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THE ASYMPTOTIC BEHAVIORS OF A STOCHASTIC SOCIAL EPIDEMIC MODEL WITH MULTI-PERTURBATION

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Abstract Alcohol abuse is a major social problem, which is often called social epidemic, for the some similarities to the classical infectious diseases. In this paper, we formulated a new stochastic alcoholism model based on the deterministic model proposed in [24], with the mortalities of all populations as well as the contact infected coefficient are all perturbed. Based on this model, we investigate the long-term stochastic dynamics behaviors of two equilibria of the corresponding deterministic model and point out the effect of random disturbance on the stability of the system. Finally, we carry out numerical simulations to support our theoretical results.

Keywords Stochastic differential equation, Ergodic, positive recurrence, s-tochastic Lyapunov functional.

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1. Introduction

As we all know, currently, some social behaviors such as drinking and smoking as well as drug abuse et al are more frequently occurred than any previous historical period although the rapid development of social economy and technology, with part of the reasons are probably that the pressure of people's life is increasing day by day and the way of life is diversified [10]. Among of these irrational social behaviors, drinking is particularly serious in doing harm to society and individuals, for example, drunk driving and drunk dangerous sexual behaviors frequently occurred [1, 15, 21].

There are studies show that drinking and smoking as well as drug abuse and so forth are social infectious disease, since their transmission characteristics are just similar to the infectious diseases in the common sense [26], such as influenza, maria, et al. Based on such facts, we can use the compartment theory of infectious disease to formulate appropriate mathematical model to investigate the law of the spread of alcoholism. Recently, there appeared many research papers on modeling alcoholism from several aspects. Specifically speaking, one is the prediction of the development trend of alcoholism [9, 14, 28], the second is the control of alcohol abuse [11, 24], the third is complications caused by alcohol abuse [17, 19, 22, 23], the forth is taking

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the complexity of alcohol abuse into consideration, and formulate complex network alcoholism models [2,8], and so on(please see the references therein).

We can easily found that almost all the models formulated in the above-mentioned references are deterministic. Actually, it doesn't agree with the reality completely, since drinking behavior is influenced by many random factors, such as climate, customs, habits, individual mood and so on. Considering these random factors, based on alcoholism model proposed in [25] as

$$\begin{cases} S' = \mu N - (1 - u_1(t))\frac{\beta SA}{N} - \mu S, \\ A' = (1 - u_1(t))\frac{\beta SA}{N} + \xi T - (u_2(t) + \mu)A, \\ T' = u_2(t)A - (\mu + \xi + \delta)T, \\ Q' = \delta T - \mu Q, \end{cases} \tag{1.1}$$

we modify the standard contact infection rate $\frac{\beta SA}{N}$ into saturated form as $\frac{\beta S(t)A(t)}{1+\gamma A(t)}$ and omit the corresponding control factors (please see the reasons described in [25]), we formulate a new deterministic alcoholism model as

$$\begin{cases} S' = \Pi - \frac{\beta S(t)A(t)}{1+\gamma A(t)} - \mu_s S(t), \\ A' = \frac{\beta S(t)A(t)}{1+\gamma A(t)} - \xi_1 A(t) + \xi_2 T(t) - \mu_a A(t), \\ T' = \xi_1 A(t) - \xi_2 T(t) - \eta T(t) - \mu_t T(t), \\ Q' = \eta T(t) - \mu_q Q(t). \end{cases} \tag{1.2}$$

Furthermore, we perturbed the mortality rates of all kinds of population involved due to the influence of alcohol abuse as that [25]:

$$\widetilde{\mu_s} = \mu_s + \mathrm{error}_1, \widetilde{\mu_a} = \mu_a + \mathrm{error}_2, \widetilde{\mu_t} = \mu_t + \mathrm{error}_3, \widetilde{\mu_q} = \mu_q + \mathrm{error}_4.$$

By the central limit theorem, the error terms may be approximated by normal distributions with mean zero and variance $\sigma_i^2 dt, 1 \leq i \leq 4$ respectively, so we represent them as

$$\mathrm{error}_i dt = \sigma_i dB_i(t), \quad 1 \leq i \leq 4.$$

Hence, we formerly got a stochastic alcoholism model in [25] as follows:

$$\begin{cases} dS(t) = (\Pi - \frac{\beta S(t)A(t)}{1+\gamma A(t)} - \mu_s S(t))dt + \sigma_1 S(t)dB_1(t), \\ dA(t) = (\frac{\beta S(t)A(t)}{1+\gamma A(t)} - \xi_1 A(t) + \xi_2 T(t) - \mu_a A(t))dt + \sigma_2 A(t)dB_2(t), \\ dT(t) = (\xi_1 A(t) - \xi_2 T(t) - \eta T(t) - \mu_t T(t))dt + \sigma_3 T(t)dB_3(t), \\ dQ(t) = (\eta T(t) - \mu_q Q(t))dt + \sigma_4 Q(t)dB_4(t). \end{cases} \tag{1.3}$$

The model (1.3) is obviously more objective and reasonable than the models (1.1) and (1.2). However, the model (1.3) only embodies the effect of alcohol on population mortality. Then, besides, in this paper, we will further continue to consider the random effect of alcoholism on contact infection coefficient. Hence, we let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets), and let $B_i(t), 0 \leq i \leq 4$ be 5 independent standard Brownain motion. In practice, we usually estimate a parameter by an average value plus an error term. In this case, the parameters β, μ_s, μ_a, μ_t and μ_q in equations (1.1) change to random variables $\widetilde{\beta}$, $\widetilde{\mu_s}, \widetilde{\mu_a}, \widetilde{\mu_t}$ and $\widetilde{\mu_q}$ respectively such that

$$\widetilde{\beta} = \beta + \mathrm{error}_0,\ \widetilde{\mu_s} = \mu_s + \mathrm{error}_1,\ \widetilde{\mu_a} = \mu_a + \mathrm{error}_2,\ \widetilde{\mu_t} = \mu_t + \mathrm{error}_3,\ \widetilde{\mu_q} = \mu_q + \mathrm{error}_4.$$

By the central limit theorem, the error terms may be approximated by normal distributions with mean zero and variance $\sigma_i^2 dt, 0 \leq i \leq 4$ respectively, so we represent them as follows

$$\text{error}_i dt = \sum_{j=1}^{d} \sigma_{ij} dB_j(t), \quad 0 \leq i \leq 4,$$

where $\{B_j(t), t \geq 0\}, 1 \leq j \leq d$ is a sequence of independent scaled Brownian motion.

To this end, we finally formulate a new stochastic alcoholism model as

$$\begin{cases} dS(t) = (\Pi - \dfrac{\beta S(t)A(t)}{1+\gamma A(t)} - \mu_s S(t))dt - \dfrac{S(t)A(t)}{1+\gamma A(t)} \displaystyle\sum_{j=1}^{d} \sigma_{0,j} dB_j(t) - S(t) \sum_{j=1}^{d} \sigma_{1,j} dB_j(t), \\ dA(t) = (\dfrac{\beta S(t)A(t)}{1+\gamma A(t)} - \xi_1 A(t) + \xi_2 T(t) - \mu_a A(t))dt + \dfrac{S(t)A(t)}{1+\gamma A(t)} \displaystyle\sum_{j=1}^{d} \sigma_{0,j} dB_j(t) \\ \qquad\qquad - A(t) \displaystyle\sum_{j=1}^{d} \sigma_{2,j} dB_j(t), \\ dT(t) = (\xi_1 A(t) - \xi_2 T(t) - \eta T(t) - \mu_t T(t))dt - T(t) \displaystyle\sum_{j=1}^{d} \sigma_{3,j} dB_j(t), \\ dQ(t) = (\eta T(t) - \mu_q Q(t))dt - Q(t) \displaystyle\sum_{j=1}^{d} \sigma_{4,j} dB_j(t). \end{cases} \tag{1.4}$$

Since the compartment $Q(t)$ is determined by the compartment $T(t)$, for the sake of simplicity, we can omit the compartment $Q(t)$ and just analyze the following 3-dimension SDE unless explicit emphasis throughout this paper:

$$\begin{cases} dS(t) = (\Pi - \dfrac{\beta S(t)A(t)}{1+\gamma A(t)} - \mu_s S(t))dt - \dfrac{S(t)A(t)}{1+\gamma A(t)} \displaystyle\sum_{j=1}^{d} \sigma_{0,j} dB_j(t) - S(t) \sum_{j=1}^{d} \sigma_{1,j} dB_j(t), \\ dA(t) = (\dfrac{\beta S(t)A(t)}{1+\gamma A(t)} - \xi_1 A(t) + \xi_2 T(t) - \mu_a A(t))dt + \dfrac{S(t)A(t)}{1+\gamma A(t)} \displaystyle\sum_{j=1}^{d} \sigma_{0,j} dB_j(t) \\ \qquad\qquad - A(t) \displaystyle\sum_{j=1}^{d} \sigma_{2,j} dB_j(t), \\ dT(t) = (\xi_1 A(t) - \xi_2 T(t) - \eta T(t) - \mu_t T(t))dt - T(t) \displaystyle\sum_{j=1}^{d} \sigma_{3,j} dB_j(t). \end{cases} \tag{1.5}$$

The parameters involved in (1.4) and their explanations, please see Table 1. In this paper, we will utilize stochastic analysis method and technique to investigate the behaviors of the sub-dynamic (1.5).

This paper is arranged as follows. In section 2, we put forward preliminaries including some tools for stochastic analysis and fundamental results of the corresponding deterministic model (1.3); In section 3, we discuss the existence and

Table 1. The parameters description of model(1.3).

Parameter	Description
$\mu_s, \mu_a, \mu_t, \mu_q$	Death rate in $S(t), A(t), T(t), Q(t)$ accordingly
β	Transmission coefficient between $S(t)$ and $A(t)$
Π	The recruitment constant of population
γ	Half saturation coefficient
ξ_1	The ratio of alcoholics in the treatment
ξ_2	The ratio of the treatment population who are failed and return to be alcoholisms
η	The ratio of successfully treated population and never drink hereafter
$\sigma_i, i = 0, 1, 2, 3, 4$	Disturbance intensity according to $\beta, \mu_s, \mu_a, \mu_t, \mu_q$ respectively
$B_j, j = 1, \cdots, d$	a sequence of independent scaled Brownian motions

uniqueness of the positive solution of (1.4); In section 4, we discuss the stochastic stability of alcohol free equilibrium point E_0; In section 5, we discuss the stochastic stability of the internal alcoholism equilibrium point E^*; In section 6, we carry out some simulations to support our theoretical results; In last section, we give some conclusions to end this paper.

2. Preliminaries

Firstly, we give some criteria on the ergodic property. Denote

$$R_+^l = \{x \in R^l : x_i > 0 \text{ for all } 1 \leq i \leq l\}.$$

In general, let X be a regular temporally homogeneous Markov process in $E_l \subset R_+^l$ described by the stochastic differential equation

$$dX(t) = b\left(X(t)\right) dt + \sum_{r=1}^{d} \sigma_r \left(X(t)\right) dB_r(t), \tag{2.1}$$

with initial value $X(t_0) = x_0 \in E_l$ and $B_r(t), 1 \leq r \leq d$, are standard Brownian motion defined on the above probability space. The diffusion matrix is defined as follows

$$A(x) = \left(A_{ij}(x)\right)_{1 \leq i,j \leq l}, \quad A_{ij}(x) = \sum_{r=1}^{d} \sigma_r^i(x) \sigma_r^j(x).$$

Define the differential operator L associated with equation (2.1) by

$$L = \sum_{i=1}^{l} b_i(x) \frac{\partial}{\partial x_i} + \frac{1}{2} \sum_{i,j=1}^{l} A_{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j}.$$

If L acts on a function $V \in C^{2,1}(E_l \times R_+; R)$, then

$$LV(x) = \sum_{i=1}^{l} b_i(x) \frac{\partial V}{\partial x_i} + \frac{1}{2} \sum_{i,j=1}^{l} A_{ij}(x) \frac{\partial^2 V}{\partial x_i \partial x_j},$$

where $V_x = (\frac{\partial V}{\partial x_1}, \cdots, \frac{\partial V}{\partial x_l})$ and $V_{xx} = \left(\frac{\partial^2 V}{\partial x_i \partial x_j}\right)_{l\times l}$. By Itô's formula, we have

$$dV(X(t)) = LV(X(t))dt + \sum_{r=1}^{d} V_x(X(t))\sigma_r(X(t))\,dB_r(t).$$

Lemma 2.1 ([6]). *We assume that there exists a bounded domain $U \subset E_l$ with regular boundary, having the following properties:*

(i) In the domain U and some neighborhood thereof, the smallest eigenvalue of the diffusion matrix $A(x)$ is bounded away from zero.

(ii) If $x \in E_l \backslash U$, the mean time τ at which a path issuing from x reaches the set U is finite, and $\sup_{x\in K} E_x\tau < \infty$ for every compact subset $K \subset E_l$.

Then, the Markov process $X(t)$ has a stationary distribution $\upsilon(\cdot)$ with density in E_l such that for any Borel set $B \subset E_l$

$$\lim_{t\to\infty} P(t, x, B) = \upsilon(B),$$

and

$$P_x \left\{ \lim_{T\to\infty} \frac{1}{T} \int_0^T f\big(x(t)\big)dt = \int_{E_l} f(x)\upsilon(dx) \right\} = 1,$$

for all $x \in E_l$ and $f(x)$ being a function integrable with respect to the probability measure υ.

Remark 2.1. (i) The existence of the stationary distribution with density is referred to Theorem 4.1 on page 119 and Lemma 9.4 on page 138 in [6] while the ergodicity and the weak convergence are referred to Theorem 5.1 on page 121 and Theorem 7.1 on page 130 in [6].

(ii) To verify Assumptions (B.1) and (B.2), it suffices to show that there exists a bounded domain U with regular boundary and a non-negative C^2-function V such that $A(x)$ is uniformly elliptical in U and for any $x \in E_l \backslash U$, $LV(x) \le -C$ for some $C > 0$ (See e.g. [30, pp. 1163]).

To facilitate subsequent analysis, we can derived the alcohol-free equilibrium E_0 of deterministic model (1.2), that is, $E_0 = (\frac{\Pi}{\mu_s}, 0, 0, 0)$, and get the fundamental reproduction number of alcoholism as

$$R_0 = \beta \frac{\Pi}{\mu_s} \frac{\xi_2 + \eta + \mu_t}{[\xi_1(\eta + \mu_t) + \mu_a(\xi_2 + \eta + \mu_t)]}.$$

Simultaneously, the following results is given in [25].

Theorem 2.1 ([25]). *For the system (1.2), when $R_0 < 1$, it only exists an alcohol-free equilibrium $E_0 = (\frac{\Pi}{\mu_s}, 0, 0, 0)$ and it is globally asymptoticly stable; when $R_0 > 1$, besides E_0(it is not stable), there also exists an internal equilibrium $E^* = (S^*, A^*, T^*, Q^*)$, which is globally asymptoticly stable.*

3. The existence and uniqueness of the positive solution of (1.4)

In this section, we will investigate the existence and uniqueness of the global solution of system (1.4) under the given initial conditions (i.e., it won't blow up in a limited time), we generally require that the coefficient of stochastic system (1.4) satisfy the linear growth condition and the local Lipschitz condition [16]. However, the coefficient of the system (1.4) does not satisfy the linear growth condition, therefore, the solution is likely to blow up in a finite time. In the following, we will use the analysis method of Lyapunov functional to prove global existence of positive solution.

Theorem 3.1. *For arbitrary initial value $(S(0), A(0), T(0), Q(0)) \in R_+^4$, as $t \geq 0$, the system (1.4) exists a unique positive solution $(S(t), A(t), T(t), Q(t))$, which fall in R_+^4 almost with probability 1, i.e., $(S(t), A(t), T(t), Q(t)) \in R_+^4$, a.s.*

Proof. We firstly prove that system (1.4) exists a unique local solution. Noticing that the right hand of the system (1.4) does not satisfy the local Lipschitz condition, we introduce variable transformations $S = e^u$, $A = e^v$, thus, the system (1.4) can be converted into the one whose coefficients satisfy the local Lipschitz condition.

Next, we prove that the only solution is positive and non-explosive. We assume that system (1.4) only exists the unique local solution $(S(t), A(t), T(t), Q(t))$, $t \in [0, \tau_e)$, where τ_e means the explosive time. To show that the solution is global, we should prove $\tau_e = \infty$, *a.e.*. Exists a sufficient large m_0 such that $(S(0), A(0), T(0), Q(0)) \in [\frac{1}{m_0}, m_0]$. For arbitrary positive integer $m \geq m_0$, we define the stopping time

$$\tau_m = \inf\{t \in [0, \tau_e) : \min\{S(t), A(t), T(t), Q(t)\} \leq \frac{1}{m} \quad \text{or} \\ \max\{S(t), A(t), T(t), Q(t)\} \geq m\},$$

here, $\inf \phi = \infty$(ϕ means empty set). Obviously, as $m \to \infty$, τ_m is monotonically increasing. Due to $\tau_m \leq \tau_e$, sequence $\{\tau_m\}$ is convergent. Let $\tau_\infty = \lim_{m\to\infty} \tau_m$, so $\tau_\infty \leq \tau_e$ a.s.. If we can show $\tau_\infty = \infty$, so is $\tau_e = \infty$ obviously.

We take apagoge to prove this conclusion. Assume $\tau_\infty \neq \infty$ (i.e., $\tau_\infty < \infty$), then there must be a large enough positive constant T and a small enough positive constant $\epsilon \in (0, 1)$ such that

$$P\{\tau_\infty \leq T\} > \epsilon.$$

Hence, there exists a positive integer $m_1 \geq m_0$ such that

$$P\{\tau_m \leq T\} \geq \epsilon, \forall m \geq m_1. \tag{3.1}$$

We define a Lyapunov functional $V_1 : R_+^4 \to R_+$ as follows

$$V_1(S, A, T, Q) = (S - 1 - \ln S) + (A - 1 - \ln A) + (T - 1 - \ln T) + (Q - 1 - \ln Q).$$

By the simple inequality of $u - 1 - \ln u \geq 0, \forall u > 0$, it's easy to know $V(S, A, T, Q)$ is nonnegative. In virtue of Itô formula, we calculate the stochastic differential of $V(S, A, T, Q)$ along with (1.4) as follows

$$LV_1 = \Pi - \mu_s S - \mu_a A - \mu_t T - \mu_q Q - \frac{\Pi}{S} + \frac{\beta A}{1 + \gamma A} + \mu_s - \frac{\beta S}{1 + \gamma A} + \xi_1 - \xi_2 \frac{T}{A}$$

$$
\begin{aligned}
&+\mu_a-\xi_1\frac{A}{T}+\xi_2+\eta+\mu_t-\eta\frac{T}{Q}+\mu_q+\sum_{j=1}^{d}\sigma_{0,j}^2\frac{A^2}{(1+\gamma A)^2}+\sum_{j=1}^{d}\sigma_{1,j}^2\\
&+\sum_{j=1}^{d}\sigma_{0,j}^2\frac{S^2}{(1+\gamma A)^2}+\sum_{j=1}^{d}\sigma_{2,j}^2+\sum_{j=1}^{d}\sigma_{3,j}^2+\sum_{j=1}^{d}\sigma_{3,j}^2\\
&\le C(1+S+A+Q+T)+\sum_{j=1}^{d}\sigma_{0,j}^2\frac{A^2}{(1+\gamma A)^2}+\sum_{j=1}^{d}\sigma_{0,j}^2\frac{S^2}{(1+\gamma A)^2}\\
&\le C(1+S+A+Q+T)+\sum_{j=1}^{d}\sigma_{0,j}^2A^2+\sum_{j=1}^{d}\sigma_{0,j}^2S^2\\
&\le C(1+S+A+Q+T+A^2+S^2)
\end{aligned}
\tag{3.2}
$$

for some positive constant C.

Define another Lyapunov functional $V_2 : R_+^4 \to R_+$ as follows

$$V_2(S, A, T, Q) = (S + A + T + Q)^2.$$

By calculation, we note that

$$
\begin{aligned}
LV_2 &= 2(S+A+Q+T)(\Pi-\Pi-\mu_sS-\mu_aA-\mu_tT-\mu_qQ)\\
&\quad+\sum_{j=1}^{d}\sigma_{1,j}^2S^2+\sum_{j=1}^{d}\sigma_{2,j}^2A^2+\sum_{j=1}^{d}\sigma_{3,j}^2T^2+\sum_{j=1}^{d}\sigma_{4,j}^2Q^2\\
&\le C(1+V_2),
\end{aligned}
\tag{3.3}
$$

where the last inequality is derived by the Hölder's inequality and the definition of V_2 for some positive constant C.

Combining V_1 with V_2 and define

$$V = V_1 + V_2.$$

By virtue of (3.2), (3.3), the definition of V_1, V_2 and Hölder's inequality again, we have

$$LV \le C(1 + V). \tag{3.4}$$

In fact, we define $\widetilde{V} := \exp(-Ct)(1 + V)$, then

$$L\widetilde{V} = \exp(-Ct)[-C + L(1 + V)] \le \exp(-Ct)[-CV + CV] = 0. \tag{3.5}$$

Integrating the two sides of $d\widetilde{V}$ above from 0 to $\tau_m \wedge T$, we get

$$\int_0^{\tau_m\wedge T} d\widetilde{V}(l, S(l), A(l), T(l), Q(l)) \le \int_0^{\tau_m\wedge T} LV dl + M_{\tau_m\wedge T} \le M_{\tau_m\wedge T},$$

where $\{M(t \wedge \tau_m), t \ge 0\}$ is a martingale.

Taking expectation of the two sides in the above expression to get

$$E[\widetilde{V}(\tau_m, S(\tau_m\wedge T), A(\tau_m\wedge T), T(\tau_m\wedge T), Q(\tau_m\wedge T))] \le \widetilde{V}(0, S(0), A(0), T(0), Q(0)).$$

Let $\Omega_m = \{\tau_m \le T\}$, there exists $m_1 > 0$, for $\forall m > m_1$, by (3.1), we know $P(\Omega_m) \ge \epsilon$.

For $\forall\omega \in \Omega_m$, at least one of $S(\tau_m), A(\tau_m), T(\tau_m)$ and (τ_m) equals to m or $\frac{1}{m}$. Specifically, when $S(\tau_m, \omega) = m$ or $\frac{1}{m}$; or $A(\tau_m, \omega) = m$ or $\frac{1}{m}$; or $T(\tau_m, \omega) = m$ or $\frac{1}{m}$; or $Q(\tau_m, \omega) = m$ or $\frac{1}{m}$, there will always be

$$\widetilde{V}(\tau_m \wedge T, S(\tau_m \wedge T), A(\tau_m \wedge T), T(\tau_m \wedge T), Q(\tau_m \wedge T)) \geq (m-1-\ln m) \wedge (\frac{1}{m}-1-\ln\frac{1}{m}).$$

Comprehensive consideration of (3.1) and (3.2), we can get

$$\begin{aligned}
&\widetilde{V}(0, S(0), A(0), T(0), Q(0))\\
\geq &E[1_{\Omega_m(\omega)} V(S(\tau_m \wedge T), A(\tau_m \wedge T), T(\tau_m \wedge T), Q(\tau_m \wedge T))]\\
\geq &\epsilon[(m-1-\ln m) \wedge (\frac{1}{m}-1-\ln\frac{1}{m})],
\end{aligned}$$

where 1_{Ω_m} denotes the indicator function of Ω_m. By letting $m \to \infty$, it's easy to get the following contradiction

$$\infty > V(0, S(0), A(0), T(0), Q(0)) = \infty.$$

Thus, $\tau_\infty = \infty$ a.e.. The proof is complete. □

4. Asymptotic behavior near alcohol free equilibrium point E_0

Seen from Theorem 2.1, for deterministic system (1.3), as $R_0 < 1$, alcohol free equilibrium point $E_0 = (\frac{\Pi}{\mu_s}, 0, 0, 0)$ is globally asymptotically stable, which means the alcohol population will disappear in the appropriate conditions finally, and the alcohol behavior will be effectively controlled. However, there is no any equilibrium point in the stochastic system (1.5), therefore, in this section, we will discuss the disturbance behavior of system (1.5) by investigating the stability and the ergodic property of E_0.

Theorem 4.1. *Assume $(S(t), A(t), T(t))$ is the solution of system (1.5) with the initial value $(S(0), A(0), T(0)) \in R^3_+$. If $\sum_{j=1}^d (\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2 < \frac{m}{6}, c_1 < \frac{\mu_s}{2}, c_2 < \frac{\mu_a}{2}, c_3 < \frac{\eta+\mu_t}{2}$, and*

$$R_0 = \beta\frac{\Pi}{\mu_s} \cdot \frac{\xi_2+\eta+\mu_t}{[\xi_1(\eta+\mu_t)+\mu_a(\xi_2+\eta+\mu_t)]} \leq 1,$$

then

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\int_0^T E[(\mu_s-2c_1)(S(t)-S_0)^2 + (\frac{\mu_a}{2}-c_2)A^2(t) + (\frac{\eta+\mu_t}{2}-c_3)T^2(t)]dt\\
\leq &c_0 + 2c_1 S_0^2,
\end{aligned} \tag{4.1}$$

where $m, c_i, i = 1, 2, 3$ are defined as follows

$$m = \mu_s \wedge \mu_a \wedge (\mu_t + \eta),$$

$$c_0 = \frac{\Pi^4 \sum_{j=1}^d \sigma_{0,j}^2}{m^{\frac{3}{4}}[m - 6\sum_{j=1}^d (\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2]} \cdot [\frac{1}{\mu_s}(\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)})]$$

$$c_1 = [\frac{1}{\mu_s}(\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)}) + \frac{3}{2}]\sum_{j=1}^{d}\sigma_{1,j}^2,$$

$$c_2 = \frac{3}{2}\sum_{j=1}^{d}\sigma_{2,j}^2, \qquad c_3 = (\frac{3}{2} + \frac{\mu_a+\eta+\mu_t}{2\xi_1})\sum_{j=1}^{d}\sigma_{3,j}^2.$$

Proof. Define nonnegative C^2-function $V_1 : R_+ \to R_+$ as

$$V_1(S) = \frac{(S-S_0)^2}{2},$$

then

$$\begin{aligned} LV_1 &= (S-S_0)(\Pi - \frac{\beta SA}{1+\gamma A} - \mu_s S) + \frac{1}{2}\sum_{j=1}^{n}(\frac{\sigma_{0,j}SA}{1+\gamma A} + \sigma_{1,j}S)^2 \\ &= (S-S_0)(\mu_s S_0 - \frac{\beta SA}{1+\gamma A} - \mu_s S) + \frac{1}{2}(\sigma_1 S)^2 \\ &\le -\mu_s(S-S_0)^2 - \frac{\beta S_0(S-S_0)A}{1+\gamma A} + S^2A^2\sum_{j=1}^{d}\sigma_{0,j}^2 + S^2\sum_{j=1}^{d}\sigma_{1,j}^2. \end{aligned}$$

Let

$$V_2(A,T) = A + \frac{\xi_2}{\eta+\mu_t+\xi_2}T,$$

then

$$\begin{aligned} LV_2 &= \frac{\beta SA}{1+\gamma A} - \xi_1 A + \xi_2 T - \mu_a A + \frac{\xi_2}{\eta+\mu_t+\xi_2}(\xi_1 A - \xi_2 T - \eta T - \mu_t T) \\ &= \frac{\beta(S-S_0)A}{1+\gamma A} + \frac{\beta S_0 A}{1+\gamma A} - (\xi_1+\mu_a)A + \frac{\xi_1\xi_2 A}{\eta+\mu_t+\xi_2} \\ &= \frac{\beta(S-S_0)A}{1+\gamma A} + A[\frac{\beta S_0}{1+\gamma A} - (\xi_1+\mu_a - \frac{\xi_1\xi_2}{\eta+\mu_t+\xi_2})] \\ &\le \frac{\beta(S-S_0)A}{1+\gamma A} + A[\beta S_0 - \frac{\xi_1(\eta+\mu_t)}{\eta+\mu_t+\xi_2} - \mu_a] \\ &\le \frac{\beta(S-S_0)A}{1+\gamma A}. \end{aligned}$$

The last inequality is attribute to the following relation

$$\beta S_0 - \frac{\xi_1(\eta+\mu_t)}{\eta+\mu_t+\xi_2} - \mu_a \le 0 \Longleftrightarrow R_0 = \beta\frac{\Pi}{\mu_s}\cdot\frac{\xi_2+\eta+\mu_t}{[\xi_1(\eta+\mu_t)+\mu_a(\xi_2+\eta+\mu_t)]} \le 1.$$

Consider linear combination $V_1 + S_0V_2$, then

$$L(V_1+S_0V_2) \le -\mu_s(S-S_0)^2 + S^2A^2\sum_{j=1}^{d}\sigma_{0,j}^2 + S^2\sum_{j=1}^{d}\sigma_{1,j}^2.$$

Let

$$V_3 = \frac{T^2}{2},$$

then

$$\begin{aligned}LV_3 &= T(\xi_1 A - \xi_2 T - \eta T - \mu_t T) + \frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2 \\ &= \xi_1 AT - (\eta + \mu_t + \xi_2)T^2 + \frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2.\end{aligned}$$

Let

$$V_4 = \frac{(S - S_0 + A + T)^2}{2},$$

then

$$\begin{aligned}LV_4 &= (S - S_0 + A + T)(\Pi - \mu_s S - \mu_a A - \mu_t T - \eta T) + \frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S + \sigma_{2,j}A + \sigma_{3,j}T)^2 \\ &= -\mu_s(S - S_0)^2 - \mu_a A^2 - (\eta + \mu_t)T^2 - (\mu_s + \mu_a)(S - S_0)A \\ &\quad - (\mu_s + \eta + \mu_t)(S - S_0)T - (\mu_a + \eta + \mu_t)AT + \frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S + \sigma_{2,j}A + \sigma_{3,j}T)^2 \\ &\le -\mu_s(S - S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{(\eta + \mu_t)}{2}T^2 + \frac{(\mu_s + \mu_a)^2}{2\mu_a}(S - S_0)^2 \\ &\quad + \frac{(\mu_s + \eta + \mu_t)^2}{2(\eta + \mu_t)}(S - S_0)^2 - (\mu_a + \eta + \mu_t)AT \\ &\quad + \frac{3}{2}[\sum_{j=1}^{d}(\sigma_{1,j}S)^2 + (\sigma_{2,j}A)^2 + (\sigma_{3,j}T)^2] \\ &= [-\mu_s + \frac{(\mu_s + \mu_a)^2}{2\mu_a} + \frac{(\mu_s + \eta + \mu_t)^2}{2(\eta + \mu_t)}](S - S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{\eta + \mu_t}{2}T^2 \\ &\quad - (\mu_a + \eta + \mu_t)AT + \frac{3}{2}[\sum_{j=1}^{d}(\sigma_{1,j}S)^2 + (\sigma_{2,j}A)^2 + (\sigma_{3,j}T)^2].\end{aligned}$$

In order to eliminate the cross term AT, we consider linear combination of V_3 and V_4 as $\frac{\mu_a+\eta+\mu_t}{\xi_1}V_3 + V_4$, then

$$\begin{aligned}&L[\frac{\mu_a + \eta + \mu_t}{\xi_1}V_3 + V_4] \\ \le &[-\mu_s + \frac{(\mu_s + \mu_a)^2}{2\mu_a} + \frac{(\eta + \mu_t + \mu_s)^2}{2(\eta + \mu_t)}](S - S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{\eta + \mu_t}{2}T^2 \\ &+ \frac{3}{2}[\sum_{j=1}^{d}(\sigma_{1,j}S)^2 + (\sigma_{2,j}A)^2 + (\sigma_{3,j}T)^2] + \frac{\mu_a + \eta + \mu_t}{\xi_1}\cdot\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2.\end{aligned}$$

At last, to eliminate the term $(S - S_0)^2$, we consider linear combination

$$\frac{1}{\mu_s}[\frac{(\mu_s + \mu_a)^2}{2\mu_a} + \frac{(\eta + \mu_t + \mu_s)^2}{2(\eta + \mu_t)}](V_1 + S_0V_2) + \frac{\mu_a + \eta + \mu_t}{\xi_1}V_3 + V_4.$$

Let

$$V_5 = \frac{1}{\mu_s}[\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)}](V_1+S_0V_2) + \frac{\mu_a+\eta+\mu_t}{\xi_1}V_3 + V_4.$$

Integrated all the formulas of LV_i, then

$$\begin{aligned}
LV_5 \leq & -\mu_s(S-S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{\eta+\mu_t}{2}T^2 + \frac{1}{\mu_s}[\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)}] \\
& \times (S^2A^2\sum_{j=1}^{d}\sigma_{0,j}^2 + S^2\sum_{j=1}^{d}\sigma_{1,j}^2) + \frac{3}{2}[\sum_{j=1}^{d}(\sigma_{1,j}S)^2 + (\sigma_{2,j}A)^2 + (\sigma_{3,j}T)^2] \\
& + \frac{\mu_a+\eta+\mu_t}{\xi_1}\cdot\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2 \\
:= & -\mu_s(S-S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{\eta+\mu_t}{2}T^2 + \tilde{c}_0(S+A)^4 + c_1S^2 + c_2A^2 + c_3T^2,
\end{aligned}$$

where $c_i(i=0,1,2,3)$ is defined as follows

$$\begin{aligned}
\tilde{c}_0 &= [\frac{1}{\mu_s}(\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)})]\sum_{j=1}^{d}\sigma_{0,j}^2, \\
c_1 &= [\frac{1}{\mu_s}(\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)}) + \frac{3}{2}]\sum_{j=1}^{d}\sigma_{1,j}^2, \\
c_2 &= \frac{3}{2}\sum_{j=1}^{d}\sigma_{2,j}^2, \qquad c_3 = (\frac{3}{2} + \frac{\mu_a+\eta+\mu_t}{2\xi_1})\sum_{j=1}^{d}\sigma_{3,j}^2.
\end{aligned}$$

Consider

$$V_6 = \frac{(S+A+T)^4}{4},$$

then

$$\begin{aligned}
LV_6 = & (S+A+T)^3[\Pi - \mu_sS - \mu_aA - (\mu_t+\eta)T] \\
& + \frac{3}{2}(S+A+T)^2\sum_{j=1}^{d}(\sigma_{1,j}S + \sigma_{2,j}A + \sigma_{3,j}T)^2 \\
\leq & \Pi(S+A+T)^3 - m(S+A+T)^4 + \frac{3}{2}(S+A+T)^4\sum_{j=1}^{d}(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2 \\
\leq & \frac{\Pi^4}{4m^{\frac{3}{4}}} - \frac{m}{4}(S+A+T)^4 + \frac{3}{2}(S+A+T)^4\sum_{j=1}^{d}(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2 \\
= & \frac{\Pi^4}{4m^{\frac{3}{4}}} - [\frac{m}{4} - \frac{3}{2}\sum_{j=1}^{d}(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2](S+A+T)^4,
\end{aligned}$$

where $m = \mu_s \wedge \mu_a \wedge (\mu_t+\eta)$. Seen from the definition of $V_i, i=1,2,3,4,5$,

$$V = V_5 + \frac{c_0V_6}{\frac{m}{4} - \frac{3}{2}\sum_{j=1}^{d}(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2}$$

is obviously positive definite, it's easily to know that

$$
\begin{aligned}
LV &= -\mu_s(S-S_0)^2 - \frac{\mu_a}{2}A^2 - \frac{\eta+\mu_t}{2}T^2 + c_0 + c_1S^2 + c_2A^2 + c_3T^2 \\
&\leq -(\mu_s - 2c_1)(S-S_0)^2 - (\frac{\mu_a}{2} - c_2)A^2 - (\frac{\eta+\mu_t}{2} - c_3)T^2 + c_0 + 2c_1S_0^2,
\end{aligned}
$$

where $c_0 = \frac{\Pi^4 \sum_{j=1}^d \sigma_{0,j}^2}{m^{\frac{3}{4}}[m-6\sum\limits_{j=1}^d(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2]} \cdot [\frac{1}{\mu_s}(\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\eta+\mu_t)^2}{2(\eta+\mu_t)})]$.

$$
\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\int_0^T E[(\mu_s-2c_1)(S(t)-S_0)^2 + (\frac{\mu_a}{2}A^2 - c_2)A^2(t) + (\frac{\eta+\mu_t}{2} - c_3)T^2(t)]dt \\
&\leq c_0 + 2c_1S_0^2.
\end{aligned}
$$

The proof is complete. □

Remark 4.1. In the proof, if $\sigma_0 = \sigma_1 = 0$, we see

$$LV \leq -\mu_s(S-S_0)^2 - (\frac{\mu_a}{2} - c_2)A^2 - (\frac{\eta+\mu_t}{2} - c_3)T^2,$$

Thus, the solution of system is stochastically asymptotically stable in the large ([16]) as the conditions in the theorem are satisfied.

5. Stochastic behaviors near the internal equilibrium point E^*

In this section, we will investigate the stochastic behavior of (1.5) around the internal equilibrium point E^* in deterministic model (2.2) from two aspects, that is, the one is concerned with the stability of its distribution, the other is concerned with ergodic property of equilibrium point E^*.

Theorem 5.1. *If the matrix $(\sigma_{i,j})_{0\leq i\leq 3, 1\leq j\leq d}$ are full rank in row,*

$$
\begin{aligned}
&\sum_{j=1}^d(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2 < \frac{m}{6}, \quad \mu_s > \frac{C}{{S^*}^2} + c_1, \quad \mu_a > 2\left(\frac{C}{{A^*}^2} + c_2\right), \\
&\eta + \mu_t > 2\left(\frac{C}{{T^*}^2} + c_3\right),
\end{aligned}
$$

where

$$
\begin{aligned}
&m = \mu_s \wedge \mu_a \wedge (\mu_t + \eta), \\
&c_1 = 3\sum_{j=1}^d \sigma_{1,j}^2 + [\frac{(\mu_s+\mu_a)^2}{\mu_a} + \frac{(\mu_s+\mu_t)^2}{(\eta+\mu_t)}]\frac{\sum_{j=1}^d\sigma_{1,j}^2 + \frac{S^*}{1+\gamma A^*}\sum_{j=1}^d\sigma_{0,j}^2}{\mu_s}, \\
&c_2 = 3\sum_{j=1}^d\sigma_{2,j}^2, \quad c_3 = (3 + \frac{\mu_a+\mu_t}{\xi_1})\sum_{j=1}^d\sigma_{3,j}^2, \\
&c_4 = \frac{1}{\mu_s}[\frac{(\mu_s+\mu_a)^2}{\mu_a} + \frac{(\mu_s+\mu_t)^2}{(\eta+\mu_t)}]\Bigg(\frac{S^*\sum_{j=1}^d\sigma_{2,j}^2}{1+\gamma A^*} + \frac{\xi_2{T^*}^2}{\xi_1 A^*}\frac{S^*\sum_{j=1}^d\sigma_{3,j}^2}{1+\gamma A^*}
\end{aligned}
$$

$$+\frac{\Pi^4 \sum_{j=1}^d \sigma_{0,j}^2}{m}^{\frac{3}{4}}(m-6\sum_{j=1}^d(\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2)\Bigg),$$

and

$$C = c_4 + c_1(S^*)^2 + c_2(A^*)^2 + c_3(T^*)^2,$$

then Eq. (1.5) is ergodic and positive recurrent.

Proof. Constructing Lyapunov function U_1 as

$$U_1 = \frac{(S-S^*)^2}{2},$$

calculating to get

$$\begin{aligned}
LU_1 &= (S-S^*)(\Pi - \frac{\beta SA}{1+\gamma A} - \mu_s S) + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 \\
&= (S-S^*)[\frac{\beta S^*A^*}{1+\gamma A^*} - \frac{\beta SA}{1+\gamma A} - \mu_s(S-S^*)] + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 \\
&= \beta(S-S^*)(\frac{S^*A^*}{1+\gamma A^*} - \frac{SA}{1+\gamma A}) - \mu_s(S-S^*)^2 + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 \\
&= \beta(S-S^*)[S^*A^*(\frac{1}{1+\gamma A^*} - \frac{1}{1+\gamma A}) + \frac{S^*A^*-SA}{1+\gamma A}] - \mu_s(S-S^*)^2 \\
&\quad + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 \\
&= \beta\gamma S^*A^*\frac{(A-A^*)(S-S^*)}{(1+\gamma A^*)(1+\gamma A)} - \frac{\beta S^*(S-S^*)(A-A^*)}{1+\gamma A} \\
&\quad - \frac{\beta A(S-S^*)^2}{1+\gamma A} - \mu_s(S-S^*)^2 + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 \\
&\leq -\beta S^*\frac{(S-S^*)(A-A^*)}{(1+\gamma A)(1+\gamma A^*)} + \frac{1}{2}\sum_{j=1}^d(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 - \mu_s(S-S^*)^2.
\end{aligned}$$

Next, we let

$$U_2 = A - A^* - A^*\ln\frac{A}{A^*},$$

and calculate to get

$$\begin{aligned}
LU_2 &= (1-\frac{A^*}{A})(\frac{\beta SA}{1+\gamma A} - \xi_1 A + \xi_2 T - \mu_a A) + \frac{1}{2}\sum_{j=1}^d(\frac{\sigma_{0,j}S}{1+\gamma A} - \sigma_{2,j})^2 \\
&= (1-\frac{A^*}{A})[\frac{\beta SA}{1+\gamma A} - (\xi_1+\mu_a)A] + (1-\frac{A^*}{A})\xi_2 T + +\frac{1}{2}\sum_{j=1}^d(\frac{\sigma_{0,j}S}{1+\gamma A} - \sigma_{2,j})^2 \\
&= (A-A^*)[\frac{\beta S}{1+\gamma A} - (\xi_1+\mu_a)] + \xi_2 T - \xi_2 T\frac{A^*}{A} + \frac{1}{2}\sum_{j=1}^d(\frac{\sigma_{0,j}S}{1+\gamma A} - \sigma_{2,j})^2
\end{aligned}$$

$$
\begin{aligned}
&= (A-A^*)[\frac{\beta S}{1+\gamma A}-(\frac{\beta S^*}{1+\gamma A^*}+\xi_2\frac{T^*}{A^*})]+\xi_2 T-\xi_2 T\frac{A^*}{A}+\frac{1}{2}\sum_{j=1}^{d}(\frac{\sigma_{0,j}S}{1+\gamma A}-\sigma_{2,j})^2\\
&= \beta(A-A^*)(\frac{S}{1+\gamma A}-\frac{S^*}{1+\gamma A^*})+\frac{\xi_2 T^*}{A^*}(A-A^*)+\xi_2 T-\xi_2 T\frac{A^*}{A}\\
&\quad +\frac{1}{2}\sum_{j=1}^{d}(\frac{\sigma_{0,j}S}{1+\gamma A}-\sigma_{2,j})^2\\
&= \frac{\beta(A-A^*)(S-S^*)}{1+\gamma A}+\beta S^*(A-A^*)(\frac{1}{1+\gamma A}-\frac{1}{1+\gamma A^*})\\
&\quad +\frac{\xi_2 T^*}{A^*}(A-A^*)+\xi_2 T-\xi_2 T\frac{A^*}{A}+\frac{1}{2}\sum_{j=1}^{d}(\frac{\sigma_{0,j}S}{1+\gamma A}-\sigma_{2,j})^2\\
&\le \frac{\beta(A-A^*)(S-S^*)}{1+\gamma A}+\frac{\xi_2 T^*}{A^*}(A-A^*)+\xi_2 T-\xi_2 T\frac{A^*}{A}+S^2\sum_{j=1}^{d}\sigma_{0,j}^2+\sum_{j=1}^{d}\sigma_{2,j}^2.
\end{aligned}
$$

Then, it's easy to know

$$
\begin{aligned}
&L(U_1+\frac{S^*}{1+\gamma A^*}U_2)\\
\le& -\mu_s(S-S^*)^2+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\frac{\sigma_{0,j}SA}{1+\gamma A})^2+\frac{S^*}{1+\gamma A^*}\frac{\xi_2 T^*}{A^*}(A-A^*)\\
&+\frac{S^*}{1+\gamma A^*}\xi_2 T-\frac{S^*}{1+\gamma A^*}\xi_2 T\frac{A^*}{A}+\frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2+\sum_{j=1}^{d}\sigma_{2,j}^2)\\
=& -\mu_s(S-S^*)^2+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\frac{\sigma_{0,j}SA}{1+\gamma A})^2+\frac{S^*}{1+\gamma A^*}\frac{\xi_2 T^*}{A^*}A\\
&+\frac{S^*}{1+\gamma A^*}\xi_2(T-T^*)-\frac{S^*}{1+\gamma A^*}\xi_2 T\frac{A^*}{A}+\frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2+\sum_{j=1}^{d}\sigma_{2,j}^2)\\
=& -\mu_s(S-S^*)^2-\frac{S^*}{1+\gamma A^*}\xi_2 T^*[(\frac{A}{A^*}+\frac{T}{T^*}+\frac{T}{T^*}\frac{A^*}{A}-1)]\\
&+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\frac{\sigma_{0,j}SA}{1+\gamma A})^2+\frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2+\sum_{j=1}^{d}\sigma_{2,j}^2)\\
\le& -\mu_s(S-S^*)^2-\frac{S^*}{1+\gamma A^*}\xi_2 T^*(\frac{A}{A^*}-\log\frac{A^*}{A}-1+\frac{T}{T^*}-\log\frac{T^*}{T}-1)\\
&+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\frac{\sigma_{0,j}SA}{1+\gamma A})^2+\frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2+\sum_{j=1}^{d}\sigma_{2,j}^2),
\end{aligned}
$$

where the last inequality is derived from $x \ge \log x+1, x>0$.

Let

$$U_3=\frac{(S-S^*+A-A^*+T-T^*)^2}{2},$$

we calculate to get

$$LU_3=(S-S^*+A-A^*+T-T^*)[\Pi-\mu_s S-\mu_a A-(\eta+\mu_t)T]$$

$$
\begin{aligned}
&+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\sigma_{2,j}A+\sigma_{3,j}T)^2\\
&=(S-S^*+A-A^*+T-T^*)[-\mu_s(S-S^*)-\mu_a(A-A^*)\\
&\quad-(\eta+\mu_t)(T-T^*)]+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\sigma_{2,j}A+\sigma_{3,j}T)^2\\
&=-\mu_s(S-S^*)^2-\mu_a(A-A^*)^2-(\eta+\mu_t)(T-T^*)^2\\
&\quad-(\mu_s+\mu_a)(S-S^*)(A-A^*)-(\mu_s+\mu_t)(S-S^*)(T-T^*)\\
&\quad-(\mu_a+\mu_t)(A-A^*)(T-T^*)+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\sigma_{2,j}A+\sigma_{3,j}T)^2.
\end{aligned}
$$

Let

$$U_4=\frac{(T-T^*)^2}{2},$$

similar to the technique in computing LU_1, we calculate to get

$$
\begin{aligned}
LU_4&=(T-T^*)[\xi_1(A-A^*)-(\xi_2+\eta+\mu_t)(T-T^*)]+\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2\\
&=\xi_1(A-A^*)(T-T^*)-(\xi_2+\eta+\mu_t)(T-T^*)^2+\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2.
\end{aligned}
$$

Considering the linear combination of U_3 and U_4 as

$$U_3+\frac{\mu_a+\mu_t}{\xi_1}U_4,$$

then

$$
\begin{aligned}
&L[U_3+\frac{\mu_a+\mu_t}{\xi_1}U_4]\\
&=-\mu_s(S-S^*)^2-\frac{\mu_a}{2}(A-A^*)^2-(\frac{\eta+\mu_t}{2}+\frac{\mu_a+\mu_t}{\xi_1}(\xi_2+\eta+\mu_t))(T-T^*)^2\\
&\quad+\frac{(\mu_s+\mu_a)^2}{2\mu_a}(S-S^*)^2+\frac{(\mu_s+\mu_t)^2}{2(\eta+\mu_t)}(S-S^*)^2+\frac{\mu_a+\mu_t}{\xi_1}\cdot\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2\\
&\quad+\frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S+\sigma_{2,j}A+\sigma_{3,j}T)^2\\
&\le-\mu_s(S-S^*)^2-\frac{\mu_a}{2}(A-A^*)^2-(\frac{\eta+\mu_t}{2}+\frac{\mu_a+\mu_t}{\xi_1}(\xi_2+\eta+\mu_t))(T-T^*)^2\\
&\quad+\frac{(\mu_s+\mu_a)^2}{2\mu_a}(S-S^*)^2+\frac{(\mu_s+\mu_t)^2}{2(\eta+\mu_t)}(S-S^*)^2+\frac{\mu_a+\mu_t}{\xi_1}\cdot\frac{T^2}{2}\sum_{j=1}^{d}\sigma_{3,j}^2\\
&\quad+\frac{3}{2}\sum_{j=1}^{d}(\sigma_{1,j}^2S^2+\sigma_{2,j}^2A^2+\sigma_{3,j}^2T^2).
\end{aligned}
$$

Next, we define

$$U_5 = T - T^* - T^* \log \frac{T}{T^*},$$

then $\xi_1 A^* = (\xi_2 + \eta + \mu_t)T^*$ implies

$$\begin{aligned}
LU_5 &= (1 - \frac{T^*}{T})(\xi_1 A - (\xi_2 + \eta + \mu_t)T) + \frac{T^*}{2}\sum_{j=1}^{d}\sigma_{3,j}^2 \\
&= \xi_1 A^*(\frac{A}{A^*} - \frac{A}{A^*}\frac{T^*}{T} - \frac{T}{T^*} + 1) + \frac{T^*}{2}\sum_{j=1}^{d}\sigma_{3,j}^2 \\
&\le \xi_1 A^*[\frac{A}{A^*} - \log\frac{A}{A^*} - 1 - (\frac{T}{T^*} - \log\frac{T}{T^*} - 1)] + \frac{T^*}{2}\sum_{j=1}^{d}\sigma_{3,j}^2.
\end{aligned}$$

We let

$$U_6 = U_1 + \frac{S^*}{1+\gamma A^*}U_2 + \frac{\xi_2 T^*}{\xi_1 A^*}\frac{S^*}{1+\gamma A^*}U_5,$$

it's easy to prove that

$$\begin{aligned}
LU_6 \le &- \mu_s(S-S^*)^2 + \frac{1}{2}\sum_{j=1}^{d}(\sigma_{1,j}S + \frac{\sigma_{0,j}SA}{1+\gamma A})^2 + \frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2 + \sum_{j=1}^{d}\sigma_{2,j}^2) \\
&+ \frac{\xi_2 T^{*2}}{\xi_1 A^*}\frac{S^*}{1+\gamma A^*}\frac{\sum_{j=1}^{d}\sigma_{3,j}^2}{2} \\
\le &- \mu_s(S-S^*)^2 + \sum_{j=1}^{d}\sigma_{1,j}^2 S^2 + \sum_{j=1}^{d}\sigma_{0,j}^2(S+A)^4 + \frac{S^*}{1+\gamma A^*}(S^2\sum_{j=1}^{d}\sigma_{0,j}^2 \\
&+ \sum_{j=1}^{d}\sigma_{2,j}^2) + \frac{\xi_2 T^{*2}}{\xi_1 A^*}\frac{S^*}{1+\gamma A^*}\frac{\sum_{j=1}^{d}\sigma_{3,j}^2}{2}
\end{aligned}$$

Consider

$$U_7 = \frac{(S+A+T)^4}{4},$$

then

$$\begin{aligned}
LU_7 =&(S+A+T)^3[\Pi - \mu_s S - \mu_a A - (\mu_t + \eta)T] + \frac{3}{2}(S+A+T)^2\sum_{j=1}^{d}(\sigma_{1,j}S \\
&+ \sigma_{2,j}A + \sigma_{3,j}T)^2 \\
\le&\Pi(S+A+T)^3 - m(S+A+T)^4 + \frac{3}{2}(S+A+T)^4\sum_{j=1}^{d}(\sigma_{1,j} + \sigma_{2,j} + \sigma_{3,j})^2 \\
\le&\frac{\Pi^4}{4m^{\frac{3}{4}}} - \frac{m}{4}(S+A+T)^4 + \frac{3}{2}(S+A+T)^4\sum_{j=1}^{d}(\sigma_{1,j} + \sigma_{2,j} + \sigma_{3,j})^2 \\
=&\frac{\Pi^4}{4m^{\frac{3}{4}}} - [\frac{m}{4} - \frac{3}{2}\sum_{j=1}^{d}(\sigma_{1,j} + \sigma_{2,j} + \sigma_{3,j})^2](S+A+T)^4,
\end{aligned}$$

where $m = \mu_s \wedge \mu_a \wedge (\mu_t + \eta)$.

We consider the linear combination of

$$U = [\frac{(\mu_s+\mu_a)^2}{2\mu_a} + \frac{(\mu_s+\mu_t)^2}{2(\eta+\mu_t)}]\frac{U_6 + \frac{\sum_{j=1}^d \sigma_{0,j}^2 U_7}{\frac{m}{4} - \frac{3}{2}\sum_{j=1}^d (\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2}}{\mu_s} + U_3 + \frac{\mu_a+\mu_t}{\xi_1}U_4,$$

then

$$\begin{aligned} LU \le& -\mu_s(S-S^*)^2 - \frac{\mu_a}{2}(A-A^*)^2 - (\frac{\eta+\mu_t}{2} + \frac{\mu_a+\mu_t}{\xi_1}(\xi_2+\eta+\mu_t))(T-T^*)^2 \\ &+ \frac{c_1 S^2}{2} + \frac{c_2 A^2}{2} + \frac{c_3 T^2}{2} + c_4 \\ \le& -(\mu_s - c_1)(S-S^*)^2 - (\frac{\mu_a}{2} - c_2)(A-A^*)^2 - (\frac{\eta+\mu_t}{2} - c_3)(T-T^*)^2 + C, \end{aligned}$$

where $c_i, i = 1,2,3,4$ is represented as

$$\begin{aligned} c_1 &= 3\sum_{j=1}^d \sigma_{1,j}^2 + [\frac{(\mu_s+\mu_a)^2}{\mu_a} + \frac{(\mu_s+\mu_t)^2}{(\eta+\mu_t)}]\frac{\sum_{j=1}^d \sigma_{1,j}^2 + \frac{S^*}{1+\gamma A^*}\sum_{j=1}^d \sigma_{0,j}^2}{\mu_s}, \\ c_2 &= 3\sum_{j=1}^d \sigma_{2,j}^2, \quad c_3 = (3 + \frac{\mu_a+\mu_t}{\xi_1})\sum_{j=1}^d \sigma_{3,j}^2, \\ c_4 &= \frac{1}{\mu_s}[\frac{(\mu_s+\mu_a)^2}{\mu_a} + \frac{(\mu_s+\mu_t)^2}{(\eta+\mu_t)}]\Bigg(\frac{S^*\sum_{j=1}^d \sigma_{2,j}^2}{1+\gamma A^*} + \frac{\xi_2 T^{*2}}{\xi_1 A^*}\frac{S^*\sum_{j=1}^d \sigma_{3,j}^2}{1+\gamma A^*} \\ &\quad + \frac{\Pi^4 \sum_{j=1}^d \sigma_{0,j}^2}{m}^{\frac{3}{4}}(m - 6\sum_{j=1}^d (\sigma_{1,j}+\sigma_{2,j}+\sigma_{3,j})^2)\Bigg), \end{aligned}$$

and

$$C = c_4 + c_1(S^*)^2 + c_2(A^*)^2 + c_3(T^*)^2.$$

Choosing appropriate parameters in system (1.2) to satisfy

$$(\mu_s - c_1)(S^*)^2 > C, (\frac{\mu_a}{2} - c_2)(A^*)^2 > C, (\frac{\eta+\mu_t}{2} - c_3)(T^*)^2 > C,$$

i.e.,

$$\mu_s > \frac{C}{(S^*)^2} + c_1, \mu_a > 2(\frac{C}{(A^*)^2} + c_2), \eta + \mu_t > 2(\frac{C}{(T^*)^2} + c_3).$$

Then $LU \le -C$ for some $C > 0$ and the conditions are satisfied if $(\sigma_{i,j})_{0\le i\le 3, 1\le j\le d}$ are full rank in row. The proof is complete. □

Next, we will discuss the mean convergence of system (1.5) based on the condition in Theorem 4.1.

Theorem 5.2. *If the condition in Theorem 4.1 holds, then the following conclusions are established*

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \frac{\beta S(u)A(u)}{1+\gamma A(u)}du + \lim_{T\to\infty}\frac{1}{T}\int_0^T \mu_s S(u)du = \Pi, \ a.e.,$$

and

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T\frac{\beta S(u)A(u)}{1+\gamma A(u)}du+\lim_{T\to\infty}\frac{1}{T}\int_0^T\xi_2T(u)du$$
$$=\lim_{T\to\infty}\frac{1}{T}\int_0^T[\xi_1A(u)+\mu_aA(u)]du,\ a.e,$$

and

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T\xi_1A(u)du=\lim_{T\to\infty}\frac{1}{T}\int_0^T[\xi_2T(u)+\eta T(u)+\mu_tT(u)]du,\ a.e.$$

Proof. By ergodic theorem, for arbitrary function f, if it satisfies

$$\int|f(x,y,z)m(dx,dy,dz)|<+\infty,$$

then

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T f(S(u),A(u),T(u))du\to\int f(x,y,z)m(dx,dy,dz)<+\infty,\ a.e.$$

Note $C=\min\{\mu_s,\mu_a,\eta+\mu_t\}$, since

$$\begin{aligned}d(S(t)+A(t)+T(t))=&\Pi-\mu_sS(t)-\mu_aA(t)-(\eta+\mu_t)T(t)\\&+\sum_{j=1}^d(\sigma_{1,j}S(t)+\sigma_{2,j}A(t)+\sigma_{3,j}T(t))dB_j(t),\end{aligned}$$

we take stochastic integral of the above formula over $[0,t], 0<t<T$,

$$\begin{aligned}S(t)+A(t)+T(t)=&S(0)+A(0)+T(0)+\Pi t\\&-\int_0^t[\mu_sS(u)+\mu_aA(u)+(\eta+\mu_t)T(u)]du\\&+\int_0^t\sum_{j=1}^d(\sigma_{1,j}S(u)+\sigma_{2,j}A(u)+\sigma_{3,j}T(u))dB_j(u)\\E[S(t)+A(t)+T(t)]\le&S(0)+A(0)+T(0)-C\int_0^tE[S(u)+A(u)+T(u)]du,\end{aligned}$$

by Grownwall inequality, we can derive

$$0\le E[S(t)+A(t)+T(t)]<M<+\infty.$$

Furthermore,

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T[S(t)+A(t)+T(t)]\wedge n\,dt=\int(x+y+z)\wedge n\,m(dx,dy,dz).$$

After taking expectation of this formula, by the control convergence theorem, we know

$$\int_{R_+^3}(x+y+z)\wedge n\,m(dx,dy,dz)\le M<+\infty.$$

Let $n \to +\infty$, then

$$\int_{R_+^3} (x+y+z)m(dx,dy,dz) < +\infty.$$

Therefore, if we define $f = \frac{\beta SA}{1+\gamma A}$ or S or A or T respectively, we can prove the corresponding convergence.

By the use of (1.3), we can get

$$S(t)=\int_0^t [\Pi-\frac{\beta S(u)A(u)}{1+\gamma A(u)}-\mu_s S(u))du+\sigma_1 S(u)dB_1(u)].$$

Hence,

$$\frac{S(t)}{t}=\frac{1}{t}\int_0^t [\Pi-\frac{\beta S(u)A(u)}{1+\gamma A(u)}-\mu_s S(u))]du+\frac{1}{t}\int_0^t \sum_{j=1}^{d}[\sigma_{1,j}S(u)+\frac{\sigma_{0,j}S(u)A(u)}{1+\gamma A}]dB_j(u). \tag{5.1}$$

By the law of large numbers of martingale,

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t \sigma_{1,j}S(u)dB_j(u) \to 0.$$

Considering that $\lim\limits_{t\to\infty}\frac{S(t)}{t}$ exists, then

$$E\lim_{t\to\infty}\frac{S(t)}{t} = \lim_{t\to\infty} E\frac{S(t)}{t}.$$

Since $ES(t) \le M, \forall t$, then

$$E\lim_{t\to\infty}\frac{S(t)}{t} = 0.$$

Thus, we proved

$$\frac{S(t)}{t} \to 0, \quad a.e.$$

Similarly, we can prove

$$\frac{A(t)}{t} \to 0, \qquad \frac{T(t)}{t} \to 0, \quad a.e.$$

From (5.1), we can conclude

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T [\Pi-\frac{\beta S(u)A(u)}{1+\gamma A(u)}-\mu_s S(u)]du = 0.$$

The others conclusions can be similarly proved. The proof is completed. □

6. Numerical simulations

In this section, to verify the results of stochastic stability, we use Milstein's numerical method to make corresponding simulations based on stochastic model (1.5) with some given initial values and parameters [7]. Thus, discreting the system (1.5), we get the following iterative scheme

$$\begin{cases} S_{i+1} = & S_i + (\Pi - \dfrac{\beta S_i A_i}{1+\gamma A_i} - \mu_s S_i) \bigtriangleup t - \displaystyle\sum_{j=1}^{d} (\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A_i} + \sigma_{1,j} S_i) c(j,i) \sqrt{\bigtriangleup t} \\ & + 0.5 \displaystyle\sum_{j=1}^{d} [(\sigma_{1,j} + \dfrac{\sigma_{0,j} A_i}{1+\gamma A_i})(\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A_i} + \sigma_{1,j} S_i) - \dfrac{\sigma_{0,j} S_i}{1+\gamma A_i}(\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A_i} \\ & - \sigma_{2,j} A_i)](c(j,i)^2 - 1) \bigtriangleup t, \\ A_{i+1} = & A_i + (\dfrac{\beta S_i A_i}{1+\gamma A_i} - \xi_1 A_i + \xi_2 T_i - \mu_a A_i) \bigtriangleup t + \displaystyle\sum_{j=1}^{d} (\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A_i} \\ & - \sigma_{2,j} A_i) c(j,i) \sqrt{\bigtriangleup t} + 0.5 \displaystyle\sum_{j=1}^{d} [\dfrac{\sigma_{0,j} A_i}{1+\gamma A_i}(\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A(i)} + \sigma_{1,j} S_i) + (\dfrac{\sigma_{0,j} S_i}{1+\gamma A_i} \\ & - \sigma_{2,j})(\dfrac{\sigma_{0,j} S_i A_i}{1+\gamma A_i} - \sigma_{2,j} A_i)](c(j,i)^2 - 1) \bigtriangleup t, \\ T_{i+1} = & T_i + (\xi_1 A_i - \xi_2 T_i - \eta T_i - \mu_t T_i) \bigtriangleup t \\ & - \displaystyle\sum_{j=1}^{d} \sigma_{3,j} T_i c(j,i) \sqrt{\bigtriangleup t} + 0.5 \displaystyle\sum_{j=1}^{d} \sigma_{3,j}^2 T_i (c(j,i)^2 - 1) \bigtriangleup t, \end{cases} \tag{6.1}$$

where $c(l,i), l = 1,2,3,4; i = 1,2,\cdots,n$ are independent Gaussian random variables $N(0,1)$, $\sigma_i, i = 1,2,3,4$ are intensities of white noise.

We choose appropriate initial values $(S(0), A(0), T(0), Q(0)) = (6000, 70, 20, 5)$ and system parameter values as $\gamma = 0.2$ as well as population mortality are $\mu_s = 0.001, \mu_a = 0.005, \mu_t = 0.003, \mu_q = 0.0015$ respectively by the latest report from WHO [26]. In addition, some parameters are reasonably estimated as $\eta = 0.3, \xi_1 = \xi_2 = 0.2$. For the sake of simplicity, we will only consider the evolution curves of $S(t)$ and $A(t)$. Furthermore, we will select different critical parameters to discuss stability of the stochastic solution of (1.5) as follows.

(1) Choosing population recruitment rate as $\Pi = 0.1$ and alcohol infection rate as $\beta = 0.001$ to make $R_0 = 0.797 < 1$. In addition, we let disturbance intensity be $\sigma_i = 0.05(i = 1,2,3,4.)$(see figs.1-2).

Synthesizing and comparing the information in Figures 1-2, we can conclude that when $R_0 < 1$, the solution of stochastic system (1.5) will randomly tend to the alcohol-free equilibrium $E_0 = (\frac{\Pi}{\mu_s}, 0, 0, 0)$ of deterministic system (1.2). Specifically speaking, if we can take some effective measures, for example, controlling the population recruitment rate Π or reducing the infection rate β, the population in $A(t), T(t), Q(t)$ will eventually tends to zero while the population in $S(t)$ will eventually tends to a constant $\frac{\Pi}{\mu_s}$. Figures 1-2 agree to the conclusions in Theorem 4.1.

(2) Choosing population recruitment rate as $\Pi = 0.2$ and alcohol infection rate as $\beta = 0.002$ to make $R_0 = 1.59 > 1$. To compare, we still let disturbance intensity be $\sigma_i = 0.05(i = 1,2,3,4)$ respectively(see figs.3-4).

Similarly, by synthesizing and comparing the information in Figures 3-4, we can conclude that in one hand, when $R_0 > 1$, the solution of stochastic system (1.5) will randomly tend to the internal equilibrium point $E^* = (S^*, A^*, T^*, Q^*)$

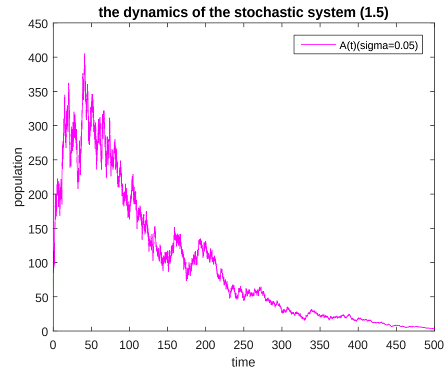

Figure 1. $R_0 < 1$, population evolution curve of $A(t)$ with disturbance intensity $\sigma_i = 0.05 (i = 1, 2, 3, 4.)$

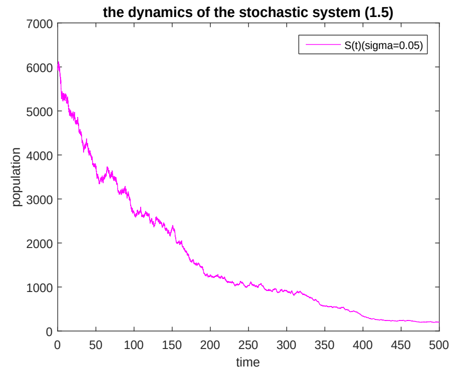

Figure 2. $R_0 < 1$, population evolution curve of $S(t)$ with disturbance intensity $\sigma_i = 0.05 (i = 1, 2, 3, 4.)$

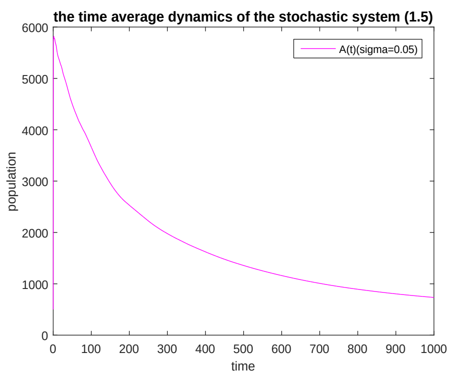

Figure 3. $R_0 > 1$, population evolution curve of $A(t)$ with disturbance intensity $\sigma_i = 0.05 (i = 1, 2, 3, 4.)$

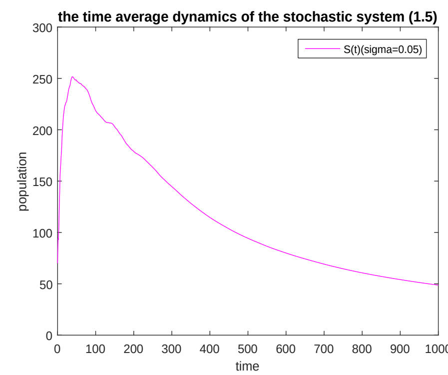

Figure 4. $R_0 > 1$, population evolution curve of $S(t)$ with disturbance intensity $\sigma_i = 0.05 (i = 1, 2, 3, 4.)$

of deterministic system corresponding to $\sigma_i = 0 (i = 1, 2, 3, 4)$ in (1.5). Specifically speaking, if the population recruitment rate Π or the infection rate β is rather large, the population in $A(t), T(t), Q(t)$ will increase with time going and eventually tends to a fixed level, while he population in $S(t)$ will randomly vary with time going and eventually tends to a constant value. Figures 3-4 also reveals the fact that if system parameters meet the conditions to make $R_0 > 1$, then the model is ergodic and has a uniqueness stationary distribution which illustrates the persistence. All in all, Figures 3-4 agree to the conclusions in Theorem 5.1 and Theorem 5.2.

7. Competing Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

8. Acknowledgements

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