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DYNAMIC ANALYSIS OF A NON-AUTONOMOUS RATIO-DEPENDENT PREDATOR-PREY MODEL WITH ADDITIONAL FOOD*

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Abstract In this paper, a non-autonomous ratio-dependent three species predator-prey system with additional food to top predator was proposed. The permanence of the model is obtained. Based on the continuation theorem, the sufficient conditions for the existence of a periodic solution are obtained. By using the method of Lyapunov function, we prove that the system exists a unique positive almost periodic solution under some certain conditions.

Keywords Non-autonomous predator-prey model, permanence, periodic solutions, almost periodic solutions.

MSC(2010) 34C27, 92D25, 34D20.

1. Introduction

Traditional Lotka-Volterra type predator-prey model with Michaelis-Menten or Holling type II functional response is described by the following system:

$$\begin{cases} \dfrac{dx}{dt} = ax(1-bx) - \dfrac{cxy}{m+x}, \\ \dfrac{dy}{dt} = \dfrac{fcxy}{m+x} - dy, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ denote the densities of the prey and predator. We denote that a is the prey intrinsic growth rate and b is the carrying capacity of prey x population. Here, c and f express the capture rate and the conversion rate of prey to predator, respectively. m is the half saturation constant and t is time, d is the death rate for predator.

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Holling type II functional responses [14] are often used to model predator prey interactions. Most of predator-prey models have two or three species. Dynamics of three species predator-prey models are much more complex and interesting than two species models. Hastings and Powell [7] showed the existence of chaos in a three species food chain model. Later, Freedman and Waltman [4] proved persistence in three interacting predator-prey population models. Usually, top predator can eat only the middle predator and middle predator can eat only prey. However, top predator not only consume the middle predator but also prey in the real world. In 2014, Pal et al. [13] assumed that top predator consumes both prey and middle predator by considering the following control system:

$$\begin{cases} \dfrac{dx}{dt} = ax(1-bx) - \dfrac{c_1xy}{m_1+x} - \dfrac{c_3xz}{m_3+x}, \\ \dfrac{dy}{dt} = \dfrac{f_1c_1xy}{m_1+x} - \dfrac{c_2yz}{m_2+y} - d_1y, \\ \dfrac{dz}{dt} = \dfrac{f_3c_3xz}{m_3+x} + \dfrac{f_2c_2yz}{m_2+y} - d_2z, \end{cases} \tag{1.2}$$

where $z(t)$ denotes the density of top predator. We define that c_1, f_1 are the capture rate and the conversion rate of prey to middle predator; c_2, f_2 are the capture rate and the conversion rate of middle predator to top predator; c_3, f_3 are the capture rate and the conversion rate of prey to top predator. In this model, m_i, $i = 1,\ 2,\ 3$ are the half saturation constant of prey, middle predator and top predator, respectively; d_i, $i = 1,\ 2$ are death rate of middle predator and top predator, respectively.

Arditi and Ginzburg [1] proposed ratio-dependent predator-prey model. For its advantages, one can refer to Yang Kuang [11] and Lundberg and Fryxell [12]. Replacing the functional response $\frac{x}{m+x}$ in system (1.2) by a ratio-dependent response $\frac{\frac{x}{y}}{m+\frac{x}{y}}$. Then, we consider the following model:

$$\begin{cases} \dfrac{dx}{dt} = ax(1-bx) - \dfrac{c_1xy}{m_1y+x} - \dfrac{c_3xz}{m_3z+x}, \\ \dfrac{dy}{dt} = \dfrac{f_1c_1xy}{m_1y+x} - \dfrac{c_2yz}{m_2z+y} - d_1y, \\ \dfrac{dz}{dt} = \dfrac{f_3c_3xz}{m_3z+x} + \dfrac{f_2c_2yz}{m_2z+y} - d_2z. \end{cases} \tag{1.3}$$

In some real world ecological systems, top predator have additional food except prey and middle predator. Sahoo and Poria [17–19] introduced the concept of additional food to the top predator. Recently, Panja et al. [15] proposed a three species predator-prey model where top predator eats both prey and middle predator in which the additional food is supplied for top predator. A $(0 \leq A \leq 1)$ is the additional food. As you see, if $A = 0$, there will be no additional food in this system. If $A = 1$, all items associated with it will become 0. In fact, the environment is affected by the perturbation. We should consider the time-varying parameters.

Motivated by above mentioned words, we propose the following model:

$$\begin{cases} \dfrac{dx}{dt} = a(t)x(1-b(t)x) - \dfrac{c_1(t)xy}{m_1(t)y+x} - \dfrac{(1-A(t))c_3(t)xz}{m_3(t)z+x}, \\ \dfrac{dy}{dt} = \dfrac{f_1(t)c_1(t)xy}{m_1(t)y+x} - \dfrac{(1-A(t))c_2(t)yz}{m_2(t)z+y} - d_1(t)y, \\ \dfrac{dz}{dt} = \dfrac{(1-A(t))f_3(t)c_3(t)xz}{m_3(t)z+x} + \dfrac{(1-A(t))f_2(t)c_2(t)yz}{m_2(t)z+y} - d_2(t)z. \end{cases} \tag{1.4}$$

In what follows, we focus on the mathematical analysis of the boundedness, existence of periodic (almost periodic) solutions of system (1.4).

2. General Type

Let $\mathbb{R}^3_+ := \{(x,y,z) \in \mathbb{R}^3 \mid x \geq 0,\ y \geq 0,\ z \geq 0\}$. For a continuous bounded function $f(t)$ on $\mathbb{R}$, denote

$$f^u := \sup_{t\in\mathbb{R}} f(t),\ f^l := \inf_{t\in\mathbb{R}} f(t).$$

For the biological view, we assume the initial conditions satisfying $x(t_0) > 0,\ y(t_0) > 0,\ z(t_0) > 0$.

Definition 2.1. If a positive solution $(x(t),\ y(t),\ z(t))$ of system (1.4) satisfies

$$\min\{\lim_{t\to\infty}\inf x(t),\ \lim_{t\to\infty}\inf y(t),\ \lim_{t\to\infty}\inf z(t)\} = 0,$$

then system (1.4) is non-persistent.

Definition 2.2. If there exist two positive constants ϕ and φ $(0 < \phi < \varphi)$ with

$$\min\{\lim_{t\to\infty}\inf x(t),\ \lim_{t\to\infty}\inf y(t),\ \lim_{t\to\infty}\inf z(t)\} \geq \phi,$$

$$\max\{\lim_{t\to\infty}\sup x(t),\ \lim_{t\to\infty}\sup y(t),\ \lim_{t\to\infty}\sup z(t)\} \leq \varphi,$$

then system (1.4) is permanent.

Theorem 2.1. *Assume the parameters are all continuous and bounded by positive constants. If* $A^u < 1,\ f_1^u c_1^u > d_1^l,\ a^l m_1^l m_3^l > c_1^u m_3^l + (1-A^l)c_3^u m_1^l,\ f_1^l c_1^l m_2^l > (1-A^l)c_2^u + d_1^u m_2^l,\ (1-A^u)(f_3^l c_3^l + f_2^l c_2^l) > d_2^u$, *the set* Γ *defined by*

$$\Gamma = \{(x,\ y,\ z) \in \mathbb{R}^3 \mid g_1 \leq x \leq G_1,\ g_2 \leq y \leq G_2,\ g_3 \leq z \leq G_3\} \tag{2.1}$$

is a positively invariant and bounded region with respect to system (1.4), *where*

$$\begin{aligned} G_1 &= \frac{1}{b^l}, & g_1 &= \frac{a^l m_1^l m_3^l - [c_1^u m_3^l + (1-A^l)c_3^u m_1^l]}{a^l b^u m_1^l m_3^l}, \\ G_2 &= \frac{(f_1^u c_1^u - d_1^l)G_1}{d_1^l m_1^l}, & g_2 &= \frac{[f_1^l c_1^l m_2^l - (1-A^l)c_2^u - d_1^u m_2^l]g_1}{[(1-A^l)c_2^u + d_1^u m_2^l]m_1^u}, \\ G_3 &= \frac{(1-A^l)(m_2^l f_3^u c_3^u G_1 + m_3^l f_2^u c_2^u G_2)}{d_2^l m_2^l m_3^l}, & g_3 &= \frac{[(1-A^u)(f_3^l c_3^l + f_2^l c_2^l) - d_2^u]F}{d_2^u E}, \\ E &= \max\{m_2^u,\ m_3^u\},\ F = \max\{g_1,\ g_2\}. \end{aligned} \tag{2.2}$$

Proof. Let $(x(t), y(t), z(t))$ be any solution of system (1.4) satisfying $(x(t_0), y(t_0), z(t_0)) \in \Gamma$. It follows from the first equation of system (1.4) that

$$\dot{x}(t) \le a^u b^l x(t)[\frac{1}{b^l} - x(t)] = a^u b^l x(t)[G_1 - x(t)], \quad t \ge t_0,$$

which implies

$$0 \le x(t_0) \le G_1 \Rightarrow x(t) \le G_1, \quad t \ge t_0.$$

Similarly, we have

$$\begin{aligned}
\dot{x}(t) &\ge x(t)\left[a^l(1 - b^u x(t)) - \frac{c_1^u}{m_1^l} - \frac{(1 - A^l)c_3^u}{m_3^l}\right] \\
&= x(t)\left[a^l - a^l b^u x(t) - \frac{c_1^u m_3^l + m_1^l(1 - A^l)c_3^u}{m_1^l m_3^l}\right] \\
&= \frac{x(t)}{m_1^l m_3^l}\left\{a^l m_1^l m_3^l - a^l b^u m_1^l m_3^l x(t) - \left[c_1^u m_3^l + (1 - A^l)c_3^u m_1^l\right]\right\} \\
&= \frac{x(t)a^l b^u m_1^l m_3^l}{m_1^l m_3^l}\left\{\frac{a^l m_1^l m_3^l - \left[c_1^u m_3^l + (1 - A^l)c_3^u m_1^l\right]}{a^l b^u m_1^l m_3^l} - x(t)\right\} \\
&= x(t)a^l b^u[g_1 - x(t)],
\end{aligned}$$

which leads to

$$x(t_0) \ge g_1 \Rightarrow x(t) \ge g_1, \quad t \ge t_0.$$

From the second equation

$$\begin{aligned}
\dot{y}(t) &\le y(t)\left[\frac{f_1^u c_1^u G_1}{m_1^l y(t) + G_1} - d_1^l\right] = \frac{y(t)}{m_1^l y(t) + G_1}\left[f_1^u c_1^u G_1 - d_1^l(m_1^l y(t) + G_1)\right] \\
&= \frac{y(t)d_1^l m_1^l}{m_1^l y(t) + G_1}\left[\frac{(f_1^u c_1^u - d_1^l)G_1}{d_1^l m_1^l} - y(t)\right] = \frac{y(t)d_1^l m_1^l}{m_1^l y(t) + G_1}[G_2 - y(t)],
\end{aligned}$$

which implies

$$0 \le y(t_0) \le G_2 \Rightarrow y(t) \le G_2, \quad t \ge t_0.$$

In the same way, we get

$$\begin{aligned}
\dot{y}(t) &\ge y(t)\left[\frac{f_1^l c_1^l g_1}{m_1^u y(t) + g_1} - \frac{(1 - A^l)c_2^u}{m_2^l} - d_1^u\right] \\
&= y(t)\left[\frac{f_1^l c_1^l g_1}{m_1^u y(t) + g_1} - \frac{(1 - A^l)c_2^u + d_1^u m_2^l}{m_2^l}\right] \\
&= y(t)\left\{\frac{f_1^l c_1^l g_1 m_2^l - (m_1^u y(t) + g_1)[(1 - A^l)c_2^u + d_1^u m_2^l]}{m_2^l[m_1^u y(t) + g_1]}\right\} \\
&= \frac{y(t)}{m_2^l[m_1^u y(t) + g_1]}\{f_1^l c_1^l g_1 m_2^l - \left[(1 - A^l)c_2^u + d_1^u m_2^l\right]g_1 - \left[(1 - A^l)c_2^u + d_1^u m_2^l\right]m_1^u y(t)\} \\
&= \frac{y(t)\left[(1 - A^l)c_2^u + d_1^u m_2^l\right]m_1^u}{m_2^l(m_1^u y(t) + g_1)}\left\{\frac{\left[f_1^l c_1^l m_2^l - (1 - A^l)c_2^u - d_1^u m_2^l\right]g_1}{[(1 - A^l)c_2^u + d_1^u m_2^l]m_1^u} - y(t)\right\} \\
&= \frac{y(t)\left[(1 - A^l)c_2^u + d_1^u m_2^l\right]m_1^u}{m_2^l(m_1^u y(t) + g_1)}[g_2 - y(t)],
\end{aligned}$$

and hence,

$$y(t_0) \geq g_2 \Rightarrow y(t) \geq g_1, \quad t \geq t_0.$$

Moreover, it follows from the top predator equation that

$$\begin{aligned}
\dot{z}(t) &\leq z(t)\left[\frac{(1-A^l)f_3^u c_3^u G_1}{m_3^l z} + \frac{(1-A^l)f_2^u c_2^u G_2}{m_2^l z} - d_2^l\right] \\
&= z(t)\left\{\frac{(1-A^l)(m_2^l f_3^u c_3^u G_1 + m_3^l f_2^u c_2^u G_2)}{m_3^l m_2^l z(t)} - \frac{d_2^l m_3^l m_2^l z(t)}{m_3^l m_2^l z(t)}\right\} \\
&= \frac{1}{m_3^l m_2^l}[(1-A^l)(m_2^l f_3^u c_3^u G_1 + m_3^l f_2^u c_2^u G_2) - d_2^l m_2^l m_3^l z(t)] \\
&= \frac{d_2^l m_2^l m_3^l}{m_2^l m_3^l}\left[\frac{(1-A^l)(m_2^l f_3^u c_3^u G_1 + m_3^l f_2^u c_2^u G_2)}{d_2^l m_2^l m_3^l} - z(t)\right] \\
&= d_2^l[G_3 - z(t)],
\end{aligned}$$

which leads to

$$0 \leq z(t_0) \leq G_3 \Rightarrow z(t) \leq G_3, \quad t \geq t_0.$$

Furthermore, we have

$$\begin{aligned}
\dot{z}(t) &\geq z(t)\left[\frac{(1-A^u)f_3^l c_3^l g_1}{m_3^u z(t) + g_1} + \frac{(1-A^u)f_2^l c_2^l g_2}{m_2^u z(t) + g_2} - d_2^u\right] \\
&= z(t)\left[\frac{(1-A^u)(f_3^l c_3^l F + f_2^l c_2^l F)}{Ez(t) + F} - d_2^u\right] \\
&= \frac{z(t)}{Ez(t) + F}\left[(1-A^u)(f_3^l c_3^l + f_2^l c_2^l)F - d_2^u F - d_2^u E z(t)\right] \\
&= \frac{z(t) d_2^u E}{Ez(t) + F}\left\{\frac{[(1-A^u)(f_3^l c_3^l + f_2^l c_2^l) - d_2^u]F}{d_2^u E} - z(t)\right\} \\
&= \frac{z(t) d_2^u E}{Ez(t) + F}[g_3 - z(t)],
\end{aligned}$$

which implies

$$z(t_0) \geq g_3 \Rightarrow z(t) \geq g_3, \quad t \geq t_0.$$

This completes the proof of Theorem 2.1. □

Theorem 2.2. *Assume that conditions in Theorem 2.1 are satisfied, then the set* Γ *defined by system* (2.1) *is the ultimately bounded region of system* (1.4).

3. Periodic Solution

For the autonomous system, stability and bifurcation theory plays a great role in qualitative analysis of differential equations (see e.g. [2, 3, 8–10, 16, 20, 21, 24, 26]). Correspondingly, when we consider the non-autonomous periodic system, we focus on obtaining the existence of positive periodic solutions. To do this, we assume that the parameters of system (1.4) are periodic in t of period $\omega > 0$.

We adopt the notations and definitions, lemmas from [5, 6, 22, 23, 27]. We denote $\bar{f} := \frac{1}{\omega}\int_0^\omega f(t)dt$ where $f(t)$ is a periodic and continuous function with period ω.

Theorem 3.1. *If* $\overline{(1-A)} > 0$, $\bar{a}\bar{m}_1\bar{m}_3 > \bar{m}_3\bar{c}_1 + \overline{(1-A)}\bar{c}_3\bar{m}_1$, $\bar{f}_1\bar{c}_1\bar{m}_2 > \bar{d}_1\bar{m}_2 + \overline{(1-A)}\bar{c}_2$ *and* $\overline{(1-A)}(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2) > \bar{d}_2$, *then system* (1.4) *has at least one positive* ω *periodic solution, namely,* $(x^*(t),\ y^*(t),\ z^*(t))$.

Proof. Change the variables as follows,

$$x(t) = \exp\{\tilde{x}(t)\},\ y(t) = \exp\{\tilde{y}(t)\},\ z(t) = \exp\{\tilde{z}(t)\},$$

then, system (1.4) becomes

$$\begin{cases} \tilde{x}'(t) = a(t)(1 - b(t)\exp\{\tilde{x}(t)\}) - \dfrac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}} \\ \qquad\qquad - \dfrac{(1 - A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{x}(t)\}}, \\ \tilde{y}'(t) = \dfrac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}} - \dfrac{(1 - A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{y}(t)\}} - d_1(t), \\ \tilde{z}'(t) = \dfrac{(1 - A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{x}(t)\}} + \dfrac{(1 - A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{y}(t)\}} - d_2(t). \end{cases} \tag{3.1}$$

Let

$$\mathcal{X} = \mathcal{Y} = \big\{(\tilde{x},\ \tilde{y},\ \tilde{z})^T \in C(\mathbb{R},\ \mathbb{R}^2,\ \mathbb{R}^3) \mid \tilde{x}(t+\omega) = \tilde{x},\ \tilde{y}(t+\omega) = \tilde{y},\ \tilde{z}(t+\omega) = \tilde{z}\big\},$$
$$\|\ (\tilde{x},\ \tilde{y},\ \tilde{z})\ \| = \max_{t\in[0,\omega]}\big(|\tilde{x}(t)| + |\tilde{y}(t)| + |\tilde{z}(t)|\big),\ (\tilde{x},\ \tilde{y},\ \tilde{z}) \in \mathcal{X}\ (\text{or } \mathcal{Y}).$$

Clearly, $\mathcal{X}$ and $\mathcal{Y}$ are Banach spaces. Let

$$N\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix} a(t)\big(1 - b(t)\exp\{\tilde{x}(t)\}\big) - \frac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} \\ \frac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_1(t) \\ \frac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} + \frac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_2(t) \end{bmatrix},$$

$$L\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\tilde{x}'\\ \tilde{y}'\\ \tilde{z}'\end{bmatrix},$$

$$P\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = Q\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\frac{1}{\omega}\int_0^\omega \tilde{x}(t)dt\\ \frac{1}{\omega}\int_0^\omega \tilde{y}(t)dt\\ \frac{1}{\omega}\int_0^\omega \tilde{z}(t)dt\end{bmatrix} \text{ and } \begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} \in X = Y.$$

Similar arguments to [5, 8, 22], we easily prove that $Kp : ImL \to DomL \cap \ker P$ exists and is given by

$$Kp\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix}\int_0^t \tilde{x}(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t \tilde{x}(s)dsdt\\ \int_0^t \tilde{y}(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t \tilde{y}(s)dsdt\\ \int_0^t \tilde{z}(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t \tilde{z}(s)dsdt\end{bmatrix}.$$

It is easy to see that

$$QN\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix} \frac{1}{\omega}\int_0^\omega \big[a(t)\big(1-b(t)\exp\{\tilde{x}(t)\}\big) - \frac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}}\big]dt \\ \frac{1}{\omega}\int_0^\omega \big[\frac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_1(t)\big]dt \\ \frac{1}{\omega}\int_0^\omega \big[\frac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} + \frac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_2(t)\big]dt \end{bmatrix}$$

and

$$Kp(I-Q)N\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix} \int_0^t N_1(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t N_1(s)dsdt - (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_1(s)ds \\ \int_0^t N_2(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t N_2(s)dsdt - (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_2(s)ds \\ \int_0^t N_3(s)ds - \frac{1}{\omega}\int_0^\omega\int_0^t N_3(s)dsdt - (\frac{t}{\omega}-\frac{1}{2})\int_0^\omega N_3(s)ds \end{bmatrix}.$$

It is not difficult to prove that N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are in a position to find an appropriate open bounded subset Ω for the application of the continuation theorem of [5,6,22]. According to the equation $Lx = \lambda Nx,\ \lambda \in (0,1)$, we get

$$\begin{cases} \tilde{x}'(t) = \lambda\big[a(t)(1-b(t)\exp\{\tilde{x}(t)\}) - \dfrac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} \\ \qquad\qquad - \dfrac{(1-A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}}\big], \\ \tilde{y}'(t) = \lambda\big[\dfrac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \dfrac{(1-A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_1(t)\big], \\ \tilde{z}'(t) = \lambda\big[\dfrac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} + \dfrac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}} - d_2(t)\big]. \end{cases} \tag{3.2}$$

Suppose that $(\tilde{x}(t),\ \tilde{y}(t),\ \tilde{z}(t))$ is an arbitrary solution of system (3.1) with a certain $\lambda \in (0,\ 1)$ on both side of system (3.2) over the interval $[0,\ \omega]$, such that

$$\begin{cases} \bar{a}\omega = \displaystyle\int_0^\omega \big[a(t)b(t)\exp\{\tilde{x}(t)\} + \frac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} \\ \qquad + \dfrac{(1-A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}}\big]dt, \\ \bar{d_1}\omega = \displaystyle\int_0^\omega \big[\frac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}\big]dt, \\ \bar{d_2}\omega = \displaystyle\int_0^\omega \big[\frac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} + \frac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}\big]dt. \end{cases} \tag{3.3}$$

According to equation (3.2) and (3.3), we have

$$\begin{cases} \int_0^\omega |\tilde{x}'(t)|dt \le \lambda\big[\int_0^\omega a(t)dt + \int_0^\omega a(t)b(t)\exp\{\tilde{x}(t)\}dt \\ \qquad\qquad + \int_0^\omega \dfrac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}}dt \\ \qquad\qquad + \int_0^\omega \dfrac{(1-A)c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}}dt\big] \\ \qquad\qquad < 2\bar{a}\omega, \\ \int_0^\omega |\tilde{y}'(t)|dt \le \lambda\big[\int_0^\omega d_1(t)dt + \int_0^\omega \dfrac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}}dt \\ \qquad\qquad - \int_0^\omega \dfrac{(1-A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}dt\big] \\ \qquad\qquad < 2\bar{d}_1\omega, \\ \int_0^\omega |\tilde{z}'(t)|dt \le \lambda\big[\int_0^\omega d_2(t)dt + \int_0^\omega \dfrac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\}} \\ \qquad\qquad + \int_0^\omega \dfrac{(1-A(t))f_2c_2\exp\{\tilde{y}(t)\}}{m_2\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}dt\big] \\ \qquad\qquad < 2\bar{d}_2\omega. \end{cases} \tag{3.4}$$

There exist ξ_i and $\eta_i \in [0,\omega]$, $i = 1,\ 2,\ 3$ such that

$$\begin{aligned} &\tilde{x}(\xi_1) = \min_{t\in[0,\omega]}\tilde{x}(t), \quad \tilde{x}(\eta_1) = \max_{t\in[0,\omega]}\tilde{x}(t), \\ &\tilde{y}(\xi_2) = \min_{t\in[0,\omega]}\tilde{y}(t), \quad \tilde{y}(\eta_2) = \max_{t\in[0,\omega]}\tilde{y}(t), \\ &\tilde{z}(\xi_3) = \min_{t\in[0,\omega]}\tilde{z}(t), \quad \tilde{z}(\eta_3) = \max_{t\in[0,\omega]}\tilde{z}(t). \end{aligned} \tag{3.5}$$

According to the first equation of system (3.3), we find that

$$\bar{a}\omega \ge \int_0^\omega a(t)b(t)\exp\{\tilde{x}(\xi_1)\}dt, \quad \tilde{x}(\xi_1) \le \ln\frac{\bar{a}}{\bar{a}\bar{b}}.$$

From system (3.4) and (3.5), we obtain

$$\tilde{x}(t) \le \tilde{x}(\xi_1) + \int_0^\omega |\tilde{x}'(t)|dt < \ln\frac{\bar{a}}{\bar{a}\bar{b}} + 2\bar{a}\omega := H_1. \tag{3.6}$$

Now, we consider the first equation of system (3.4)

$$\begin{aligned} \bar{a}\omega &\le \int_0^\omega \big[a(t)b(t)\exp\{\tilde{x}(\eta_1)\} + \frac{c_1(t)}{m_1(t)} + \frac{(1-A(t))c_3(t)}{m_3(t)}\big]dt \\ &= \int_0^\omega \big[a(t)b(t)\exp\{\tilde{x}(\eta_1)\} + \frac{c_1(t)m_3(t)+(1-A(t))c_3(t)m_1(t)}{m_1(t)m_3(t)}\big]dt, \end{aligned}$$

which implies

$$\bar{a} \le \bar{a}\bar{b}\exp\{\tilde{x}(\eta_1)\} + \frac{\bar{c}_1\bar{m}_3 + \overline{(1-A)}\bar{c}_3\bar{m}_1}{\bar{m}_1\bar{m}_3},$$

$$\bar{a}\bar{b}\exp\{\tilde{x}(\eta_1)\} \geq \frac{\bar{a}\bar{m}_1\bar{m}_3 - \bar{c}_1\bar{m}_3 - \overline{(1-A)}\bar{c}_3\bar{m}_1}{\bar{m}_1\bar{m}_3},$$

$$\tilde{x}(\eta_1) \geq \ln\left\{\frac{\bar{a}\bar{m}_1\bar{m}_3 - [\bar{c}_1\bar{m}_3 + \overline{(1-A)}\bar{c}_3\bar{m}_1]}{\bar{a}\bar{b}\bar{m}_1\bar{m}_3}\right\},$$

and hence,

$$\tilde{x}(t) \geq \tilde{x}(\eta_1) - \int_0^\omega |\tilde{x}'(t)|dt > \ln\left\{\frac{\bar{a}\bar{m}_1\bar{m}_3 - [\bar{m}_3\bar{c_1} + \overline{(1-A)}\bar{c_3}\bar{m_1}]}{\bar{a}\bar{b}\bar{m}_1\bar{m}_3}\right\} - 2\bar{a}\omega := H_2. \tag{3.7}$$

According to system (3.5) and the second equation of system (3.3), we have

$$\bar{d_1}\omega \leq \int_0^\omega \frac{f_1(t)c_1(t)\exp\{H_1\}}{m_1(t)\exp\{\tilde{y}(\xi_2)\}}dt,$$

$$\bar{d_1} \leq \frac{\bar{f_1}\bar{c}_1\exp\{H_1\}}{\bar{m}_1\exp\{\tilde{y}(\xi_2)\}},$$

which implies

$$\exp\{\tilde{y}(\xi_2)\} \leq \frac{\bar{f_1}\bar{c}_1\exp\{H_1\}}{\bar{d_1}\bar{m}_1},$$

$$\tilde{y}(\xi_2) \leq \ln\left\{\frac{\bar{f_1}\bar{c}_1\exp\{H_1\}}{\bar{d_1}\bar{m}_1}\right\},$$

thus,

$$\tilde{y}(t) \leq \tilde{y}(\xi_2) + \int_0^\omega |\tilde{y}'(t)|dt < \ln\left\{\frac{\bar{f_1}\bar{c}_1\exp\{H_1\}}{\bar{d_1}\bar{m}_1}\right\} + 2\bar{d_1}\omega := H_3. \tag{3.8}$$

From the second equation of system (3.3), we obtain

$$\bar{d_1}\omega \geq \int_0^\omega \left[\frac{f_1(t)c_1(t)\exp\{H_2\}}{m_1(t)\exp\{\tilde{y}(\eta_2)\} + \exp\{H_2\}} - \frac{(1-A(t))c_2(t)}{m_2(t)}\right]dt.$$

This leads to

$$\bar{d_1} \geq \frac{\bar{f_1}\bar{c}_1\exp\{H_2\}}{m_1^u\exp\{\tilde{y}(\eta_2)\} + \exp\{H_2\}} - \frac{\overline{(1-A)}\bar{c}_2}{\bar{m}_2},$$

and therefore,

$$\tilde{y}(\eta_2) \geq \ln\left\{\frac{\bar{f_1}\bar{c}_1\exp\{H_2\}\bar{m}_2}{(\bar{d_1}\bar{m}_2 + \overline{(1-A)}\bar{c}_2)m_1^u} - \frac{\exp\{H_2\}}{m_1^u}\right\},$$

$$\tilde{y}(\eta_2) \geq \ln\left\{\frac{[\bar{f_1}\bar{c}_1\bar{m}_2 - (\bar{d_1}\bar{m}_2 + \overline{(1-A)}\bar{c}_2)]\exp\{H_2\}}{(\bar{d_1}\bar{m}_2 + \overline{(1-A)}\bar{c}_2)m_1^u}\right\},$$

furthermore,

$$\begin{aligned}\tilde{y}(t) &\geq \tilde{y}(\eta_2) - \int_0^\omega |\tilde{y}'(t)|dt \\ &> \ln\left\{\frac{[\bar{f_1}\bar{c}_1\bar{m}_2 - (\bar{d_1}\bar{m}_2 + \overline{(1-A)}\bar{c}_2)]\exp\{H_2\}}{(\bar{d_1}\bar{m}_2 + \overline{(1-A)}\bar{c}_2)m_1^u}\right\} - 2\bar{d_1}\omega := H_4.\end{aligned} \tag{3.9}$$

From system (3.5) and the third equation of system (3.3), we get

$$\bar{d}_2\omega \leq \int_0^\omega \left[\frac{(1-A(t))f_3(t)c_3(t)\exp\{H_1\}}{m_3(t)\exp\{\tilde{z}(\xi_3)\}} + \frac{(1-A(t))f_2(t)c_2(t)\exp\{H_3\}}{m_2(t)\exp\{\tilde{z}(\xi_3)\}}\right]dt,$$

$$\bar{d}_2 \leq \frac{\overline{(1-A)}[\bar{f}_3\bar{c}_3\bar{m}_2\exp\{H_1\} + \bar{f}_2\bar{c}_2\bar{m}_3\exp\{H_3\}]}{\bar{m}_2\bar{m}_3\exp\{\tilde{z}(\xi_3)\}},$$

$$\tilde{z}(\xi_3) \leq \ln\left\{\frac{\overline{(1-A)}[\bar{f}_3\bar{c}_3\bar{m}_2\exp\{H_1\} + \bar{f}_2\bar{c}_2\bar{m}_3\exp\{H_3\}]}{\bar{m}_2\bar{m}_3\bar{d}_2}\right\},$$

then, it follows that

$$\begin{aligned}\tilde{z}(t) &\leq \tilde{z}(\xi_3) + \int_0^\omega |\tilde{z}'(t)|dt \\ &< \ln\left\{\frac{\overline{(1-A)}[\bar{f}_3\bar{c}_3\bar{m}_2\exp\{H_1\} + \bar{f}_2\bar{c}_2\bar{m}_3\exp\{H_3\}]}{\bar{m}_2\bar{m}_3\bar{d}_2}\right\} + 2\bar{d}_2\omega := H_5.\end{aligned} \tag{3.10}$$

The third equation of system (3.3) gives

$$\bar{d}_2\omega \geq \int_0^\omega \left[\frac{(1-A(t))f_3(t)c_3(t)\exp\{H_2\}}{m_3^u\exp\{\tilde{z}(\eta_3)\} + \exp\{H_2\}} + \frac{(1-A(t))f_2(t)c_2(t)\exp\{H_4\}}{m_2^u\exp\{\tilde{z}(\eta_3)\} + \exp\{H_4\}}\right]dt,$$

$$\bar{d}_2 \geq \frac{\overline{(1-A)}\bar{f}_3\bar{c}_3\exp\{H_2\}}{m_3^u\exp\{\tilde{z}(\eta_3)\} + \exp\{H_2\}} + \frac{\overline{(1-A)}\bar{f}_2\bar{c}_2\exp\{H_4\}}{m_2^u\exp\{\tilde{z}(\eta_3)\} + \exp\{H_4\}}.$$

Denote that $W = \max\{m_3^u,\ m_2^u\}$ and $V = \max\{\exp\{H_2\},\ \exp\{H_4\}\}$, we have

$$\bar{d}_2 \geq \frac{\overline{(1-A)}V(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2)}{W\exp\{\tilde{z}(\eta_3)\} + V},$$

$$\overline{(1-A)}V(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2) \leq \bar{d}_2 W\exp\{\tilde{z}(\eta_3)\} + \bar{d}_2 V,$$

then,

$$\exp\{\tilde{z}(\eta_3)\} \geq \frac{[\overline{(1-A)}(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2) - \bar{d}_2]V}{\bar{d}_2 W},$$

$$\tilde{z}(\eta_3) \geq \ln\left\{\frac{[\overline{(1-A)}(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2) - \bar{d}_2]V}{\bar{d}_2 W}\right\},$$

consequently,

$$\tilde{z}(t) \geq \tilde{z}(\eta_3) - \int_0^\omega |\tilde{z}'(t)|dt > \ln\left\{\frac{[\overline{(1-A)}(\bar{f}_3\bar{c}_3 + \bar{f}_2\bar{c}_2) - \bar{d}_2]V}{\bar{d}_2 W}\right\} - 2\bar{d}_2\omega := H_6. \tag{3.11}$$

It follows from (3.6)-(3.11) that

$$\begin{cases}\max\limits_{t\in[0,\omega]}|\tilde{x}(t)| \leq \max\{|H_1|,\ |H_2|\} := C_1, \\ \max\limits_{t\in[0,\omega]}|\tilde{y}(t)| \leq \max\{|H_3|,\ |H_4|\} := C_2, \\ \max\limits_{t\in[0,\omega]}|\tilde{z}(t)| \leq \max\{|H_5|,\ |H_6|\} := C_3.\end{cases} \tag{3.12}$$

We choose $C > 0$ such that $C > C_1 + C_2 + C_3$. Let $\Omega = \{(\tilde{x},\ \tilde{y},\ \tilde{z}) \in X \mid \| (\tilde{x},\ \tilde{y},\ \tilde{z}) \| < C\}$, then it is easy to verify that the requirement (1) in the continuation theorem of [5,6,22] is satisfied. Also,

$$QN\begin{bmatrix}\tilde{x}\\ \tilde{y}\\ \tilde{z}\end{bmatrix} = \begin{bmatrix} \bar{a}(1-\bar{b}\exp\{\tilde{x}\}) - \frac{1}{\omega}\int_0^\omega [\frac{c_1\exp\{\tilde{x}(t)\}\exp\{\tilde{y}(t)\}}{m_1\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A)c_3\exp\{\tilde{z}(t)\}}{m_3\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}}]dt \\ -\bar{d}_1 + \frac{1}{\omega}\int_0^\omega [\frac{f_1c_1\exp\{\tilde{x}(t)\}}{m_1\exp\{\tilde{y}(t)\}+\exp\{\tilde{x}(t)\}} - \frac{(1-A)c_2\exp\{\tilde{z}(t)\}}{m_2\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}]dt \\ -\bar{d}_2 + \frac{1}{\omega}\int_0^\omega [\frac{(1-A)f_3c_3\exp\{\tilde{x}(t)\}}{m_3\exp\{\tilde{z}(t)\}+\exp\{\tilde{x}(t)\}} + \frac{(1-A)f_2c_2\exp\{\tilde{y}(t)\}}{m_2\exp\{\tilde{z}(t)\}+\exp\{\tilde{y}(t)\}}]dt \end{bmatrix}$$
$$\neq \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

In addition, we have

$$\deg\{JQN,\ \Omega \cap KerL, 0\} \neq 0.$$

Thus, we have proved that Ω meets all the conditions in the continuation theorem of [5,6,22]. Hence, system (3.1) has at least one ω periodic solution $(\tilde{x}^*(t), \tilde{y}^*(t), \tilde{z}^*(t))$. Set $x^*(t) = \exp\{\tilde{x}^*(t)\}, y^*(t) = \exp\{\tilde{y}^*(t)\}, z^*(t) = \exp\{\tilde{z}^*(t)\}$, then $(x^*(t),\ y^*(t),\ z^*(t))$ is an ω periodic solution of system (1.4). The proof of Theorem 3.1 is completed. □

4. Almost Periodic Case

Let

$$x(t) = \exp\{\tilde{x}(t)\},\ y(t) = \exp\{\tilde{y}(t)\},\ z(t) = \exp\{\tilde{z}(t)\},$$

then system (1.4) becomes

$$\begin{cases} \tilde{x}'(t) = a(t)(1 - b(t)\exp\{\tilde{x}(t)\}) - \dfrac{c_1(t)\exp\{\tilde{y}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}} \\ \qquad\qquad - \dfrac{(1 - A(t))c_3(t)\exp\{\tilde{z}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{x}(t)\}}, \\ \tilde{y}'(t) = \dfrac{f_1(t)c_1(t)\exp\{\tilde{x}(t)\}}{m_1(t)\exp\{\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}} - \dfrac{(1 - A(t))c_2(t)\exp\{\tilde{z}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{y}(t)\}} - d_1(t), \\ \tilde{z}'(t) = \dfrac{(1 - A(t))f_3(t)c_3(t)\exp\{\tilde{x}(t)\}}{m_3(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{x}(t)\}} + \dfrac{(1 - A(t))f_2(t)c_2(t)\exp\{\tilde{y}(t)\}}{m_2(t)\exp\{\tilde{z}(t)\} + \exp\{\tilde{y}(t)\}} - d_2(t). \end{cases} \tag{4.1}$$

Theorem 4.1. *Assume that the conditions in Theorem 2.1 are satisfied, system* (4.1) *has a positively invariant and ultimately bounded region* Γ^*. *Here* Γ^* *denotes* $\{(x,\ y,\ z) \in \mathbb{R}^3 \mid \ln\{g_1\} \le \tilde{x} \le \ln\{G_1\},\ \ln\{g_2\} \le \tilde{y} \le \ln\{G_2\},\ \ln\{g_3\} \le \tilde{z} \le \ln\{G_3\}\}$ *and* $g_i,\ G_i,\ i = 1,\ 2,\ 3$ *are defined in Theorem 2.1.*

We need Theorem 19.1 of [25] to prove the existence of the almost periodic solutions. Consider ordinary differential equation

$$x' = f(t, x),\ f(t, x) \in C(\mathbb{R} \times D,\ \mathbb{R}^n), \tag{4.2}$$

where D is an open set in $\mathbb{R}^n$, $f(t, x)$ is almost periodic in t uniformly with respect to $x \in D$.

Theorem 4.2. *Assume that the conditions in Theorem 2.1 are satisfied. If further assume that*

$$\begin{cases} \inf\limits_{t\in\mathbb{R}}\{a(t)b(t)-\dfrac{[f_1(t)m_1(t)-1]c_1(t)G_2}{(m_1(t)g_2+g_1)^2}-\dfrac{[f_3(t)m_3(t)-1]c_3(t)G_3(1-A(t))}{(m_3(t)g_3+g_1)^2}\}>0,\\ \inf\limits_{t\in\mathbb{R}}\{\dfrac{[f_1(t)m_1(t)-1]c_1(t)g_1}{(m_1(t)G_2+G_1)^2}+\dfrac{[1-f_2(t)m_2(t)]c_2(t)g_3(1-A(t))}{(m_2(t)G_3+G_2)^2}\}>0,\\ \inf\limits_{t\in\mathbb{R}}\{\dfrac{[f_3(t)m_3(t)-1]c_3(t)g_1(1-A(t))}{(m_3(t)G_3+G_1)^2}+\dfrac{[f_2(t)m_2(t)-1]c_2(t)g_2(1-A(t))}{(m_2(t)G_3+G_2)^2}\}>0, \end{cases} \tag{4.3}$$

then, system (1.4) *has a unique uniformly asymptotically stable almost periodic solution in* Γ.

Proof. To prove that system (1.4) has a unique uniformly asymptotically stable position almost periodic solution in Γ. It suffices to show that system (4.1) has a unique uniformly asymptotically stable almost periodic solution in Γ^*.

Consider the product system (4.1)

$$\begin{cases} \tilde{x}_1'(t)=a(t)(1-b(t)\exp\{\tilde{x}_1(t)\})-\dfrac{c_1(t)\exp\{\tilde{y}_1(t)\}}{m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}}\\ \qquad -\dfrac{(1-A(t))c_3(t)\exp\{\tilde{z}_1(t)\}}{m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\}},\\ \tilde{y}_1'(t)=\dfrac{f_1(t)c_1(t)\exp\{\tilde{x}_1(t)\}}{m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}}-\dfrac{(1-A(t))c_2(t)\exp\{\tilde{z}_1(t)\}}{m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\}}-d_1(t),\\ \tilde{z}_1'(t)=\dfrac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}_1(t)\}}{m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\}}+\dfrac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}_1(t)\}}{m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\}}-d_2(t),\\ \tilde{x}_2'(t)=a(t)(1-b(t)\exp\{\tilde{x}_2(t)\})-\dfrac{c_1(t)\exp\{\tilde{y}_2(t)\}}{m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}}\\ \qquad -\dfrac{(1-A(t))c_3(t)\exp\{\tilde{z}_2(t)\}}{m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\}},\\ \tilde{y}_2'(t)=\dfrac{f_1(t)c_1(t)\exp\{\tilde{x}_2(t)\}}{m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}}-\dfrac{(1-A(t))c_2(t)\exp\{\tilde{z}_2(t)\}}{m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\}}-d_1(t),\\ \tilde{z}_2'(t)=\dfrac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}_2(t)\}}{m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\}}+\dfrac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}_2(t)\}}{m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\}}-d_2(t), \end{cases} \tag{4.4}$$

and the Lyapunov function

$$V(t,\tilde{x}_1,\tilde{y}_1,\tilde{z}_1,\tilde{x}_2,\tilde{y}_2,\tilde{z}_2)=|\tilde{x}_1(t)-\tilde{x}_2(t)|+|\tilde{y}_1(t)-\tilde{y}_2(t)|+|\tilde{z}_1(t)-\tilde{z}_2(t)|,$$

then, condition (i) in Theorem 19.1 of [25] is satisfied when $\alpha(\gamma)=\beta(\gamma)=\gamma$, $\gamma\geq 0$. In addition

$$\begin{aligned} &|V(t,\tilde{x}_1,\tilde{y}_1,\tilde{z}_1,\tilde{x}_2,\tilde{y}_2,\tilde{z}_2)-V(t,\tilde{x}_3,\tilde{y}_3,\tilde{z}_3,\tilde{x}_4,\tilde{y}_4,\tilde{z}_4)|\\ =&\big(|\tilde{x}_1(t)-\tilde{x}_2(t)|+|\tilde{y}_1(t)-\tilde{y}_2(t)|+|\tilde{z}_1(t)-\tilde{z}_2(t)|\big)\\ &-\big(|\tilde{x}_3(t)-\tilde{x}_4(t)|+|\tilde{y}_3(t)-\tilde{y}_4(t)|+|\tilde{z}_3(t)-\tilde{z}_4(t)|\big)\\ \leq&|\tilde{x}_1(t)-\tilde{x}_3(t)|+|\tilde{y}_1(t)-\tilde{y}_3(t)|+|\tilde{z}_1(t)-\tilde{z}_3(t)|+|\tilde{x}_2(t)-\tilde{x}_4(t)|\\ &+|\tilde{y}_2(t)-\tilde{y}_4(t)|+|\tilde{z}_2(t)-\tilde{z}_4(t)| \end{aligned}$$

$$\leq \| (\tilde{x}_1(t), \tilde{y}_1(t), \tilde{z}_1(t),) - (\tilde{x}_3(t), \tilde{y}_3(t), \tilde{z}_3(t)) \| + \| (\tilde{x}_2(t), \tilde{y}_2(t), \tilde{z}_2(t)) - (\tilde{x}_4(t), \tilde{y}_4(t), \tilde{z}_4(t)) \|,$$

which indicates that condition (ii) in Theorem 19.1 of [25] is also satisfied.

Let $(\tilde{x}_i,\ \tilde{y}_i,\ \tilde{z}_i)^T,\ i = 1,\ 2$ be any two solutions of system (4.1). Calculating the upper right derivative of $V(t)$ along the solution of system (4.1), we get

$$\begin{aligned}
&D^+V(t)\\
&= \Big[- a(t)b(t)\big(\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}\big) - \Big(\frac{c_1(t)\exp\{\tilde{y}_1(t)\}}{m_1(t)\exp\{\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}}\\
&- \frac{c_1(t)\exp\{\tilde{y}_2(t)\}}{m_1(t)\exp\{\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big) - \Big(\frac{(1-A(t))c_3(t)\exp\{\tilde{z}_1(t)\}}{m_3(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{x}_1(t)\}}\\
&- \frac{(1-A(t))c_3(t)\exp\{\tilde{z}_2(t)\}}{m_3(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big)\Big] sgn(\tilde{x}_1(t) - \tilde{x}_2(t))\\
&+ \Big[\Big(\frac{f_1(t)c_1(t)\exp\{\tilde{x}_1(t)\}}{m_1(t)\exp\{\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}} - \frac{f_1(t)c_1(t)\exp\{\tilde{x}_2(t)\}}{m_1(t)\exp\{\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big)\\
&- \Big(\frac{(1-A(t))c_2(t)\exp\{\tilde{z}_1(t)\}}{m_2(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{y}_1(t)\}} - \frac{(1-A(t))c_2(t)\exp\{\tilde{z}_2(t)\}}{m_2(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{y}_2(t)\}}\Big)\Big]\\
&\times sgn(\tilde{y}_1(t) - \tilde{y}_2(t))\\
&+ \Big[\Big(\frac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}_1(t)\}}{m_3(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{x}_1(t)\}} - \frac{(1-A(t))f_3(t)c_3(t)\exp\{\tilde{x}_2(t)\}}{m_3(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big)\\
&+ \Big(\frac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}_1(t)\}}{m_2(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{y}_1(t)\}} - \frac{(1-A(t))f_2(t)c_2(t)\exp\{\tilde{y}_2(t)\}}{m_2(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{y}_2(t)\}}\Big)\Big]\\
&\times sgn(\tilde{z}_1(t) - \tilde{z}_2(t))\\
&= \Big[- a(t)b(t)\big(\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}\big)\\
&- \frac{c_1(t)\big(\exp\{\tilde{y}_1(t) + \tilde{x}_2(t)\} - \exp\{\tilde{y}_2(t) + \tilde{x}_1(t)\}\big)}{\big(m_1(t)\exp\{\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}\big)\big(m_1(t)\exp\{\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}\big)}\\
&- \frac{(1-A(t))c_3(t)\big(\exp\{\tilde{z}_1(t) + \tilde{x}_2(t)\} - \exp\{\tilde{z}_2(t) + \tilde{x}_1(t)\}\big)}{\big(m_3(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{x}_1(t)\}\big)\big(m_3(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{x}_2(t)\}\big)}\Big]\\
&\times sgn(\tilde{x}_1(t) - \tilde{x}_2(t))\\
&+ \Big[\frac{f_1(t)c_1(t)m_1(t)\big(\exp\{\tilde{x}_1(t) + \tilde{y}_2(t)\} - \exp\{\tilde{x}_2(t) + \tilde{y}_1(t)\}\big)}{\big(m_1(t)\exp\{\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}\big)\big(m_1(t)\exp\{\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}\big)}\\
&- \frac{(1-A(t))c_2(t)\big(\exp\{\tilde{z}_1(t) + \tilde{y}_2(t)\} - \exp\{\tilde{z}_2(t) + \tilde{y}_1(t)\}\big)}{\big(m_2(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{y}_1(t)\}\big)\big(m_2(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{y}_2(t)\}\big)}\Big]\\
&\times sgn(\tilde{y}_1(t) - \tilde{y}_2(t))\\
&+ \Big[\frac{(1-A(t))f_3(t)c_3(t)m_3(t)\big(\exp\{\tilde{x}_1(t) + \tilde{z}_2(t)\} - \exp\{\tilde{x}_2(t) + \tilde{z}_1(t)\}\big)}{\big(m_3(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{x}_1(t)\}\big)\big(m_3(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{x}_2(t)\}\big)}\\
&+ \frac{(1-A(t))f_2(t)c_2(t)m_2(t)\big(\exp\{\tilde{y}_1(t) + \tilde{z}_2(t)\} - \exp\{\tilde{y}_2(t) + \tilde{z}_1(t)\}\big)}{\big(m_2(t)\exp\{\tilde{z}_1(t)\} + \exp\{\tilde{y}_1(t)\}\big)\big(m_2(t)\exp\{\tilde{z}_2(t)\} + \exp\{\tilde{y}_2(t)\}\big)}\Big]\\
&\times sgn(\tilde{z}_1(t) - \tilde{z}_2(t))\\
&\leq -a(t)b(t)|\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}|
\end{aligned}$$

$$
\begin{aligned}
&+\frac{c_1(t)\exp\{\tilde{x}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&-\frac{c_1(t)\exp\{\tilde{y}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&+\frac{(1-A(t))c_3(t)\exp\{\tilde{x}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|\\
&-\frac{(1-A(t))c_3(t)\exp\{\tilde{z}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&+\frac{f_1(t)c_1(t)m_1(t)\exp\{\tilde{y}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&-\frac{f_1(t)c_1(t)m_1(t)\exp\{\tilde{x}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(y)\}|\\
&+\frac{(1-A(t))c_2(t)\exp\{\tilde{y}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|\\
&-\frac{(1-A(t))c_2(t)\exp\{\tilde{z}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&+\frac{(1-A(t))f_3(t)c_3(t)m_3(t)\exp\{\tilde{z}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&-\frac{(1-A(t))f_3(t)c_3(t)m_3(t)\exp\{\tilde{x}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|\\
&+\frac{(1-A(t))f_2(t)c_2(t)m_2(t)\exp\{\tilde{z}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&-\frac{(1-A(t))f_2(t)c_2(t)m_2(t)\exp\{\tilde{y}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|\\
=&\left\{-a(t)b(t)+\frac{\big[f_1(t)c_1(t)m_1(t)-c_1(t)\big]\exp\{\tilde{y}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\right.\\
&\left.+\frac{\big[(1-A(t))f_3(t)c_3(t)m_3(t)-(1-A(t))c_3(t)\big]\exp\{\tilde{z}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\right\}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|
\end{aligned}
$$

$$
\begin{aligned}
&+\left\{\frac{\big[c_1(t)-f_1(t)c_1(t)m_1(t)\big]\exp\{\tilde{x}_2(t)\}}{(m_1(t)\exp\{\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_1(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\right.\\
&\left.+\frac{\big[(1-A(t))f_2(t)c_2(t)m_2(t)-(1-A(t))c_2(t)\big]\exp\{\tilde{z}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\right\}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&+\left\{\frac{\big[(1-A(t))c_3(t)-(1-A(t))f_3(t)c_3(t)m_3(t)\big]\exp\{\tilde{x}_2(t)\}}{(m_3(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{x}_1(t)\})(m_3(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{x}_2(t)\})}\right.\\
&\left.+\frac{\big[(1-A(t))c_2(t)-(1-A(t))f_2(t)c_2(t)m_2(t)\big]\exp\{\tilde{y}_2(t)\}}{(m_2(t)\exp\{\tilde{z}_1(t)\}+\exp\{\tilde{y}_1(t)\})(m_2(t)\exp\{\tilde{z}_2(t)\}+\exp\{\tilde{y}_2(t)\})}\right\}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|\\
\le&\left\{-a(t)b(t)+\frac{\big[f_1(t)m_1(t)-1\big]c_1(t)G_2}{(m_1(t)g_2+g_1)^2}+\frac{\big[f_3(t)m_3(t)-1\big]c_3(t)G_3(1-A(t))}{(m_3(t)g_3+g_1)^2}\right\}\\
&\times|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&-\left\{\frac{\big[f_1(t)m_1(t)-1\big]c_1(t)g_1}{(m_1(t)G_2+G_1)^2}+\frac{\big[1-f_2(t)m_2(t)\big]c_2(t)g_3(1-A(t))}{(m_2(t)G_3+G_2)^2}\right\}\\
&\times|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&-\left\{\frac{\big[f_3(t)m_3(t)-1\big]c_3(t)g_1(1-A(t))}{(m_3(t)G_3+G_1)^2}+\frac{\big[f_2(t)m_2(t)-1\big]c_2(t)g_2(1-A(t))}{(m_2(t)G_3+G_2)^2}\right\}\\
&\times|\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}|.
\end{aligned}
$$

Not that

$$
\begin{cases}
\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}=\exp\{\zeta(t)\}(\tilde{x}_1(t)-\tilde{x}_2(t)),\\
\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}=\exp\{\eta(t)\}(\tilde{y}_1(t)-\tilde{y}_2(t)),\\
\exp\{\tilde{z}_1(t)\}-\exp\{\tilde{z}_2(t)\}=\exp\{\theta(t)\}(\tilde{z}_1(t)-\tilde{z}_2(t)),
\end{cases}
\tag{4.5}
$$

where $\zeta(t)$ lies between $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$; $\eta(t)$ lies between $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$; $\theta(t)$ lies between $\tilde{z}_1(t)$ and $\tilde{z}_2(t)$. Then, we have

$$
\begin{aligned}
&D^+V(t)\\
\le&-\left\{a(t)b(t)-\frac{\big[f_1(t)m_1(t)-1\big]c_1(t)G_2}{(m_1(t)g_2+g_1)^2}-\frac{\big[f_3(t)m_3(t)-1\big]c_3(t)G_3(1-A(t))}{(m_3(t)g_3+g_1)^2}\right\}\\
&\times g_1|\tilde{x}_1(t)-\tilde{x}_2(t)|\\
&-\left\{\frac{\big[f_1(t)m_1(t)-1\big]c_1(t)g_1}{(m_1(t)G_2+G_1)^2}+\frac{\big[1-f_2(t)m_2(t)\big]c_2(t)g_3(1-A(t))}{(m_2(t)G_3+G_2)^2}\right\}g_2|\tilde{y}_1(t)-\tilde{y}_2(t)|\\
&-\left\{\frac{\big[f_3(t)m_3(t)-1\big]c_3(t)g_1(1-A(t))}{(m_3(t)G_3+G_1)^2}+\frac{\big[f_2(t)m_2(t)-1\big]c_2(t)g_2(1-A(t))}{(m_2(t)G_3+G_2)^2}\right\}\\
&\times g_3|\tilde{z}_1(t)-\tilde{z}_2(t)|\\
\le&-\mu\big(|\tilde{x}_1(t)-\tilde{x}_2(t)|+|\tilde{y}_1(t)-\tilde{y}_2(t)|+|\tilde{z}_1(t)-\tilde{z}_2(t)|\big)\\
=&-\mu\big\|\big(\tilde{x}_1(t),\tilde{y}_1(t),\tilde{z}_1(t)\big)-\big(\tilde{x}_2(t),\tilde{y}_2(t),\tilde{z}_2(t)\big)\big\|,
\end{aligned}
$$

where

$$
\mu=\min\left\{\inf_{t\in\mathbb{R}}\left\{a(t)b(t)-\frac{\big[f_1(t)m_1(t)-1\big]c_1(t)G_2}{(m_1(t)g_2+g_1)^2}-\frac{\big[f_3(t)m_3(t)-1\big]c_3(t)G_3(1-A(t))}{(m_3(t)g_3+g_1)^2}\right\}g_1,\right.
$$

$$\inf_{t\in\mathbb{R}}\Big\{\frac{[f_1(t)m_1(t)-1]c_1(t)g_1}{(m_1(t)G_2+G_1)^2}+\frac{[1-f_2(t)m_2(t)]c_2(t)g_3(1-A(t))}{(m_2(t)G_3+G_2)^2}\Big\}g_2,$$
$$\inf_{t\in\mathbb{R}}\Big\{\frac{[f_3(t)m_3(t)-1]c_3(t)g_1(1-A(t))}{(m_3(t)G_3+G_1)^2}+\frac{[f_2(t)m_2(t)-1]c_2(t)g_2(1-A(t))}{(m_2(t)G_3+G_2)^2}\Big\}g_3\Big\}>0.$$

The condition (iii) in Theorem 19.1 of [25] is verified. We conclude that system (4.1) has a unique almost periodic solution in Γ^*. Hence, system (1.4) has a unique positive almost periodic solution in Γ. The proof of Theorem 4.2 is complete. □

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