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ON SOME NEW ANALYTICAL SOLUTIONS FOR THE (2+1)-DIMENSIONAL BURGERS EQUATION AND THE SPECIAL TYPE OF DODD-BULLOUGH-MIKHAILOV EQUATION

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Abstract Some new travelling wave transform methods are very important for obtaining analytical solutions of special type of nonlinear partial differential equations (NLPDEs). Some of these solutions of NLPDEs may be in the different forms such as rational function solutions, trigonometric function solutions, hyperbolic function solutions, exponential function solutions and Jacobi elliptic function solutions. These forms tell us the various properties of the NLPDEs from scientifical applications to engineering.

In this research, we have studied to obtain the analytical solution of the nonlinear (2+1)-dimensional Burgers equation which is named from Johannes Martinus Burgers and the nonlinear special type of the Dodd-Bullough-Mikhailov equation introduced to the literature by Roger Dodd, Robin Bullough, and Alexander Mikhailov.

Keywords The Generalized Kudryashov method, the (2+1)-dimensional Burgers equation, special type of the Dodd-Bullough-Mikhailov equation, soliton solutions,rational function solutions, trigonometric function solutions, hyperbolic function solutions.

MSC(2010): 35AXX, 35DXX, 35EXX, 65MXX.

1. Introduction

Some scientists have investigated the various solutions of some differential equations such as approximate and analytical solutions for NLPDEs which are used to describe different problems. They have attained some analytical solutions for these equations by using various methods such as the modified simple equation method, various trial equation methods, Sumudu transform method, the tanh function method, the sine-cosine method, the inverse scattering method, Hirota's bilinear transformation, the tanh-sech method, homogeneous balance method, the Darboux transformation, extended tanh-function method, homotopy perturbation method, G'/G-expansion method, exp-function method, Kudryashov method, extended trial equation method and so on [1–8, 11, 12, 15, 16, 18–20, 23, 24, 27, 28].

In this paper, we have applied the generalized Kudryashov method to the special type of Dodd-Bullough-Mikhailov equation by Roger Dodd, Robin Bullough, and Alexander Mikhailov [26] and the nonlinear (2+1)-dimensional Burgers equation [14] successfully for obtaining some new analytical solutions, after primarily we

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give the general structure of new generalized Kudryashov method in section 2. In Section 3, as an application, we have solved the nonlinear special type of the Dodd-Bullough-Mikhailov equation and the nonlinear (2+1)-dimensional Burgers equation defined as follows, respectively;

$$u_{tt} - u_{xx} + e^{u} + e^{-2u} = 0 \tag{1.1}$$

and

$$u_t - uu_x - u_{xx} - u_{yy} = 0. \tag{1.2}$$

Burgers equation is very important for describing the travelling wave in the water, air and others. Different versions such as inviscid Burgers' equation, Burgers' equations, viscous Burgers' equation and coupled Burger's equations have been submitted to literature before along with various solutions like approximate, numerical and analytical. The nonlinear (2+1)-dimensional Burgers equation which is being one of them has been investigated with the help of various methods. Khan and Akbar have applied exp-function method for obtaining solutions for Burgers equation [14]. Shen, Sun and Xiong have obtained new travelling-wave solutions for Dodd-Bullough Equation by new method [25]. Davodi, Ganji and Alipour have taken into considerations some different methods such as tan, tanh, and extended tanh and sech methods for solving the nonlinear partial differential equation, including Dodd-Bullough-Mikhailov (DBM) equation in 2009 [9]. Bahrami, Abdollahzadeh, Berijani, Ganji and Abdollahzadeh have conducted the G'/G-expansion method some travelling solutions in 2011 [1]. Rui have reached another solution of Dodd-Bullough-Mikhailov equation in 2013 [22]. This differential equation plays an important role in many scientific applications like changing between fluid flow and quantum field theory [26]. Quantum field theory, especially in theoretical physics, is a theoretical framework for forming quantum mechanical models of subatomic particles in particle physics. Moreover, this differential equation has been widely used for explaining to quantum fields such as quantum electrodynamics , quantum chromo dynamics, quantum mechanical interactions, quantum mechanical systems, gravitational field, electromagnetic field , the thermodynamics of radiation, the quantum nature of radiation, modern quantum optics, general theory of relativity, the low-energy effective field theory, superstring theory and so on.

2. Fundamental Facts of the Generalized Kudryashov Method

Recently, some authors have researched generalized Kudryashov method [10, 13, 17, 21]. But, in this work, we try to constitute generalized form of Kudryashov method. We consider the following nonlinear partial differential equation for a function of two real variables, space x and time t :

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \cdots) = 0. \tag{2.1}$$

The basic phases of the generalized Kudryashov method are expressed as being four steps following:

Step 1. First of all, we must get the travelling wave solution of Eq.(2.1) as following form;

$$u(x,t) = u(\xi),\ \xi = kx - ct, \tag{2.2}$$

where k and c are arbitrary constants. Eq.(2.1) was converted into a nonlinear ordinary differential equation of the form:

$$N(u, u', u'', u''', \cdots) = 0, \tag{2.3}$$

where the prime indicates differentiation with respect to ξ.

Step 2. Suggest that the exact solutions to the Eq.(2.3) can be written as following form;

$$u(\xi) = \frac{\sum_{i=0}^{N} a_i \Phi^i}{\sum_{j=0}^{M} b_j \Phi^j} = \frac{A[\Phi(\xi)]}{B[\Phi(\xi)]}, \tag{2.4}$$

where Φ is $\frac{1}{1 \mp e^{\xi}}$. We note that the function Φ is solution of equation [13]

$$\Phi' = \Phi_\xi = \Phi^2 - \Phi. \tag{2.5}$$

Taking into consideration Eq.(2.5), we can obtain the first, second and third derivatives of $u(\xi)$ together Eq.(2.4);

$$u'(\xi) = \frac{A'B\Phi' - AB'\Phi'}{B^2} = \Phi'[\frac{A'B - AB'}{B^2}] = (\Phi^2 - \Phi)\frac{A'B - AB'}{B^2}, \tag{2.6}$$

$$u''(\xi) = \frac{\Phi^2 - \Phi}{B^2}[(2\Phi - 1)(A'B - AB') + \frac{\Phi^2 - \Phi}{B}[B(A''B - AB'') - 2A'B'b + 2A(B')^2]], \tag{2.7}$$

$$\begin{aligned} u'''(\xi) = & \frac{(\Phi^2 - \Phi)^3}{B^4}[(A'''B - AB''' - 3A''B' - 3B''A')B^2] \\ & + \frac{(\Phi^2 - \Phi)^3}{B^4}[6B^2(AB'' + A'B') - 6A(B')^3] \\ & + 3(\Phi^2 - \Phi)^2(2\Phi - 1)[\frac{B(A''B - AB'') - 2B'A'A + 2A(B')^2}{B^3}] \\ & + (\Phi^2 - \Phi)(6\Phi^2 - 6\Phi + 1)[\frac{A'B - AB'}{B^2}]. \end{aligned} \tag{2.8}$$

Step 3. Under the terms of proposed method, we suppose that the solution of Eq.(2.3) can be explained in the form of following:

$$u(\xi) = \frac{a_0 + a_1\Phi + a_2\Phi^2 + a_3\Phi^3 + \cdots + a_N\Phi^N + \cdots}{b_0 + b_1\Phi + b_2\Phi^2 + b_3\Phi^3 + \cdots + b_M\Phi^M + \cdots}. \tag{2.9}$$

To calculate the values of M and N in Eq.(2.9) that is the pole order for the general solution of Eq.(2.3), we progress conformably as in the classical Kudryashov method on balancing the highest order nonlinear terms in Eq.(2.3) and we can determine a formula of M and N. Then, we can choose some values of M and N.

Step 4. Replacing Eq.(2.4) into Eq.(2.3) provides a polynomial of $R(\Phi)$ and Φ. Establishing the coefficients of $R(\Phi)$ to zero, we acquire a system of algebraic equations. Solving this system, we can describe ξ and the variable coefficients of $a_0, a_1, a_2, a_3, \cdots, a_N, b_0, b_1, b_2, b_3, \cdots, b_M$. In this way, we attain the exact solutions to Eq.(2.3).

3. The Implementation of the Method

In this section, we have obtained the analytical solutions of the Eq.(1.1) and Eq.(1.2) by using GKM.

Example 3.1. Let us consider to Eq.(1.1) to found the new travelling wave solutions, and, we perform the transformation $u(x,t) = u(\xi)$ and $\xi = x - ct$ where c is constant. When it comes to convert partial differential equation into ordinary differential equation, we can perform as following;

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial}{\partial t}[-c\frac{\partial u(\xi)}{\partial \xi}] = c^2 u'', \tag{3.1}$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial}{\partial x}[\frac{\partial u(\xi)}{\partial \xi}] = u'', \tag{3.2}$$

so, when we use u_{xx} and u_{tt} in the Eq.(1.1), we get the nonlinear ordinary differential equation as following;

$$(c^2 - 1)u'' + e^u + e^{-2u} = 0. \tag{3.3}$$

When we take into consideration as following transformations,

$$u = \ln v, u' = \frac{v'}{v}, u'' = \frac{v''}{v} - \frac{(v')^2}{v^2}, \tag{3.4}$$

we get quickly as following nonlinear ordinary differential equation

$$(c^2 - 1)vv'' - (c^2 - 1)(v')^2 + v^3 + 1 = 0. \tag{3.5}$$

When we rearrangement to Eq.(2.4), Eq.(2.6) and Eq.(2.7) for balance principle, we obtain the term for suitability;

$$N = M + 2. \tag{3.6}$$

This resolution procedure is applied and we obtain results as follows:

Case 1: If we take $M = 1$ and $N = 3$ for using in Eq.(2.4), then, we can write follow equations;

$$v(\xi) = \frac{\sum_{i=0}^{3} a_i \Phi^i}{\sum_{j=0}^{1} b_j \Phi^j} = \frac{a_0 + a_1\Phi + a_2\Phi^2 + a_3\Phi^3}{b_0 + b_1\Phi} = \frac{A[\Phi]}{B[\Phi]}, \tag{3.7}$$

$$v'(\xi) = (\Phi^2 - \Phi)[\frac{A'B - AB'}{B^2}] \tag{3.8}$$

and

$$\begin{aligned} v''(\xi) =& \frac{\Phi^2 - \Phi}{B^2}[(2\Phi - 1)(A'B - AB') + \frac{\Phi^2 - \Phi}{B}[B(A''B - AB'') \\ &- 2A'B'b + 2A(B')^2]], \end{aligned} \tag{3.9}$$

where $a_3 \neq 0$ and $b_1 \neq 0$. When we use Eq.(3.7), Eq.(3.8) and Eq.(3.9) into the Eq.(3.5), we get a system of algebraic equations for Eq.(3.5). Thus, we have a system of algebraic equations from the coefficients of polynomial of Φ. Solving the algebraic equation system Eq.(3.5) by using commercial Wolfram Mathematica programming 9, us yields the following coefficients:

$$c = -2, a_0 = -b_0, a_1 = 6b_0 - b_1, a_2 = 6(-b_0 + b_1), a_3 = -6b_1, b_0 = b_0, b_1 = b_1. \tag{3.10}$$

Substituting these coefficients and $\Phi = \frac{1}{1 \mp e^{\xi}}$ into Eq.(3.7) , we have

$$\begin{aligned} v(\xi) &= \frac{-b_0 + (6b_0 - b_1)\frac{1}{1\mp e^{\xi}} + 6(-b_0 + b_1)(\frac{1}{1\mp e^{\xi}})^2 - 6b_0(\frac{1}{1\mp e^{\xi}})^3}{b_0 + b_1\frac{1}{1\mp e^{\xi}}}, \\ v(\xi) &= -1 + 6\frac{1}{1 \mp e^{\xi}} - 6(\frac{1}{1 \mp e^{\xi}})^2, \end{aligned} \tag{3.11}$$

then, when we go on such process, we get the following solutions for the nonlinear ordinary differential Eq.(3.5) as following manner;

$$v(\xi) = -1 + 6\frac{1}{1 \mp e^{\xi}}(1 - \frac{1}{1 \mp e^{\xi}}). \tag{3.12}$$

When we use Eq.(3.12) into the Eq.(3.4) and $\xi = x - ct$, we attain the soliton solution as following;

$$u(x,t) = \ln[-1 + \frac{6}{1 \mp e^{x-ct}}(1 - \frac{1}{1 \mp e^{x-ct}})]. \tag{3.13}$$

When we take into consideration for rearrangement,

$$\tanh(\frac{x}{2}) = \frac{e^x - 1}{e^x + 1} \Rightarrow \frac{1}{1 + e^x} = \frac{1}{2} - \frac{1}{2}\tanh(\frac{x}{2}), \tag{3.14}$$

we can write Eq.(3.13) in the form of hyperbolic function solution as following;

$$v(\xi) = -1 + 6(\frac{1}{2} - \frac{1}{2}\tanh(\frac{\xi}{2}))(\frac{1}{2} + \frac{1}{2}\tanh(\frac{\xi}{2}), \tag{3.15}$$

$$v(\xi) = \frac{1}{2} - \frac{3}{2}\tanh^2\frac{\xi}{2}. \tag{3.16}$$

If we use Eq.(3.16) into the Eq.(3.4) and $c = 2$, for simplicity, we obtain the soliton solution as following;

$$u(x,t) = \ln[\frac{1}{2} - \frac{3}{2}\tanh^2\frac{(x - 2t)}{2}]. \tag{3.17}$$

Remark 3.1. The soliton solution Eq.(3.17) obtained by using GKM for Eq.(1.1) is the same analytical solution obtained by Abdul-Majid Wazwaz by using the Tanh Method and being solution in [26] under the special circumstance of constant for the nonlinear Special Type of The Dodd-Bullough- Mikhailov equation, [see [26], Eq.(45), for $c = -3$]. The soliton solution and *Figure 1* being surfaces of Eq.(3.13) have been checked by the programming language Mathematica 9. As we know, the application of GKM to Eq.(1.1) has not submitted to the literature before.

Case 2: In this case, especially, If we take $M = 0$, of course $N = 2$, for Eq.(3.4), it gives us the Kudryashov method, nevertheless, when we keep on in the same way, we write follow equations;

$$v(\xi) = \frac{\sum_{i=0}^{2} a_i \Phi^i}{\sum_{j=0}^{0} b_j \Phi^j} = \frac{a_0 + a_1\Phi + a_2\Phi^2}{b_0} = \frac{A[\Phi]}{B[\Phi]}, \tag{3.18}$$

$$v'(\xi) = (\Phi^2 - \Phi)[\frac{A'B - AB'}{B^2}] \tag{3.19}$$

and

$$\begin{aligned} v''(\xi) =& \frac{\Phi^2 - \Phi}{B^2}[(2\Phi - 1)(A'B - AB') + \frac{\Phi^2 - \Phi}{B}[B(A''B - AB'') \\ &- 2A'B'b + 2A(B')^2]], \end{aligned} \tag{3.20}$$

where $a_2 \neq 0$ and $b_0 \neq 0$. When we use Eq.(3.18), Eq.(3.19) and Eq.(3.20) into the Eq.(3.5), we get a system of algebraic equations for Eq.(3.5). Thus, we have a system of algebraic equations from the coefficients of polynomial of Φ. Solving the algebraic equation system Eq.(3.5) by using Mathematica programming yields the following coefficients:

$$c = \mp 2, a_0 = -b_0, a_1 = 6b_0, a_2 = -6b_0, b_0 = b_0. \tag{3.21}$$

If it substitutes these coefficients and $\Phi = \frac{1}{1 \mp e^{\xi}}, \xi = x - ct$ into Eq.(3.18) , we attain;

$$v(\xi) = -1 + 6\Phi - 6\Phi^2 = -1 + 6(\frac{1}{1 \mp e^{\xi}}) - 6(\frac{1}{1 \mp e^{\xi}})^2, \tag{3.22}$$

then, when we keep on this procedure, we get the following solutions for the nonlinear ordinary differential Eq.(3.5) as following manner;

$$v(\xi) = -1 + 6(\frac{1}{1 \mp e^{\xi}})(1 - \frac{1}{1 \mp e^{\xi}}). \tag{3.23}$$

When we consider Eq.(3.23) along with the Eq.(3.4), we attain the hyperbolic function solution as following;

$$u(x,t) = \ln[-1 + \frac{6}{1 \mp e^{-x+2t}}(1 - \frac{1}{1 \mp e^{-x+2t}})], \tag{3.24}$$

$$u(x,t) = \ln[-1 + \frac{3}{2}\text{sech}^2(\frac{-x+2t}{2})]. \tag{3.25}$$

When we take into consideration for rearrangement Eq.(3.14), we can found the another soliton solution as following for Eq.(3.25);

$$u(x,t) = \ln[\frac{1}{2} - \frac{3}{2}\tanh^2(\frac{-x+2t}{2})]. \tag{3.26}$$

Example 3.2. Let us consider the nonlinear (2+1)-dimensional Burgers equation defined by following equation for obtaining the new travelling wave solutions;

$$u_t - uu_x - u_{xx} - u_{yy} = 0. \tag{3.27}$$

When we perform the transformation $u(x,y,t)=u(\xi)$ and $\xi=k(x+y-ct)$, it gives us follows;

$$\frac{\partial u}{\partial t}=-cku', \frac{\partial u}{\partial x}=ku', \frac{\partial u}{\partial y}=ku', \frac{\partial^2 u}{\partial x^2}=k^2u'', \frac{\partial^2 u}{\partial y^2}=k^2u'', \tag{3.28}$$

so, when we use Eq.(3.28) into Eq.(3.27) by integrating the resulting equation with respect to ξ and setting the integration constant to zero, we obtain the nonlinear ordinary differential equation as following;

$$2cu+u^2+4ku'=0. \tag{3.29}$$

When we rearrangement to Eq.(2.4) and Eq.(2.6) due to considering balance principle to determine relationship between M and N , we obtain the term for suitability;

$$N=M+1. \tag{3.30}$$

This resolution procedure is applied and we obtain results as follows:

Case 1: If we take $M=1$ and $N=2$ for Eq.(2.4), then, we write follow equations;

$$v(\xi)=\frac{\sum_{i=0}^{2}a_i\Phi^i}{\sum_{j=0}^{1}b_j\Phi^j}=\frac{a_0+a_1\Phi+a_2\Phi^2}{b_0+b_1\Phi}=\frac{A[\Phi]}{B[\Phi]}, \tag{3.31}$$

$$v'(\xi)=(\Phi^2-\Phi)[\frac{A'B-AB'}{B^2}] \tag{3.32}$$

and

$$\begin{aligned} v''(\xi)=&\frac{\Phi^2-\Phi}{B^2}[(2\Phi-1)(A'B-AB')+\frac{\Phi^2-\Phi}{B}[B(A''B-AB'') \\ &-2A'B'b+2A(B')^2]], \end{aligned} \tag{3.33}$$

where $a_2\neq 0$ and $b_1\neq 0$. When we use Eq.(3.31) and Eq.(3.32) into the Eq.(3.29), we get a system of algebraic equations for Eq.(3.29). Thus, we have a system of algebraic equations from the coefficients of polynomial of Φ. Solving the algebraic equation system Eq.(3.29) by using software commercial programming Mathematica 9 yields the following coefficients:

$$c=-4k, a_0=8kb_0, a_1=-16kb_0, a_2=8kb_0, b_0=b_0, b_1=-2b_0, \tag{3.34}$$

$$c=-2k, a_0=0, a_1=4kb_1, a_2=-4kb_1, b_0=0, b_1=b_1, \tag{3.35}$$

$$c=-2k, a_0=0, a_1=a_1, a_2=-4kb_1, b_0=\frac{-a_1}{4k}, b_1=b_1. \tag{3.36}$$

Substituting Eq.(3.34), Eq.(3.35), Eq.(3.36) coefficients and $\Phi=\frac{1}{1\mp e^{\xi}}$ into Eq.(3.31), we have the traveling wave solution, hyperbolic function solution, another new traveling wave solution for the nonlinear (2+1)-dimensional Burgers equation as following, respectively;

$$u(x,y,t)=4k[1+\coth(4tk^2+kx+ky)], \tag{3.37}$$

$$u(x,y,t)=2k[1+\tanh(\frac{2tk^2+kx+ky}{2})], \tag{3.38}$$

$$u(x,y,t)=-2k[1+\tanh(\frac{-2tk^2+kx+ky}{2})]. \tag{3.39}$$

Case 2: If we take $M = 2$ and $N = 3$ for Eq.(2.4), then, we can write follow equations;

$$v(\xi) = \frac{\sum_{i=0}^{3} a_i \Phi^i}{\sum_{j=0}^{2} b_j \Phi^j} = \frac{a_0 + a_1\Phi + a_2\Phi^2 + a_3\Phi^3}{b_0 + b_1\Phi + b_2\Phi^2} = \frac{A[\Phi]}{B[\Phi]} \tag{3.40}$$

and

$$v'(\xi) = (\Phi^2 - \Phi)[\frac{A'B - AB'}{B^2}], \tag{3.41}$$

where $a_3 \neq 0$ and $b_2 \neq 0$. When we use Eq.(3.40) and Eq.(3.41) into the Eq.(3.29), we get a system of algebraic equations for Eq.(3.29). Thus, we have a system of algebraic equations from the coefficients of polynomial of Φ. Solving this algebraic equation system Eq.(3.29) by using Wolfram Mathematica programming 9 yields the following coefficients:

$$c = -6k, a_0 = \frac{-a_1}{3}, a_1 = a_1, a_2 = -a_1, a_3 = \frac{a_1}{3}, b_0 = \frac{-a_1}{36k}, b_1 = \frac{a_1}{12k}, b_2 = -b_1. \tag{3.42}$$

Substituting Eq.(3.42) coefficients and $\Phi = \frac{1}{1 \mp e^{\xi}}$ into Eq.(3.40), we have the another new hyperbolic function solution for the nonlinear (2+1)-dimensional Burgers equation as following;

$$u(x, y, t) = 6k[1 + \tanh(\frac{3k}{2}(6kt + x + y)]. \tag{3.43}$$

Case 3: If we take $M = 3$ and $N = 4$ for Eq.(2.4), then, we write follow equations;

$$v(\xi) = \frac{\sum_{i=0}^{4} a_i \Phi^i}{\sum_{j=0}^{3} b_j \Phi^j} = \frac{a_0 + a_1\Phi + a_2\Phi^2 + a_3\Phi^3 + a_4\Phi^4}{b_0 + b_1\Phi + b_2\Phi^2 + b_3\Phi^3} = \frac{A[\Phi]}{B[\Phi]} \tag{3.44}$$

and

$$v'(\xi) = (\Phi^2 - \Phi)[\frac{A'B - AB'}{B^2}]. \tag{3.45}$$

When we use Eq.(3.44) and Eq.(3.45) into the Eq.(3.29), we get a system of algebraic equations for Eq.(3.29). Thus, we have a system of algebraic equations from the coefficients of polynomial of Φ. Solving the algebraic equation system Eq.(3.29) by using Wolfram commercial computer programming Mathematica 9 yields the following coefficients:

$$\begin{aligned} &c = -4k, a_0 = 8kb_0, a_1 = 8kb_1, a_2 = -8k(3b_0 + 2b_1), a_3 = 8k(2b_0 + b_1), \\ &a_4 = 0, b_0 = b_0, b_1 = b_1, b_2 = b_2, b_3 = \frac{(2b_0 + b_1)(4b_0 + 2b_1 + b_2)}{b_0}, \end{aligned} \tag{3.46}$$

$$\begin{aligned} &c = 4k, a_0 = a_1 = 0, a_2 = 8kb_0, a_3 = 8k(2b_0 + b_1), a_4 = 8k(4b_0 + 2b_1 + b_2) \\ &b_0 = b_0, b_1 = b_1, b_2 = b_2, b_3 = -2(4b_0 + 2b_1 + b_2), \end{aligned} \tag{3.47}$$

$$\begin{aligned} &c = 4k, a_0 = a_1 = 0, a_2 = a_2, a_3 = \frac{2a_2b_0 + a_2b_1}{b_0}, a_4 = 0, b_0 = b_0, b_1 = b_1, \\ &b_2 = \frac{-a_2}{8k} - 3b_0 - 2b_1, b_3 = \frac{-(a_2 - 8kb_0)(2b_0 + b_1)}{8kb_0}. \end{aligned} \tag{3.48}$$

Substituting Eq.(3.46), Eq.(3.47), Eq.(3.48) coefficients and $\Phi = \frac{1}{1\mp e^{\xi}}$ into Eq.(3.44), we obtain the exponential function, hyperbolic function and new another exponential rational function solutions for the nonlinear (2+1)-dimensional Burgers equation as follows, respectively;

$$u(x,y,t) = \frac{8kb_0 e^{2k(4kt+x+y)}}{2b_1 + b_2 + b_0(3 + e^{2k(4kt+x+y)})}, \tag{3.49}$$

$$u(x,y,t) = 4k[-1 + \coth(k(-4tk + x + y))], \tag{3.50}$$

$$u(x,y,t) = \frac{8ka_2}{-a_2 + 8kb_0 e^{2k(-4kt+x+y)})}. \tag{3.51}$$

Remark 3.2. The analytical solutions Eq.(3.37), Eq.(3.38), Eq.(3.39), Eq.(3.43), Eq.(3.49), Eq.(3.50), Eq.(3.51) obtained by using GKM are the new exponential, rational and hyperbolic function solutions for Eq.(1.2). These analytical solutions have been checked by using the programming language Mathematica 9. As our knowledge, this application of GKM to Eq.(1.2) has not submitted to the literature before.

4. Conclusions

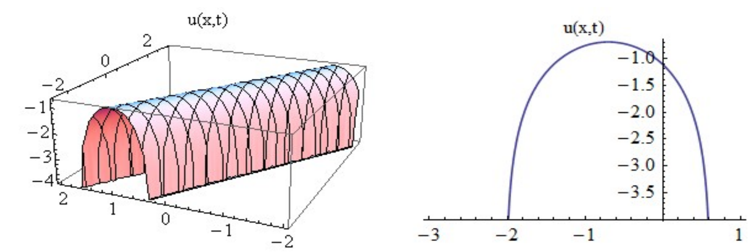

Figure 1. The 3D and 2D surfaces of the solution Eq.(3.13) by corresponding to the values $-2 < x < 3, -2 < t < 2$ for 3D graphics and $-3 < x < 1, c = -2$, t=0.35 for 2D graphics.

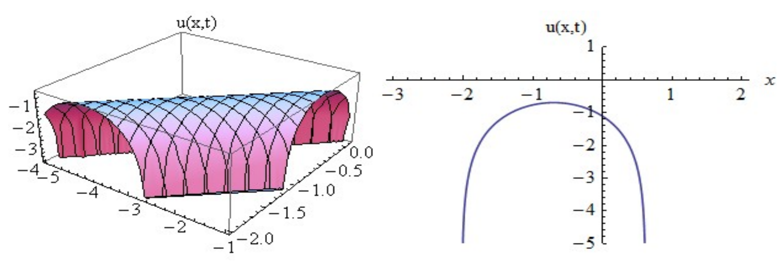

Figure 2. The 3D and 2D surfaces of the solution Eq.(3.25) by corresponding to the values $-5 < x < 1, -2 < t < 0$ for 3D graphics and $-3 < x < 2, c = -2$, t=0.35 for 2D graphics.

When we consider the analytical solutions Eq.(3.13), Eq.(3.25), Eq.(3.37), Eq.(3.38), Eq.(3.39), Eq.(3.43), Eq.(3.49), Eq.(3.50),Eq.(3.51) obtained by using GKM and Figure.1, Figure.2, Figure.3, Figure.4, Figure.5, Figure.6, Figure.7, Figure.8, Figure.9 gotten by the programming language Mathematica 9, this method supplies us with the priceless info about Eq.(1.1) and Eq.(1.2). Under the terms of these

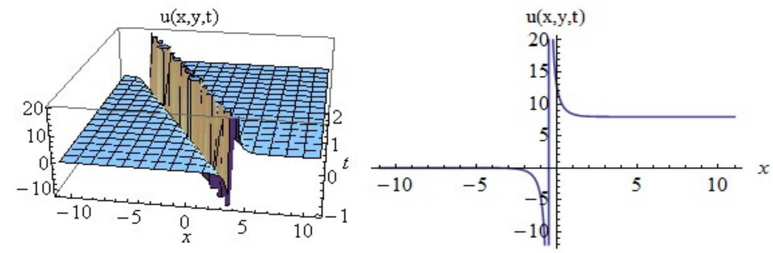

Figure 3. The 3D and 2D surfaces of the solution Eq.(3.37) by corresponding to the values $k = 1, y = 0.1, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

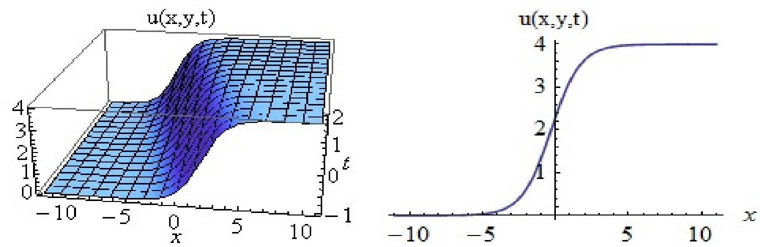

Figure 4. The 3D and 2D surfaces of the solution Eq.(3.38) by corresponding to the values $k = 1, y = 0.1, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

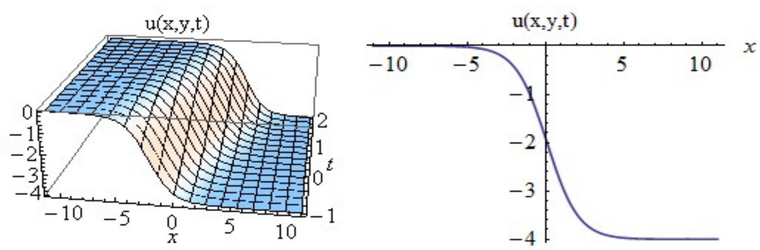

Figure 5. The 3D and 2D surfaces of the solution Eq.(3.39) by corresponding to the values $k = 1, y = 0.1, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

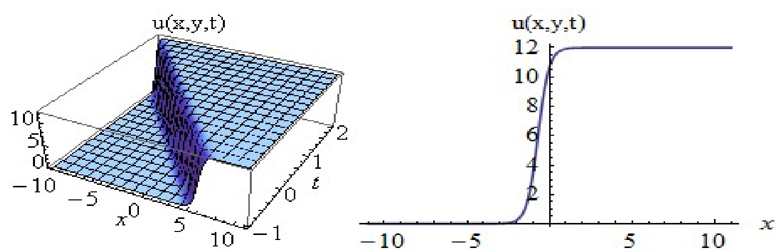

Figure 6. The 3D and 2D surfaces of the solution Eq.(3.43) by corresponding to the values $k = 1, y = 0.1, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

informations, it can be seen that several solutions obtained by GKM are new. The main goals of this work to find new analytical solutions such as rational function solutions, hyperbolic function solutions and exponential function solutions and to emphasize the power of the GKM, have been carried. These analytical solutions

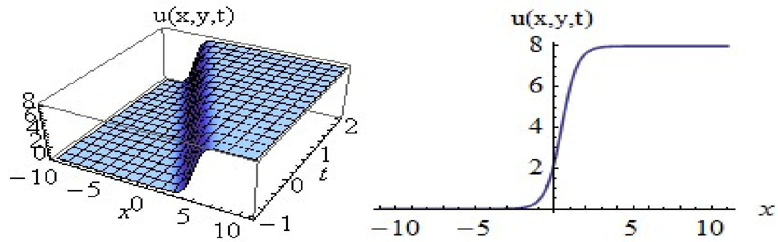

Figure 7. The 3D and 2D surfaces of the solution Eq.(3.49) by corresponding to the values $k = 1, y = 0.1, b_0 = 0.2, b_1 = 0.5, b_2 = -0.2, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

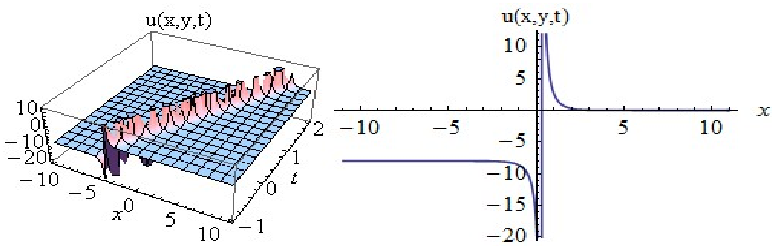

Figure 8. The 3D and 2D surfaces of the solution Eq.(3.50) by corresponding to the values $k = 1, y = 0.1, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

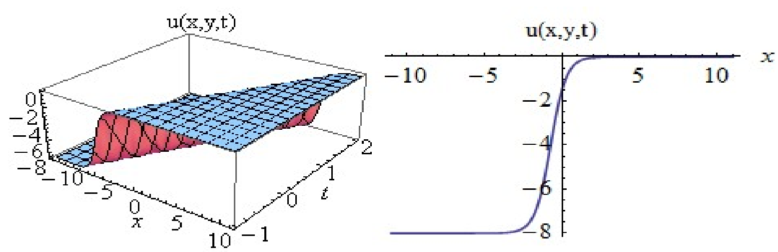

Figure 9. The 3D and 2D surfaces of the solution Eq.(3.51) by corresponding to the values $k = 1, y = 0.1, a_2 = 0.2, b_0 = -0.2, -11 < x < 11, -1 < t < 2$, for 3D graphics and t=0.1 for 2D graphics.

are important to determine the long wave of problem, range of them, and velocity of wave in terms of various aspects. The more analytical solutions can be gained by using different approaches, the more they equip us with different ideas like new versions interpretations of problems solved. The method suggested can also be conducted to many other nonlinear partial differential equations in mathematical physics because the method submitted to literature in this paper has some new advantages such as easily calculations, writing programme for being obtained coefficients and others. Therefore, we want to apply the same approach to nonlinear differential equations with fractional order (of course, other partial and ordinary differential equations with powerful nonlinearity) in the near future.

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