Journal of Applied Analysis and Computation
Volume 7, Number 4, November 2017, 1312–1322

Website:http://jaac-online.com/
DOI:10.11948/2017080

APPROXIMATE SOLUTIONS FOR TIME-FRACTIONAL TWO-COMPONENT EVOLUTIONARY SYSTEM OF ORDER 2 USING COUPLED FRACTIONAL REDUCED DIFFERENTIAL TRANSFORM METHOD*

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Abstract In this paper, Coupled Fractional Reduced Differential Transform method is extended to apply to the generalized time-fractional two-component evolutionary system of order 2. By using this method, the solutions in the form of a generalized Taylor series are obtained. The graphics of numerical solutions together with the error analysis demonstrate that the present method is effective and accurate for obtaining approximate solutions of fractional coupled equations. Moreover, the results also indicate that the solutions obtained by residual power series method in previous literature (M. Alquran, Analytical solution of time-fractional two-component evolutionary system of order 2 by residual power series method, J. Appl. Anal. Comput.,5(2015)(4), 589-599.) contain errors.

Keywords Coupled fractional reduced differential transform, generalized Taylor series, residual power series method, time-fractional two-component evolutionary system of order 2.

MSC(2010) 26A33, 35R11, 35C10.

1. Introduction

The fractional calculus, including integrals and derivatives of arbitrary order, was first founded by Leibnitz in 1695 [16]. It is a generalization of classical integer-order calculus. In the past decades, the theory of fractional calculus has been widely used in various areas of engineering, physics and other fields of applied sciences [3,6,8,12]. Due to its wide application, the fractional calculus has aroused wide concern. The fractional differential equations as mathematical tools form of fractional calculus can describe various phenomena more reasonably and reflect the physical reality better than the integer-order differential equations. For more details, see [1, 9, 13, 14] and the references therein.

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*The authors were supported by National Natural Science Foundation of China (No.11601192), Natural Science Foundation of Jiangsu Province (No. BK20140522), Startup Fund for Advanced Talents of Jiangsu University (No. 10JDG124).

It is well known that two-component evolutionary system of order 2 arises quite frequently in fluid mechanics, solid state physics, plasma physics and other mathematical physics [11]. Very recently, the author in [2] has proposed a time-fractional two-component evolutionary system of order 2 by introducing the fractional derivative of order α. The new system takes the form

$$\begin{aligned} \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} &= -3v_{xx}(x,t), \\ \frac{\partial^\alpha v(x,t)}{\partial t^\alpha} &= u_{xx}(x,t) + 4u^2(x,t), \end{aligned} \tag{1.1}$$

where $0 \leq \alpha \leq 1$. Note that $\alpha = 1$, system (1.1) becomes the standard two-component evolutionary system of order 2.

Compared to the integer-order differential equations, it is more difficult to construct and develop approximate and analytical methods to solve the fractional differential equations. Therefore, a great deal of efforts have been put to find numerical and exact solutions of the fractional differential equations, such as variation iteration method [22], Adomian decomposition method [20], collocation method [4,5], Laplace-Homotopy perturbation method [10], residual power series method (RPSM) [2,21], coupled fractional reduced differential transform method (CFRDTM) [18,19].

The CFRDTM has been proposed and developed in [18,19]. This method originated from generalized Taylor's formula [15]. Like the RPSM, it provides the power series approximate solution with fast convergence. The CFRDTM is effective and simple method to construct approximate solutions. It has been successfully implemented to get soliton solutions of time fractional coupled modified KdV equations [18], to obtain traveling wave solutions of time fractional Whitham-Broer-Kaup equations [19] and so on.

The purpose of this paper is to make use of the CFRDTM to construct approximate solutions of system (1.1). After some computations, the approximate analytical solutions with high accuracy can be acquired in the form of a generalized Taylor series. On the other hand, the results also indicate that the solutions obtained by the RPSM in the previous literature contain errors. The correct results are also proposed in this paper.

The rest of the paper has been organized as follows. In Section 2, the brief description of fractional calculus is reviewed. In Section 3, the CFRDTM has been introduced. In Section 4, using the CFRDTM, we derive the approximate solutions for system (1.1). In the mean time, we also compare the results with the solutions obtained in the previous literature. By comparing, we find the previous results contain errors. Finally, we present a short conclusion.

2. Preliminaries

In this section, we review some fundamental definitions and preliminary results of fractional calculus. Unlike classical integer-order calculus, there are different definitions of fractional operators including Riemann-Liouville fractional derivative, Caputo derivative, Riesz derivative and Grunwald-Letnikov fractional derivative [17]. In this paper, the fractional derivative is Caputo type, which was first introduced by Caputo in the late 1960s.

Definition 2.1. For m to be the smallest integer that exceeds α, the Caputo fractional derivatives of order $\alpha > 0$ is defined as

$$D_t^{\alpha} u(x,t) = \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{(m-\alpha-1)} \frac{\partial^m u(x,\tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m \in N. \end{cases} \quad (2.1)$$

We collect some properties of the Caputo fractional derivative as follows:

$$D^{\alpha} C = 0, \text{ (C is a constant)},$$
$$D^{\alpha}(\gamma f(t) + \delta g(t)) = \gamma D^{\alpha} f(t) + \delta D^{\alpha} g(t), \text{ (γ and δ are constants)},$$

and

$$D^{\alpha} t^{\beta} = \begin{cases} 0, & \beta \le \alpha - 1, \\ \frac{\Gamma(\beta+1) t^{\beta-\alpha}}{\Gamma(\beta-\alpha+1)}, & \beta \ge \alpha - 1. \end{cases}$$

Its corresponding Leibnitz's rule is following

$$D^{\alpha}(g(t)f(t)) = \sum_{k=0}^{\infty} \binom{\alpha}{k} g^{(k)}(t) D^{\alpha-k} f(t),$$

if $f(\tau)$ is continuous in $[0,t]$ and $g(\tau)$ has $n+1$ continuous derivatives in $[0,t]$.

Next, we will state Generalized Taylor's formula. For more detail, the reader is referred to [7,15].

Theorem 2.1 (Generalized Taylor's formula [15]). *Suppose that $D_a^{k\alpha} f(t) \in C(a,b]$ for $k = 0,1,...,n+1$, where $0 < \alpha \le 1$, we have*

$$f(t) = \sum_{i=0}^{n} \frac{(t-a)^{i\alpha}}{\Gamma(i\alpha+1)} [D_a^{i\alpha} f(t)]_{t=\alpha} + \mathfrak{R}_n^{\alpha}(t;a) \quad (2.2)$$

with $\mathfrak{R}_n^{\alpha}(t;a) = \frac{(t-a)^{(n+1)\alpha}}{\Gamma((n+1)\alpha+1)} [D_a^{(n+1)\alpha} f(t)]_{t=\xi}$, $a \le \xi \le t$, $\forall t \in (a,b]$, *where* $D_a^{k\alpha} = D_a^{\alpha} \cdot D_a^{\alpha} \cdot D_a^{\alpha} \cdots D_a^{\alpha}$ *(k times).*

3. Coupled fractional reduced differential transform method

For a better description of the CFRDTM, we will firstly give some interpretations. Notice that the fractional derivatives of the two equations in system (1.1) are the same α. In fact, the CFRDTM can solve more generalized time-fractional two-component evolutionary system of order 2, which has the following form:

$$\begin{aligned} \frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} &= -3 v_{xx}(x,t), \\ \frac{\partial^{\beta} v(x,t)}{\partial t^{\beta}} &= u_{xx}(x,t) + 4u^2(x,t), \end{aligned} \quad (3.1)$$

where $0 \le \alpha \le 1$, $0 \le \beta \le 1$.

In this section, we will introduce the CFRDTM to solve more generalized system (3.1). It is clear that system (1.1) is just the special case of system (3.1).

For convenience, we use $U(h, k-h)$ to denote the coupled fractional reduced differential transform of $u(x,t)$. Suppose that $u(x,t)$ is analytic and differentiated continuously with respect to time t, and then the fractional coupled reduced differential transform of $u(x,t)$ is defined as

$$U(h, k-h) = \frac{1}{\Gamma(h\alpha + (k-h)\beta + 1)}[D_t^{(h\alpha+(k-h)\beta)} u(x,t)]_{t=0}, \tag{3.2}$$

whereas the inverse transform of $U(h, k-h)$ is

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{k} U(h, k-h) t^{h\alpha+(k-h)\beta}, \tag{3.3}$$

which is one of the solution of coupled fractional differential equations.

Here, we present the import theorem of the fractional coupled reduced differential transform from [18, 19].

Theorem 3.1. *Suppose that $U(h, k-h)$ and $V(h, k-h)$ are the coupled fractional reduced differential transform of the functions $u(x,t)$ and $v(x,t)$, respectively.*

(i) If $u(x,t) = f(x,t) \pm g(x,t)$, then $U(h, k-h) = F(h, k-h) \pm G(h, k-h)$.

(ii) If $u(x,t) = af(x,t)$, where $a \in R$, then $U(h, k-h) = aF(h, k-h)$.

(iii) If $f(x,t) = u(x,t)v(x,t)$, then $F(h, k-h) = \sum_{l=0}^{h}\sum_{s=0}^{k-h} U(h-l, s)V(l, k-h-s)$.

(iv) If $f(x,t) = D_t^{\alpha} u(x,t)$, then

$$F(h, k-h) = \frac{\Gamma((h+1)\alpha + (k-h)\beta + 1)}{\Gamma(h\alpha + (k-h)\beta + 1)} U(h+1, k-h).$$

(v) If $f(x,t) = D_t^{\beta} v(x,t)$, then

$$F(h, k-h) = \frac{\Gamma(h\alpha + (k-h+1)\beta + 1)}{\Gamma(h\alpha + (k-h)\beta + 1)} V(h, k-h+1).$$

4. Application

Consider the following time-fractional two-component evolutionary system of order 2 [2]:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = -3v_{xx}(x,t), \tag{4.1}$$

$$\frac{\partial^{\alpha} v(x,t)}{\partial t^{\alpha}} = u_{xx}(x,t) + 4u^2(x,t), \tag{4.2}$$

subject to the initial conditions:

$$u(x,0) = -\frac{3}{4(1+\cos(x))}, \tag{4.3}$$

$$v(x,0) = \frac{\sqrt{3}}{4}\tan(\frac{x}{2}). \tag{4.4}$$

For the special case, where $\alpha = 1$, the exact solutions of Eqs.(4.1), (4.2) are given by

$$u(x,t) = -\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2} + \frac{\sqrt{3}}{2}t), \tag{4.5}$$

$$v(x,t) = \frac{\sqrt{3}}{4}\tan(\frac{x}{2} + \frac{\sqrt{3}}{2}t). \tag{4.6}$$

In the purpose of using the CFRDTM for solving time-fractional two-component evolutionary system of order 2, we derive the recursive formula from Eqs.(4.1), (4.2) in the first step. We suppose that $U(h, k-h)$ and $V(h, k-h)$ are the coupled fractional reduced differential transform of $u(x,t)$ and $v(x,t)$, respectively. Here, $u(x,t)$ and $v(x,t)$ are assumed to be the solutions of Eqs.(4.1), (4.2) and $U(0,0) = u(x,0)$,$V(0,0) = v(x,0)$ are the given initial conditions. Without loss of generality, we assume that

$$U(0,j) = 0,\ j = 1,2,3,\cdots \text{ and } V(i,0) = 0,\ i = 1,2,3\cdots.$$

By applying CFRDTM to Eq.(4.1), we have the following recursive formula

$$\frac{\Gamma((k+1)\alpha+1)}{\Gamma(k\alpha+1)}U(h+1,k-h) = -3\frac{\partial^2}{\partial x^2}V(h,k-h). \tag{4.7}$$

From the initial condition (4.3), we have

$$U(0,0) = u(x,0) = -\frac{3}{4(1+\cos(x))} = -\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2}). \tag{4.8}$$

Similarly, the recursive formula from Eq.(4.2) can be obtained as follows:

$$\frac{\Gamma((k+1)\alpha+1)}{\Gamma(k\alpha+1)}V(h,k-h+1) = \frac{\partial^2}{\partial x^2}U(h,k-h) + 4\sum_{i=0}^{h}\sum_{s=0}^{k-h}U(i,k-h-s)U(h-i,s). \tag{4.9}$$

Due to the initial condition of Eq.(4.4), we have

$$V(0,0) = v(x,0) = \frac{\sqrt{3}}{4}\tan(\frac{x}{2}). \tag{4.10}$$

According to the process of CFRDTM, using recursive formulas (4.7) and (4.9) together with initial conditions (4.8) and (4.10), we can easily get

$$\begin{aligned}
U(1,0) &= \frac{-3\sqrt{3}\sec^2(\frac{x}{2})\tan(\frac{x}{2})}{8\Gamma(1+\alpha)},\\
V(0,1) &= \frac{3\sec^2(\frac{x}{2})}{8\Gamma(1+\alpha)},\\
U(1,1) &= \frac{-9\sec^2(\frac{x}{2})(3\tan^2(\frac{x}{2})+1)}{16\Gamma(1+2\alpha)},\\
V(0,2) &= 0,\\
U(2,0) &= 0,\\
V(1,1) &= \frac{3\sqrt{3}(\tan(\frac{x}{2})+\tan^3(\frac{x}{2}))}{8\Gamma(1+2\alpha)}.
\end{aligned}$$

The other recursive expressions can be derived by the same manner as above.

To obtain the approximate solutions, we substitute the recursive expressions into (3.3), *i.e.*,

$$\begin{aligned}u(x,t) &= \sum_{k=0}^{\infty}\sum_{h=0}^{k} U(h,k-h)t^{k\alpha} \\ &= U(0,0) + \sum_{k=1}^{\infty}\sum_{h=1}^{k} U(h,k-h)t^{k\alpha} \\ &= -\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2}) - \frac{3\sqrt{3}\sec^2(\frac{x}{2})\tan(\frac{x}{2})}{8\Gamma(1+\alpha)}t^{\alpha} \\ &\quad - \frac{9\sec^2(\frac{x}{2})(3\tan^2(\frac{x}{2})+1)}{16\Gamma(1+2\alpha)}t^{2\alpha} + \cdots\end{aligned} \tag{4.11}$$

and

$$\begin{aligned}v(x,t) &= \sum_{k=0}^{\infty}\sum_{h=0}^{k} V(h,k-h)t^{k\alpha} \\ &= V(0,0) + \sum_{k=1}^{\infty}\sum_{h=0}^{k} V(h,k-h)t^{k\alpha} \\ &= \frac{\sqrt{3}}{4}\tan(\frac{x}{2}) + \frac{3\sec^2(\frac{x}{2})}{8\Gamma(1+\alpha)}t^{\alpha} + \frac{3\sqrt{3}(\tan(\frac{x}{2})+\tan^3(\frac{x}{2}))}{8\Gamma(1+2\alpha)}t^{2\alpha} + \cdots.\end{aligned} \tag{4.12}$$

Thus, the approximate solutions in the series form for Eqs.(4.1), (4.2) are obtained respectively. Now, let us discuss the solutions in the special case of $\alpha = 1$. When $\alpha = 1$, the solutions are given by

$$\begin{aligned}u(x,t) = &-\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2}) - \frac{3\sqrt{3}}{8}\sec^2(\frac{x}{2})\tan(\frac{x}{2})t \\ &-\frac{9}{32}\sec^2(\frac{x}{2})(3\tan^2(\frac{x}{2})+1)t^2 + \cdots\end{aligned} \tag{4.13}$$

and

$$v(x,t) = \frac{\sqrt{3}}{4}\tan(\frac{x}{2}) + \frac{3}{8}\sec^2(\frac{x}{2})t + \frac{3\sqrt{3}}{16}(\tan(\frac{x}{2}) + \tan^3(\frac{x}{2}))t^2 + \cdots. \tag{4.14}$$

We find that the solutions (4.13) and (4.14) are exactly same as the Taylor series expansions of the exact solutions

$$\begin{aligned}u(x,t) &= -\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2} + \frac{\sqrt{3}}{2}t) \\ &= -\frac{3}{8} - \frac{3}{8}\tan^2(\frac{x}{2}) - \frac{3\sqrt{3}}{8}(\sec^2(\frac{x}{2})\tan(\frac{x}{2}))t \\ &\quad -\frac{9}{32}(\sec^2(\frac{x}{2})(3\tan^2(\frac{x}{2})+1))t^2 + \cdots\end{aligned} \tag{4.15}$$

and

$$\begin{aligned} v(x,t) &= \frac{\sqrt{3}}{4}\tan(\frac{x}{2}+\frac{\sqrt{3}}{2}t) \\ &= \frac{\sqrt{3}}{4}\tan(\frac{x}{2})+\frac{3}{8}\sec^2(\frac{x}{2})t+\frac{3\sqrt{3}}{16}(\tan(\frac{x}{2})+\tan^3(\frac{x}{2}))t^2+\cdots . \end{aligned} \tag{4.16}$$

In order to illustrate the efficiency and accuracy of the present method, we construct n-th truncated series of $u(x,t)$, $v(x,t)$, which are exactly same as the n-th truncated Taylor series of the exact solutions. Let $u_n(x,t)$ and $v_n(x,t)$ denote the n-th truncated series as follows:

$$u_n(x,t) = \sum_{k=0}^{n}\sum_{h=0}^{k} U(h,k-h)t^{k\alpha},$$
$$v_n(x,t) = \sum_{k=0}^{n}\sum_{h=0}^{k} V(h,k-h)t^{k\alpha}.$$

Therefore, the 1-st truncated series solutions are

$$\begin{aligned} u_1(x,t) &= -\frac{3}{8}-\frac{3}{8}\tan^2(\frac{x}{2})-\frac{3\sqrt{3}\sec^2(\frac{x}{2})\tan(\frac{x}{2})}{8\Gamma(1+\alpha)}t^{\alpha}, \\ v_1(x,t) &= \frac{\sqrt{3}}{4}\tan(\frac{x}{2})+\frac{3\sec^2(\frac{x}{2})}{8\Gamma(1+\alpha)}t^{\alpha}. \end{aligned} \tag{4.17}$$

The 2-nd truncated series solutions have the form

$$\begin{aligned} u_2(x,t) &= u_1(x,t)-\frac{9\sec^2(\frac{x}{2})(3\tan^2(\frac{x}{2})+1)}{16\Gamma(1+2\alpha)}t^{2\alpha}, \\ v_2(x,t) &= v_1(x,t)+\frac{3\sqrt{3}(\tan(\frac{x}{2})+\tan^3(\frac{x}{2}))}{8\Gamma(1+2\alpha)}t^{2\alpha}. \end{aligned} \tag{4.18}$$

The 3-rd and the 4-th truncated series solutions are represented as

$$\begin{aligned} u_3(x,t) &= u_2(x,t)-\frac{9\sqrt{3}(2\tan(\frac{x}{2})+5\tan^3(\frac{x}{2})+3\tan^5(\frac{x}{2}))}{8\Gamma(1+3\alpha)}t^{3\alpha}, \\ v_3(x,t) &= v_2(x,t)+(\frac{9\sec^2(\frac{x}{2})(2-6\tan^2(\frac{x}{2})-12\tan^4(\frac{x}{2}))}{32\Gamma(1+3\alpha)} \\ &\quad +\frac{27\sec^4(\frac{x}{2})\tan^2(\frac{x}{2})\Gamma(1+2\alpha)}{16(\Gamma(1+\alpha))^2\Gamma(1+3\alpha)})t^{3\alpha}, \end{aligned} \tag{4.19}$$

and

$$\begin{aligned} u_4(x,t) &= u_3(x,t)+(\frac{27\sec^2(\frac{x}{2})(1+30\tan^2(\frac{x}{2})+90\tan^4(\frac{x}{2})+63\tan^6(\frac{x}{2}))}{16\Gamma(1+4\alpha)} \\ &\quad +\frac{-81\Gamma(1+2\alpha)\sec^4(\frac{x}{2})(21\tan^4(\frac{x}{2})+14\tan^2(\frac{x}{2})+1)}{32(\Gamma(1+\alpha))^2\Gamma(1+4\alpha)})t^{4\alpha}, \\ v_4(x,t) &= v_3(x,t)+(\frac{27\sqrt{3}\sec^2(\frac{x}{2})(2\tan(\frac{x}{2})+5\tan^3(\frac{x}{2})+3\tan^5(\frac{x}{2}))}{8\Gamma(1+4\alpha)} \end{aligned}$$

$$+\frac{-9\sqrt{3}\tan(\frac{x}{2})(1+\tan^2(\frac{x}{2}))(17+60\tan^2(\frac{x}{2})+45\tan^4(\frac{x}{2}))}{16\Gamma(1+4\alpha)}$$
$$+\frac{27\sqrt{3}\Gamma(1+3\alpha)\sec^4(\frac{x}{2})\tan(\frac{x}{2})(3\tan^2(\frac{x}{2})+1)}{16\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1+4\alpha)})t^{4\alpha}. \tag{4.20}$$

Now, let us compare our results with the residual power series (RPS) solutions obtained in [2]. Since both of the solutions were calculated in the form of a generalized Taylor series, it is possible to compare. It is evident that the 1-st and 2-nd RPS approximate solutions obtained by the RPSM in [2] are identical to (4.17) and (4.18), respectively. However, we find that the 3-rd and 4-th RPS approximate solutions presented in [2] are not in agreement with (4.19) and (4.20). They are the same only in the special case of $\alpha = 1$. Notice that there are errors on p.595 in [2]: when the author applied $D_t^{2\alpha}$ on both sides of Eq.(3.22), the terms in (3.23) are wrong. As a result, the expressions of $g_3(x)$ and $g_4(x)$ in (3.24) and (3.25) contain errors in [2]. The reason for the errors is that the author mixed the derivation rule of fractional derivative with integer-order derivative. By the properties of the Caputo fractional derivative, when $\beta > \alpha - 1$, we have

$$D_t^\alpha t^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}t^{\beta-\alpha}. \tag{4.21}$$

Unfortunately, the author in [2] treated it as integer-order derivative and used

$$D_t^\alpha t^\beta = \beta t^{\beta-1}D_t^\alpha t = \frac{\beta}{\Gamma(2-\alpha)}t^{\beta-\alpha}. \tag{4.22}$$

Obviously, (4.22) only holds when $\alpha = 1$. The correct expressions of $g_3(x)$ and $g_4(x)$ in [2] should be

$$g_3(x) = f_2''(x) + 8f(x)f_2(x) + 4\frac{\Gamma(1+2\alpha)}{(\Gamma(1+\alpha))^2}f_1^2(x), \tag{4.23}$$

$$g_4(x) = f_3''(x) + 8f(x)f_3(x) + 8\frac{\Gamma(1+3\alpha)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)}f_1(x)f_2(x). \tag{4.24}$$

Replacing the expressions of $g_3(x)$ and $g_4(x)$ by (4.23) and (4.24) in [2], we obtain the correct 3-rd and 4-th RPS approximate solutions, which appear to coincide with (4.19) and (4.20).

Figure 1 explores the 4-th truncated series solutions of $u(x,t)$ for different values of the fractional order α. The comparison between the approximate solution and exact solution for $\alpha = 1$ is also shown in Figure 1. The corresponding graphics of $v_4(x,t)$ and $v(x,t)$ are demonstrated in Figure 2.

When $\alpha = 1$, the absolute errors between the 4-th truncated series solutions obtained by the CFRDTM and the exact solutions for Eqs.(4.1), (4.2) are given in Tables 1 and 2. From Tables 1-2, it is not difficult to observe that we can obtain good approximate solutions by using the CFRDTM.

5. Conclusion

In this paper, we have shown the solvability of the CFRDTM for time-fractional two-component evolutionary system of order 2. In theory, we can get the exact solutions

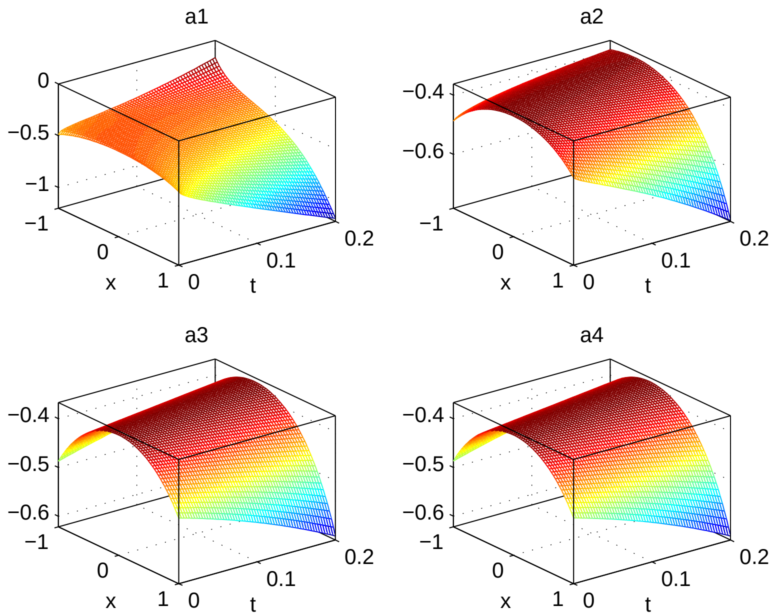

Figure 1. The 4-th truncated series solutions of $u(x,t)$: (a1) $u_4(x,t,\alpha=0.5)$, (a2) $u_4(x,t,\alpha=0.75)$, (a3) $u_4(x,t,\alpha=1)$, (a4) $u(x,t,\alpha=1)$.

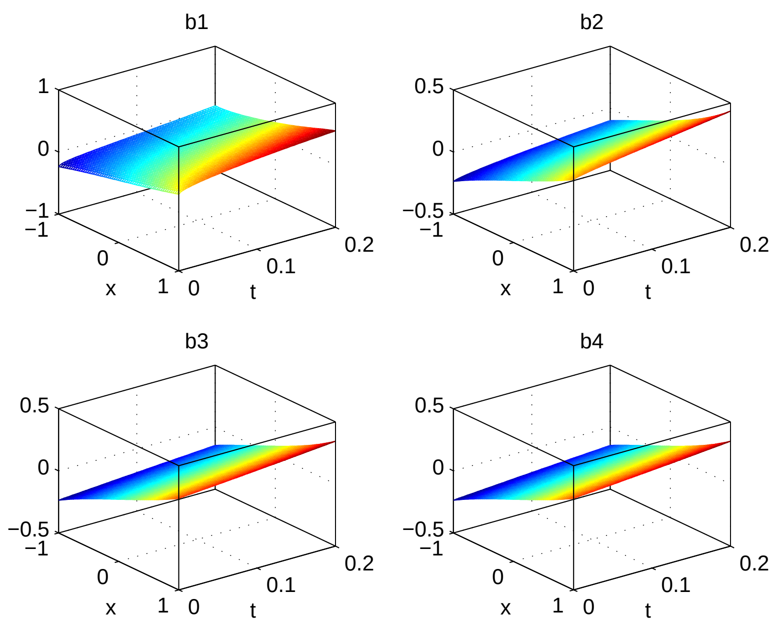

Figure 2. The 4-th truncated series solutions of $v(x,t)$: (b1) $v_4(x,t,\alpha=0.5)$, (b2) $v_4(x,t,\alpha=0.75)$, (b3) $v_4(x,t,\alpha=1)$, (b4) $v(x,t,\alpha=1)$.

Table 1. The absolute errors of $u_4(x,t)$ obtained by the CFRDTM.

t/x	0.1	0.2	0.3	0.4	0.5
0.01	2.1577×10^{-12}	4.4156×10^{-12}	7.0073×10^{-12}	1.0146×10^{-11}	1.4110×10^{-11}
0.05	7.5267×10^{-9}	1.4678×10^{-8}	2.2942×10^{-8}	3.3004×10^{-8}	4.5770×10^{-8}
0.1	2.7288×10^{-7}	5.0610×10^{-7}	7.7789×10^{-7}	1.1111×10^{-6}	1.5360×10^{-6}
0.15	2.3226×10^{-6}	4.1321×10^{-6}	6.2586×10^{-6}	8.8827×10^{-6}	1.2247×10^{-5}
0.2	1.0879×10^{-5}	1.8691×10^{-5}	2.7947×10^{-5}	3.9445×10^{-5}	5.4265×10^{-5}

Table 2. The absolute errors of $v_4(x,t)$ obtained by the CFRDTM

t/x	0.1	0.2	0.3	0.4	0.5
0.01	2.8762×10^{-12}	3.0648×10^{-12}	3.3925×10^{-12}	3.8844×10^{-12}	4.5798×10^{-12}
0.05	9.0385×10^{-9}	9.6756×10^{-9}	1.0755×10^{-8}	1.2359×10^{-8}	1.4618×10^{-8}
0.1	2.9166×10^{-7}	3.1401×10^{-7}	3.5084×10^{-7}	4.0501×10^{-7}	4.8089×10^{-7}
0.15	2.2368×10^{-6}	2.4219×10^{-6}	2.7200×10^{-6}	3.1542×10^{-6}	3.7599×10^{-6}
0.2	9.5343×10^{-6}	1.0383×10^{-5}	1.1720×10^{-5}	1.3653×10^{-5}	1.6339×10^{-5}

of the infinite series form. On the other hand, the approximate analytical solutions can be derived by truncated series. The graphical results and error analysis reveal that the CFRDTM yields a very efficient and accurate approach to solve time-fractional two-component evolutionary system of order 2. From what has been discussed above, we may safely come to a conclusion that the CFRDTM can be used as an alternative for this type of system. Our results also reveal there are errors in the expressions of $g_3(x)$ and $g_4(x)$ in [2].

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