we take limit (4.5) side by side as $m \to \infty$ we obtain that

$$\sum_k a_{nk}x_k = \sum_k d_{nk}^{r,s,u,v} y_k \tag{4.6}$$

for all $n \in \mathbb{N}$.Also by taking sup-norm (4.6) side by side, we have

$$\|Ax\|_\infty \leq \sup_{n\in\mathbb{N}}\sum_k |d_{nk}^{r,s,u,v}||y_k| \leq \|y\|_\infty . \sup_{n\in\mathbb{N}}\sum_k |d_{nk}^{r,s,u,v}| < \infty.$$

Therefore $Ax \in \ell_\infty$, namely $A \in (b_c^{r,s}(G) : \ell_\infty)$. This completes the proof of theorem. □

Theorem 4.2. *$A \in (b_c^{r,s}(G) : c)$ if and only if the conditions (4.1) - (4.4) hold, and*

$$\lim_{n\to\infty}\sum_k d_{nk}^{r,s,u,v} = \lambda, \tag{4.7}$$

$$\lim_{n\to\infty} d_{nk}^{r,s,u,v} = \lambda_k \quad \textit{for all} \quad k \in \mathbb{N}. \tag{4.8}$$

Proof. Assume that $A \in (b_c^{r,s}(G) : c)$. It is known that the inclusion $c \subset \ell_\infty$ holds. By combining the fact and Theorem 4.1, we deduce that the conditions (4.1)–(4.4) hold. Also it is obvious that Ax exists and belongs to c for all $x = (x_k) \in b_c^{r,s}(G)$. Under this fact, if we choose two sequences $x = \left(\frac{1-\left(-\frac{v}{u}\right)^{k+1}}{u+v}\right)$ and $x = d^{(k)}(r,s,u,v)$, we obtain that the conditions (4.7) and (4.8) hold, where the sequence $x = d^{(k)}(r,s,u,v)$ is defined in the Theorem 3.1.

On the contrary, for a given $x = (x_k) \in b_c^{r,s}(G)$, assume that the conditions (4.1)–(4.4), (4.7) and (4.8) hold. Then by considering Theorem 3.3 (ii), one can say that $\{a_{nk}\}_{k\in\mathbb{N}} \in \left\{b_c^{r,s}(G)\right\}^\beta$ for all $n \in \mathbb{N}$. This implies that Ax exists. From the conditions (4.1) and (4.8), we deduce that

$$\sum_{k=0}^{m} |\lambda_k| \leq \sup_{n\in\mathbb{N}}\sum_k |d_{nk}^{r,s,u,v}| < \infty$$

for every $m \in \mathbb{N}$. This shows us that $(\lambda_k) \in \ell_1$. So the series $\sum_k \lambda_k y_k$ absolute converges.

Now, we substitute $a_{nk} - \lambda_k$ instead of a_{nk} in the condition (4.6). Then, we have

$$\sum_k (a_{nk}-\lambda_k)x_k = \sum_k \frac{1}{u}\sum_{i=k}^{\infty}\sum_{j=i}^{k}\binom{j}{i}\Big(-\frac{v}{u}\Big)^{k-j}(-s)^{j-i}(r+s)^i r^{-j}\ (a_{ni}-\lambda_i)y_k \tag{4.9}$$

for all $n \in \mathbb{N}$. If we combine (4.9) and Lemma 3.1 (vi), we obtain

$$\lim_{n\to\infty} \sum_k (a_{nk} - \lambda_k) x_k = 0. \tag{4.10}$$

Lastly, if we unite the condition (4.10) and the fact $(\lambda_k y_k) \in \ell_1$, we conclude that $Ax \in c$,that is $A \in (b_c^{r,s}(G) : c)$. This completes the proof of theorem. □

Now we can give some more results by taking into account the Lemma 4.1.

Corollary 4.1. *Let us take $E = (e_{nk})$ instead of $A = (a_{nk})$ in the needed ones in Theorems 4.1 and 4.2, where $E = (e_{nk})$ is defined by*

$$e_{nk} = a_{nk} - a_{n+1,k}$$

for all $n, k \in \mathbb{N}$. Then, the necessary and sufficient conditions in order for $A = (a_{nk})$ to belong to any one of the classes $(b_c^{r,s}(G) : \ell_\infty(\Delta))$ and $(b_c^{r,s}(G) : c(\Delta))$ are obtained.

Corollary 4.2. *Let us take $Z^{\sigma,\mu} = (z_{nk}^{\sigma,\mu})$ instead of $A = (a_{nk})$ in the needed ones in Theorems 4.1 and 4.2, where $Z^{\sigma,\mu} = (z_{nk}^{\sigma,\mu})$ is defined by*

$$z_{nk}^{\sigma,\mu} = \frac{1}{(\sigma+\mu)^n} \sum_{j=0}^{n} \binom{n}{j} \mu^{n-j} \sigma^j a_{jk}$$

for all $n, k \in \mathbb{N}$, where $\sigma, \mu \in \mathbb{R}$ and $\sigma.\mu > 0$ Then, the necessary and sufficient conditions in order for $A = (a_{nk})$ to belong to any one of the classes $(b_c^{r,s}(G) : b_\infty^{\sigma,\mu})$ and $(b_c^{r,s}(G) : b_c^{\sigma,\mu})$ are obtained.

Corollary 4.3. *Let us take $S = (s_{nk})$ instead of $A = (a_{nk})$ in the needed ones in Theorems 4.1 and 4.2, where $S = (s_{nk})$ is defined by*

$$s_{nk} = \sum_{j=0}^{n} a_{jk}$$

for all $n, k \in \mathbb{N}$. Then, the necessary and sufficient conditions in order that $A = (a_{nk})$ belongs to any of the classes $(b_c^{r,s}(G) : bs)$ and $(b_c^{r,s}(G) : cs)$ are obtained.

5. Conclusion

Since the double band matrix G reduces, in the special case $u = 1$, $v = -1$, to the usual difference matrix Δ; our results are more general and more comprehensive than the corresponding results of Bişgin [7–10] and Meng and Song [17](in case of $m = 1$).

Acknowledgements. We would like to express our thanks to the anonymous reviewers for their valuable comments.

References

[1] B. Altay, F. Başar, *Some Euler sequence spaces of non-absolute type*, Ukrainian Math. J., 2005, 57(1), 1–17.

[2] B. Altay, F. Başar, M. Mursaleen, *On the Euler sequence spaces which include the spaces ℓ_p and ℓ_∞ I*, Inform. Sci., 2006, 176(10), 1450–1462.

[3] B. Altay, H. Polat, *On some new Euler difference sequence spaces*, Southeast Asian Bull. Math., 2006, 30(2), 209–220.

[4] F. Başar, B. Altay, *On the space of sequences of p-bounded variation and related matrix mappings*, Ukrainian Math. J., 2003, 55(1), 136–147.

[5] M. C. Bişgin, A. Sönmez, *Two new sequence spaces generated by the composition of m th order generalized difference matrix and lambda matrix*, J. Inequal. Appl., 2014, 274(2014).

[6] M. C. Bişgin, *Some notes on the sequence spaces* $\ell_p^\lambda(G^m)$ *and* $\ell_\infty^\lambda(G^m)$, GU J. Sci., 2017, 30(1), 381–393.

[7] M. C. Bişgin, *The Binomial sequence spaces of nonabsolute type*, J. Inequal. Appl., 2016, 309(2016).

[8] M. C. Bişgin, *The Binomial sequence spaces which include the spaces* ℓ_p *and* ℓ_∞ *and Geometric Properties*, J. Inequal. Appl., 2016, 304(2016).

[9] M. C. Bişgin, *Matrix transformations and compact operators on the binomial sequence spaces*, Under Review.

[10] M. C. Bişgin, *The binomial almost convergent and null sequence spaces*, Commun. Fac. Sci. Univ. Ank. Series A1, 2018, 67(1), 211–224.

[11] B. Choudhary, S. Nanda, *Functional Analysis with Applications*, John Wiley & sons Inc., New Delhi, 1989.

[12] M. Et, *On Some Difference Sequence Spaces*, Turkish J. Math., 1993, 17, 18–24.

[13] E. E. Kara, M. Başarır, *On compact operators and some Euler* $B^{(m)}$*-difference sequence spaces*, J. Math. Anal. Appl., 2011, 379(2), 499–511.

[14] V. Karakaya, H. Polat, *Some new paranormed sequence spaces defined by Euler difference operators*, Acta Sci. Math. (Szeged), 2010, 76, 87–100.

[15] H. Kızmaz, *On certain sequence spaces*, Canad. Math. Bull., 1981, 24(2), 169–176.

[16] M. Kirişçi, F. Başar, *Some new sequence spaces derived by the domain of generalized difference matrix*, Comput. Math. Appl., 2010, 60(5), 1299–1309.

[17] J. Meng, M. Song, *Binomial difference sequence spaces of order m*, Adv. Difference Equ., 2017, 241(2017).

[18] M. Mursaleen, F. Başar, B. Altay, *On the Euler sequence spaces which include the spaces* ℓ_p *and* ℓ_∞ *II*, Nonlinear Anal., 2006, 65(3), 707–717.

[19] P. -N. Ng, P. -Y. Lee, *Cesàro sequence spaces of non–absolute type*, Comment. Math. (Prace Mat.), 1978, 20(2), 429–433.

[20] H. Polat, F. Başar, *Some Euler spaces of difference sequences of order m*, Acta Math. Sci. Ser. B, Engl. Ed., 2007, 27(2), 254–266.

[21] M. Stieglitz, H. Tietz, *Matrix transformationen von folgenräumen eine ergebnisübersicht*, Math. Z., 1977, 154, 1–16.

[22] C. -S. Wang, *On Nörlund sequence spaces*, Tamkang J. Math., 1978, 9, 269–274.

[23] A. Wilansky, *Summability Throught Functional Analysis*, in: North-Holland Mathematics Studies, vol. 85, Elsevier Science Publishers, Amsterdam, Newyork, Oxford, 1984.