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PARAMETER ESTIMATION FOR NOISY CHAOTIC SYSTEMS BASED ON AN IMPROVED PARTICLE SWARM OPTIMIZATION ALGORITHM*

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Abstract It is of great importance to estimate the unknown parameters and time delays of chaotic systems in control and synchronization. This paper is concerned with the uncertain parameters and time delays of chaotic systems corrupted with random noise. Parameters and time delays of such chaotic systems are estimated based on the improved particle swarm optimization algorithm for its global searching ability. Numerical simulations are given to show satisfactory results.

Keywords Chaotic system, random noise, time delay, parameter estimation, particle swarm optimization algorithm.

MSC(2000) 34F05, 34G20, 65K05.

1. Introduction

Since the emergence of chaos, it has been found that chaos can be widely applied in many fields such as secure communication, data encryption, flow dynamics and biomedical engineering. Hyperchaos is usually described by a chaotic system with more than one positive Lyapunov exponent [15, 16], which was first reported by Rössler in 1979 [16]. For the reason that the dynamic behaviors of hyperchaotic systems are difficult to predict and for the great potential of hyperchaos in technological applications, increasing attention has been paid to hyperchaotic systems. Many hyperchaotic systems have been discovered and developed so far, such as the hyperchaotic Rössler system [17], hyperchaotic Chen system [9], and hyperchaotic Lü system [2].

Recently, parameter estimation of chaotic systems has been a hot research topic in the fields of nonlinear science, computer science and artificial intelligence. Usually, the parameters of chaotic systems are assumed to be fully or partially known. However, in real situations, these parameters are not exactly known. Therefore, it is necessary to estimate the unknown parameters of chaotic systems in applications. Many optimization methods [3, 7, 12, 13, 20–22] have been proposed to estimate the unknown parameters of chaotic systems. Recently, Gao et al. put forward a novel method using particle swarm intelligence and differential evolution (DE) algorithm

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to estimate the unknown parameters of chaotic systems corrupted with random initial noise [5,6]. However, to the best of our knowledge, very few researches have been done to estimate the unknown time delays and parameters of noisy hyperchaotic systems.

Since the 1990s, the study of swarm intelligence has attracted a lot of interest. As a typical implementation pattern of swarm intelligence, the particle swarm optimization (PSO) algorithm, proposed by Kennedy and Eberhart in 1995, is a kind of efficient and powerful method with great potentiality. The PSO algorithm is motivated by the behavior of organisms, such as fish schooling and bird flocking. The PSO algorithm has several significant advantages, such as the simple concept, easy implementation and quick convergence. Because of these properties, the PSO algorithm has attracted much attention and is widely applied in various fields [4, 10]. The PSO algorithm has a flexible and well-balanced mechanism to enhance the global and local exploration abilities [1]. In addition, this algorithm is capable of handling non-differentiable, nonlinear and multi-modal objective functions, with easily chosen control parameters. Moreover, because of the simplicity of the program and few parameters to be adjusted, the PSO algorithm has been further developed rapidly in recent years, and many improved PSO algorithms have been invented.

The purpose of this work is to estimate the unknown parameters and time delays of noisy chaotic systems, such as the hyperchaotic Lorenz, Lü systems without time-delay and Mackey-Glass chaotic system with time-delay. Moreover, an improved PSO algorithm is applied to a proper nonnegative multi-modal numerical optimization problem.

The rest of this paper is organized as follows. In Section 2, a brief view of an improved PSO algorithm is provided. The process of parameter estimation is introduced in Section 3 to transfer the estimation problem into a multi-modal nonnegative function's optimization. Some simulation results are given in Section 4. Finally, conclusions are presented in Section 5.

2. Improved particle swarm optimization algorithm

The PSO algorithm is a kind of evolutionary computation based on the social behavior of organisms such as fish schooling and bird flocking, which is created by Kennedy and Eberhart [8]. It can search the optimal solution through cooperation and competition among the particles in a swarm. In PSO, the solution of a specific optimization problem is regarded as a bird in the d-dimensional searching space, and it is abstracted into a particle without quality and volume. All of the particles have a fitness value determined by the optimized function, and each particle has a velocity to decide its flying distance and direction. It is assumed that each particle is aware of the experience of its own and neighboring particles, so it can adjust its velocity and position iteratively.

The PSO algorithm initializes a group of random particles at first, and then the particles could find a better solution (position) in the searching space iteratively. Each particle in the d-dimensional searching space is characterized by two factors, i.e. its position $X_i = (x_{i,1}, x_{i,2}, ..., x_{i,d})$ and velocity $V_i = (v_{i,1}, v_{i,2}, ..., v_{i,d})$, where i denotes the ith particle in the swarm. Let $P_i(t) = (p_{i,1}, p_{i,2}, ..., p_{i,d})$ denote the best position found by i within t iteration steps. $P_g(t) = (p_{g,1}, p_{g,2}, ..., p_{g,d})$ denotes the best position among all particles in the swarm so far. Particles update their positions and velocities according to the following formulas:

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_1[p_{i,j} - x_{i,j}(t)] + c_2 r_2[p_{g,j} - x_{i,j}(t)], \tag{2.1}$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1), i = 1, 2, ..., n, \tag{2.2}$$

where n denotes the number of particles in the swarm; $V_i(t)$ and $X_i(t)$ represent the velocity and position of the ith particle in the solution space at the tth iteration step, respectively; r_1 and r_2 are two random numbers uniformly distributed in the range [0,1]; c_1 and c_2 are positive learning factors; w is the inertia weight factor. Generally, the value of each component in V_i is controlled within the range $[V_{min}, V_{max}]$ to avoid excessive roaming of particles outside the searching space. Each particle updates its individual best position $P_i(t)$ and global best position $P_g(t)$ in the swarm according to Eqs. (2.1) and (2.2), and then finds the new position and velocity. This process is repeated until a user-defined stopping criterion, usually when the maximum iteration number t_{max} is reached.

According to the PSO procedure, the performance mainly depends on its parameters. It can be seen that the first part of Eq. (2.1) shows the influence of the previous velocity, which is considered as the necessary momentum for particles to roam across the searching space. The inertia weight w in Eq. (2.1) is the modulus that controls the impact of the previous velocity on the current one. It can be seen that w is a crucial parameter which influences the balance between the exploration and exploitation in the PSO algorithm. Thus, a proper control of the inertia weight is very important to find the optimum solution accurately and efficiently. In order to achieve the balance between exploration and exploitation, w can be set to vary adaptively in response to the objective function values of the particles. In this paper, we apply the adaptive inertia weight factor (AIWF) [11] to Eq. (2.1) as an improved PSO algorithm. AIWF is described as follows:

$$w = \begin{cases} w_{min} - \dfrac{(w_{max} - w_{min}) * (f - f_{min})}{(f_{avg} - f_{min})}, & f \leq f_{avg}, \\ w_{max}, & f > f_{avg}, \end{cases} \tag{2.3}$$

where w_{max} and w_{min} denote the maximum and minimum of w respectively, f denotes the current value of the particles' objective function, f_{avg} and f_{min} denote the average value and the minimum of the fitness function.

According to Eq. (2.3), w varies depending on the value of the fitness function. It shows that particles with low objective function values can be 'protected' while particles with objective values over the average objective function will be 'disrupted'. In other words, good particles tend to perform exploitation to refine results by local searching, while bad particles tend to perform large modification to explore space with a large step. Thus, AIWF provides a good way to maintain population diversity and to sustain good convergence capacity.

3. Parameter estimation for noisy chaotic systems

Generally, chaotic systems are described by a set of nonlinear differential equations. In this paper, two kinds of noisy chaotic systems are considered.

3.1. Parameter estimation for chaotic systems without time delays

Considering the following n-dimensional chaotic system described by an ordinary differential equation (ODE):

$$\dot{x} = f(x(t), x_0, \theta), \tag{3.1}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in R^n$ denotes the state vector and $x_0 = (x_{10}, x_{20}, ..., x_{n0}) + rand$ is the initial state vector with random noise, $\theta = (\theta_1, \theta_2, ..., \theta_d)^T$ is the systematic parameters. The unknown parameter θ is to be estimated.

Suppose the structure of system (3.1) is known. Then, the estimated system can be written as

$$\dot{\tilde{x}} = f(\tilde{x}(t), x_0, \tilde{\theta}), \tag{3.2}$$

where $\dot{\tilde{x}}(t) = (\dot{\tilde{x}}_1(t), \dot{\tilde{x}}_2(t), ..., \dot{\tilde{x}}_n(t))^T \in R^n$ is the state vector of system (3.1), $\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_d)^T$ is the systematic parameters to be estimated.

To estimate the unknown parameters, an objective function is defined:

$$J = \sum_{k=1}^{N} \|x_k - \tilde{x}_k\|^2, \tag{3.3}$$

where $k = 1, 2, ..., N$ is the sampling time point, N denotes the length of data used for parameter estimation, x_k and $\tilde{x}_k$ $(k = 1, 2, ..., N)$ denote the state vectors of the original and the estimated system at time k, respectively. The parameter estimation for system (3.1) can be achieved by searching suitable θ^* such that the objective function (3.3) is minimized, i.e.

$$\theta^* = \arg \min_{\theta \in \Gamma} J, \tag{3.4}$$

where Γ is the searching space admitted for parameters.

In this paper, the above improved PSO algorithm is employed to estimate the unknown parameter θ of system (3.1).

To explain the parameter estimation method, we take the hyperchaotic Lorenz system [23] for example. The new estimated system is constructed from the original hyperchaotic Lorenz system:

$$\begin{cases} \dot{x} = a(y - x) + w, \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz, \\ \dot{w} = -yz + rw, \\ X = (x, y, z, w), \end{cases} \rightarrow \begin{cases} \dot{\tilde{x}} = \tilde{a}(\tilde{y} - \tilde{x}) + \tilde{w}, \\ \dot{\tilde{y}} = \tilde{c}\tilde{x} - \tilde{y} - \tilde{x}\tilde{z}, \\ \dot{\tilde{z}} = \tilde{x}\tilde{y} - \tilde{b}\tilde{z}, \\ \dot{\tilde{w}} = -\tilde{y}\tilde{z} + \tilde{r}\tilde{w}, \\ \tilde{X} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w}), \end{cases} \tag{3.5}$$

where a, b, c, and r are unknown parameters to be estimated.

Firstly, we initialize the parameters $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{r}$ randomly within the searching range, which is set as $8 \le \tilde{a} \le 12$, $2 \le \tilde{b} \le 5$, $25 \le \tilde{c} \le 30$, $-1.5 \le \tilde{r} \le -0.1$.

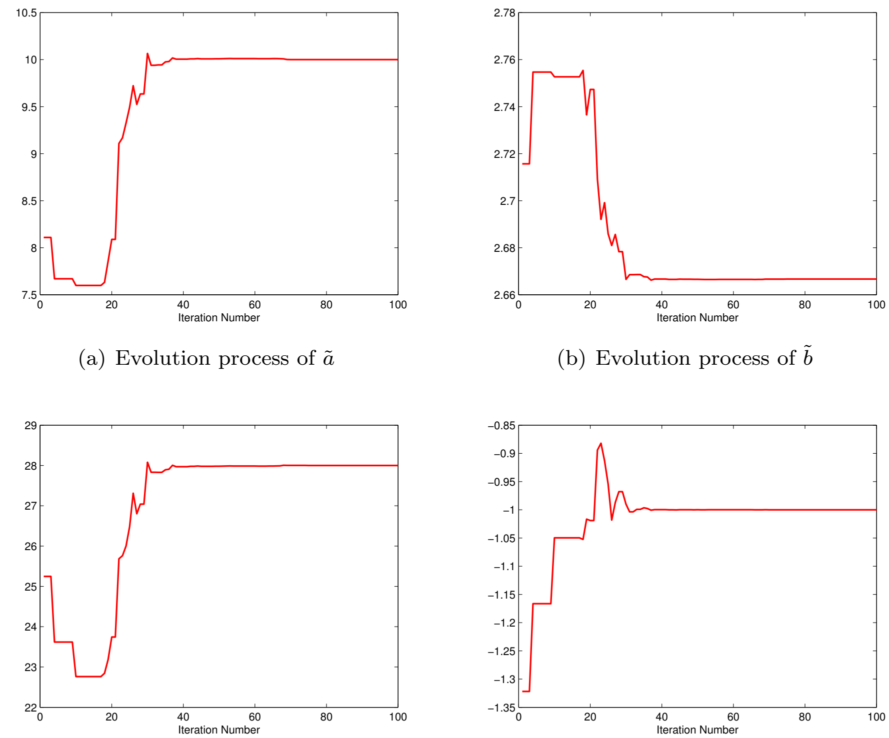
Secondly, we generate the initial state vector x_0 with random noise in $[-0.1, 0.1]^4$, which follows behind a period of transient process. Then, we use the ODE solver [18] in Matlab to solve system (3.5). Let the step size $h = 0.01$ in order to get a discrete-time series of system (3.5) at $(x(t), y(t), z(t), w(t))$, $(\tilde{x}(t), \tilde{y}(t), \tilde{z}(t), \tilde{w}(t))$, $t = 0h, h, ..., 100h$.

Thirdly, the objective function is chosen as:

$$F(\tilde{\theta}) = \sum_{t=0h}^{100h} \|\tilde{X} - X\|^2. \tag{3.6}$$

The function is optimized by the improved PSO algorithm repeatedly until the termination condition is satisfied.

The system is hyperchaotic when $a = 10, b = \frac{8}{3}, c = 28$, and $r = -1$.



(a) Evolution process of $\tilde{a}$

(b) Evolution process of $\tilde{b}$

(c) Evolution process of $\tilde{c}$

(d) Evolution process of $\tilde{r}$

Figure 1. Parameter estimation for the hyperchaotic Lorenz system (3.5) based on the improved PSO algorithm.

3.2. Parameter estimation for chaotic systems with time delays

Considering the following n-dimensional time-delay chaotic system described by delay differential equation (DDE):

$$\dot{x} = f(x(t), x(t - \tau), x_0, \theta), \tag{3.7}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in R^n$ denotes the state vector, $x_0 = (x_{10}, x_{20}, ..., x_{n0}) + rand$ denotes the initial state vector with random noise for $t > \tau$, $\theta =$

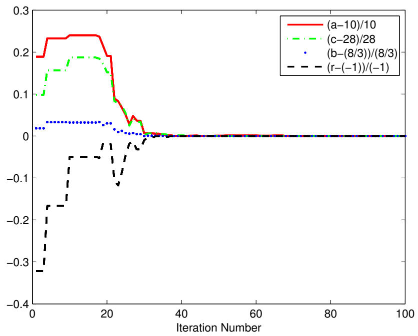


Figure 2. Evolution process of absolute errors of the unknown parameters for the hyperchaotic Lorenz system (3.5)

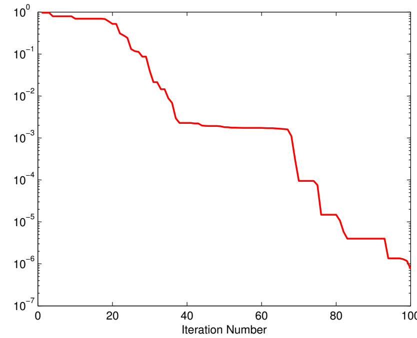


Figure 3. Convergence trajectory of the objective function of the hyperchaotic Lorenz system (3.5) based on the improved PSO algorithm

$(\theta_1, \theta_2, ..., \theta_d)^T$ is a set of original parameters. In this paper, the time-delay τ is treated as a parameter to be estimated.

Suppose the structure of system (3.7) is known. Then, the estimated system can be written as

$$\dot{\tilde{x}} = f(\tilde{x}(t), \tilde{x}(t-\tilde{\tau}), x_0, \tilde{\theta}), \tag{3.8}$$

where $\dot{\tilde{x}}(t) = (\dot{\tilde{x}}_1(t), \dot{\tilde{x}}_2(t), ..., \dot{\tilde{x}}_n(t))^T \in R^n$ is the state vector of the estimated system, $\tilde{\theta} = (\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_d)^T$ is a set of parameters to be estimated, and $\tilde{\tau}$ is the estimated time delay.

Then, the objective function is constructed as:

$$(\theta^*, \tau^*) = \arg \min_{(\theta,\tau)\in\Gamma} J, \tag{3.9}$$

where Γ is the searching space admitted for parameters. And we estimate both the parameters and time delays based on the improved PSO algorithm.

To explain the optimization process, we take the time-delay Mackey-Glass chaotic system [14] for example. The new estimated system is constructed from the original time-delay Mackey-Glass chaotic system:

$$\begin{cases} \dot{x}(t) = -\alpha x(t) + \frac{\beta x(t-\tau)}{1+x(t-\tau)^{10}}, \\ X = x, \end{cases} \rightarrow \begin{cases} \dot{\tilde{x}}(t) = -\tilde{\alpha}\tilde{x}(t) + \frac{\tilde{\beta}\tilde{x}(t-\tilde{\tau})}{1+\tilde{x}(t-\tilde{\tau})^{10}}, \\ \tilde{X} = \tilde{x}, \end{cases} \tag{3.10}$$

where τ, α, and β are unknown parameters to be estimated.

Firstly, we initialize the parameters $\tilde{\tau}, \tilde{\alpha}, \tilde{\beta}$ randomly within the searching range, which is set as $12 \le \tilde{\tau} \le 20$, $0.05 \le \tilde{\alpha} \le 1$, $0.05 \le \tilde{\beta} \le 1$.

Secondly, we generate the initial state vector x_0 with random noise in $[-0.1, 0.1]$, which follows behind a period of transient process. Then, we use the DDE solver [19] in Matlab to solve system (3.10). Let the step size $h = 0.01$ in order to get a discrete-time series of system (3.10) at $x(t)$, $\tilde{x}(t)$, $t = 0h, h, ..., 100h$.

Thirdly, the objective function is chosen as:

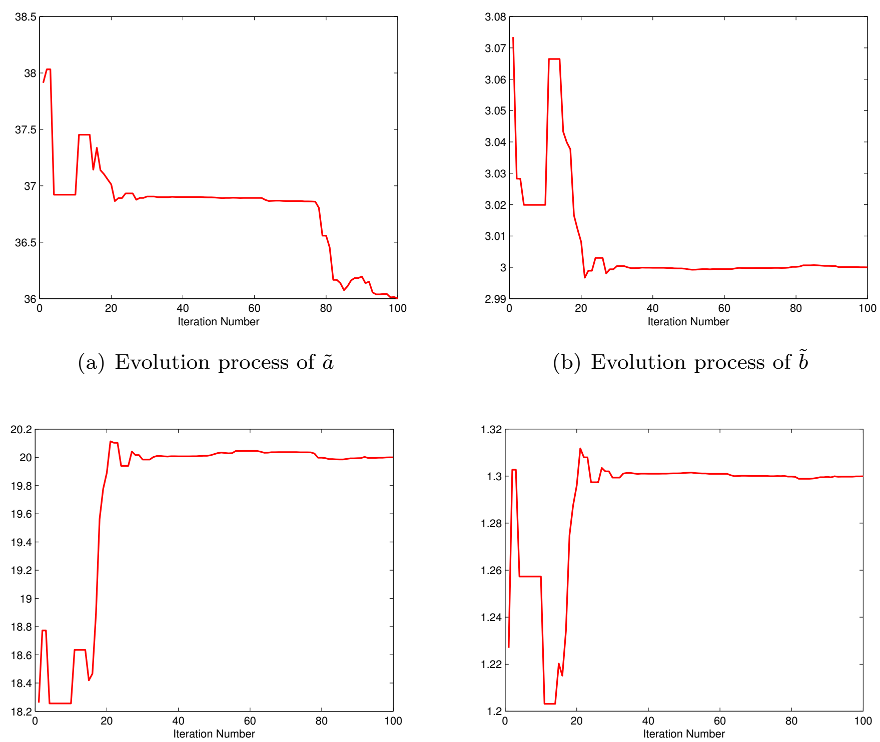
$$F(\tilde{\theta}) = \sum_{t=0h}^{100h} \|\tilde{X} - X\|^2. \tag{3.11}$$

The function is optimized by the improved PSO algorithm repeatedly until the termination condition is satisfied.

When $\tau = 17, \alpha = 0.1$, $\beta = 0.2$, system (3.10) is chaotic.

4. Numerical simulations

In this section, three simulation examples are given to show the parameter estimation for noisy chaotic systems, and to verify the effectiveness of the parameter estimation method based on the improved PSO algorithm.



(a) Evolution process of $\tilde{a}$

(b) Evolution process of $\tilde{b}$

(c) Evolution process of $\tilde{c}$

(d) Evolution process of $\tilde{d}$

Figure 4. Parameter estimation for the hyperchaotic Lü system (4.1) based on the improved PSO algorithm

4.1. Simulations on noisy chaotic systems without time delays

We select two hyperchaotic systems, free of time delays, for numerical examples.

The first one is the noisy hyperchaotic Lorenz system (3.5), as described above. In this example, the unknown parameters are estimated based on the improved PSO algorithm. The estimated results of the parameters a, b, c, and r are shown in Fig. 1. The final estimated values are $\tilde{a} = 10, \tilde{b} = \frac{8}{3}, \tilde{c} = 28$, and $\tilde{r} = -1$. Thus, the actual parameters are fully estimated. The changes of parameters' absolute errors are shown in Fig. 2. The objective function value converges to zero quickly, as

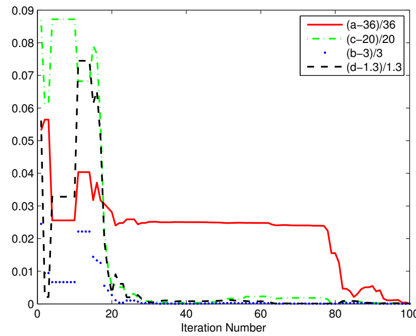


Figure 5. Evolution process of absolute errors of parameters for the hyperchaotic Lü system (4.1)

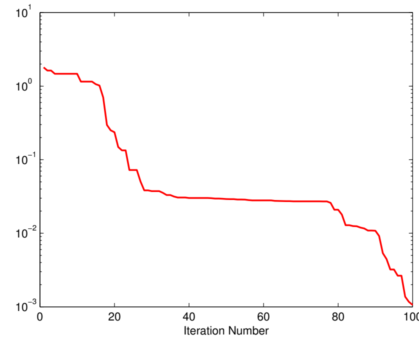


Figure 6. Convergence trajectory of the objective function of the hyperchaotic Lü system (4.1) based on the improved PSO algorithm

shown in Fig. 3. And the convergence result approaches to a value smaller than 10^{-6}.

The second example is the hyperchaotic Lü system described by

$$\begin{cases} \dot{x} = a(y-x) + w, \\ \dot{y} = -xz + cy, \\ \dot{z} = xy - bz, \\ \dot{w} = xz + dw, \end{cases} \tag{4.1}$$

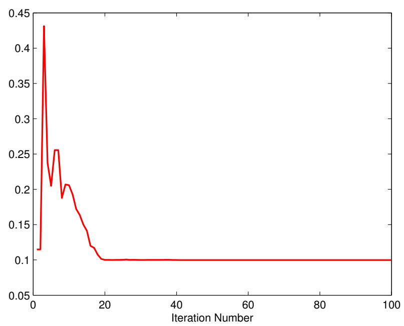
where a, b, c, and d are unknown parameters to be estimated. The system is hyperchaotic when $a = 36, b = 3, c = 20$, and $d = 1.3$. The chaotic behavior is shown in Fig. 6. Let $\tilde{\Gamma} = (\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}) \in [33, 39] \times [1, 5] \times [18, 22] \times [0.5, 2.5]$. The initial state x_0 is added with random noise in $[-0.1, 0.1]^4$.

In this example, parameters are estimated based on the improved PSO algorithm. The estimated results of parameters a, b, c, and d are shown in Fig. 4. The final estimated values are $\tilde{a} = 36, \tilde{b} = 3, \tilde{c} = 20$, and $\tilde{d} = 1.3$. Thus, the actual parameters are fully estimated. The changes of parameters' absolute errors are shown in Fig. 5. The convergence result of the objective function shown in Fig. 6 approaches 10^{-3}. It can be seen that the objective function value converges to zero quickly.

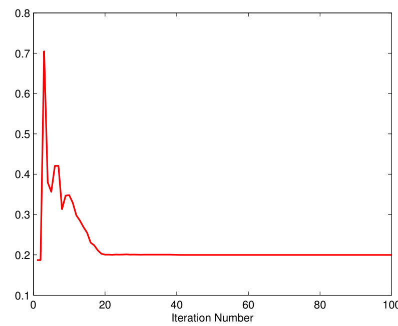
4.2. Simulations on time-delay noisy chaotic systems

We choose the time-delay Mackey-Glass chaotic system (3.10) with random noise for the last numerical example.

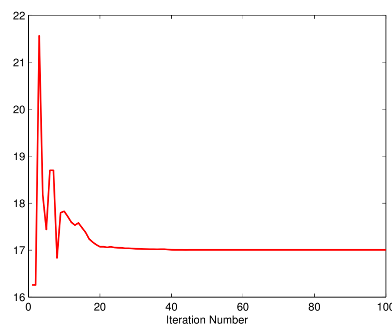
In this example, the unknown parameters are estimated based on the improved PSO algorithm. The estimated results of the parameters τ, α, and β are shown in Fig. 7. The final estimated values are $\tilde{\tau} = 17, \tilde{\alpha} = 0.1$, and $\tilde{\beta} = 0.2$. Thus, the actual parameters are fully estimated. The changes of parameters' absolute errors are shown in Fig. 8. The objective function value converges to zero quickly, as shown in Fig. 9. And the convergence result approaches a value smaller than 10^{-2}.



(a) Evolution process of $\tilde{\alpha}$



(b) Evolution process of $\tilde{\beta}$



(c) Evolution process of $\tilde{\tau}$

Figure 7. Parameter estimation for the Mackey-Glass chaotic system (3.10) based on the improved PSO algorithm

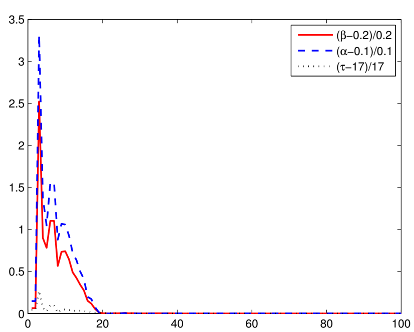


Figure 8. Evolution process of absolute errors of parameters for the noisy Mackey-Glass chaotic system (3.10)

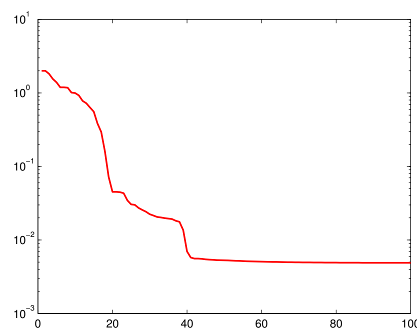


Figure 9. Convergence trajectory of the objective function of the noisy Mackey-Glass chaotic system (3.10) based on the improved PSO algorithm

5. Conclusions

In this paper, we investigated the parameter estimation problem for noisy chaotic systems with unknown time delays and parameters based on an improved PSO algorithm. Simulation results on noisy hyperchaotic Lorenz and Lü systems and the time-delay noisy Mackey-Glass chaotic system show that the improved PSO algorithm can successfully estimate the time delays and parameters. Moreover, the effectiveness and efficiency of the improved PSO algorithm are verified by numerical examples.

Acknowledgement

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