Journal of Applied Analysis and Computation
Volume 5, Number 2, May 2015, 177–188
Website:http://jaac-online.com/
doi:10.11948/2015016

A SOCIAL NETWORK MODEL WITH PROXIMITY PRESTIGE PROPERTY*

Yuli Zhao[1] , Hai Yu[1,†], Wei Zhang[1], Wenhua Zhang[1], and Zhiliang Zhu[1]

Abstract In general, many real-world networks not only possess scale-free and high clustering coefficient properties, but also have a fast information transmission capability. However, the existing network models are unable to well present the intrinsic fast information transmission feature. The initial infected nodes and the network topology are two factors that affect the information transmission capability. By using preferential attachment to high proximity prestige nodes and triad formation, we provide a proximity prestige network model, which has scale-free property and high clustering coefficient. Simulation results further indicate that the new model also possesses tunable information transmission capability archived by adjusting its parameters. Moreover, comparing with the BA scale-free network, the proximity prestige network PPNet05 achieves a higher transmission capability when messages travel based on SIR and SIS models. Our conclusions are directed to possible applications in rumor or information spreading mechanisms.

Keywords Complex network, proximity prestige, fast information spreading.

MSC(2000) 05C75, 05C20.

1. Introduction

In the past two decades, research on complex networks has captured a great deal of attentions. Complex networks are ubiquitous in nature and society, including metabolic networks [8], the World Wide Web [2], social networks and scientific citation networks [17]. For these real-world networks, lots of measurements show that some large-scale networks can self-organize into a scale-free state, which means that the probability of a randomly selected node with degree k is $Ak^{-\gamma}$, where γ is the degree exponent and A is the normalized coefficient. In many real-world networks, γ is a value in $[2.1, 3]$. Simultaneously, lots of real-world networks possess high average clustering coefficients, optimal controllability, strong resistance to attacks and fast information transmission capability, etc. However, most existing network models emphasize on the scale-free and high clustering coefficient properties, ignoring other intrinsic features such as the fast information transmission capability [15].

The BA network model, which generates a network with its degree distribution following a power-law distribution with $\gamma = 3$, is a typical scale-free network with

†The corresponding author. Email address: yuhai@mail.neu.edu.cn (H. Yu)
1Software College, Northeastern University, Shenyang, China
*The authors were supported by the National Natural Science Foundation of China (Grants Nos. 61374178 and 61402092), the Liaoning Provincial Natural Science Foundation of China (Grant No. 201202076), and the Fundamental Research Funds for Central Universities (Grant Nos. N130417004 and N130317001).

a low clustering coefficient value [3]. Further, the Extended-BA model [1] provides a scale-free network with a range of exponents γ between 2 and ∞. Recently, some scale-free network models with tunable power-law exponent and clustering coefficient have been proposed [4, 6]. All of these models are used to generate undirected networks. However, most real-world networks are directional, in which the number of incoming links into a node is its in-degree and the number of outgoing edges is its out-degree. The Price model was proposed as a directed scale-free network model constructed by selecting the existed nodes with largest in-degree as neighbors of the newly added node [12, 13, 16]. Recently, some directed network models have been proposed to investigate and model the growth of citation networks [14], [5]. In the citation network model, papers are represented by nodes, which are added sequentially and all the out-link of a paper are added once the paper joins the network. Nonetheless, in social networks, such as weibo networks and blog networks, once an individual has been added into a network, it can get or transmit information from or to the existed ones already in the network. Thus, the in-link of the newly added one may be larger than zero.

An open problem about complex networks is to figure out how the structural topology affects the epidemic or information spreading on the networks. To s-tudy the information transmission capability of various networks, we apply the susceptible-infectious-recovered (SIR) and susceptible-infectious-susceptible (SIS) models [10, 11] on complex networks. These models have been used to describe disease as well as information and rumor spreading in real-world networks [9]. It has been proven that the topological structure and the choice of the initially infected nodes are two factors which affect the spreading capacity of the network. In general, the degree, betweenness centrality, and core value of nodes are used as metrics to identify which node is more important for spreading in an undirected network [9, 11]. In directed networks, the in-degree, size of the input domain and proximity prestige are used to evaluate the prestige value of each node which reflect the importance of different nodes.

In this paper, the proximity prestige is used to select the initially infected nodes for a directed network model to accelerate the information spreading. In section 2, the concept of proximity prestige and the network model using proximity prestige are described, and in section 3, simulations are carried out and the features of the proposed proximity prestige network are reported. In section 4, a conclusion is drawn.

2. The Proximity Prestige Network

In the area of social network analysis, prestige is traditionally measured from structural metrics such as in-degree and proximity prestige values [7]. Proximity prestige considers prestige of a node in its input domain, which includes all the nodes that can directly or indirectly reach it.

For a complex network with N nodes, the proximity prestige [19] of node i is defined by

$$f(i) = \frac{|\Omega_i|^2}{(N-1)\sum_{j\in\Omega_i} d_{ji}}, \tag{2.1}$$

where Ω_i denotes the input domain of node i which contains all the nodes that have at least one path to node i in the network. The shortest path length from j to i

is denoted by d_{ji}. Figure 1 shows a network with 10 nodes and 9 edges, where the proximity prestige of each node calculated according to equation (2.1) is listed in the bracket near the node.

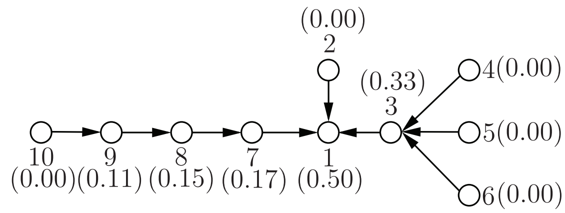


Figure 1. Proximity prestige of a directed network with 10 nodes

From Figure 1, it can be observed that although nodes 1 and 3 have the same in-degree, node 1 achieves larger proximity prestige as the input domain size of node 1 is larger than that of node 3. Figure 1 also shows that the proximity prestige of a node without in-degree is 0.

In general, the scale-free property of a complex network is accomplished by preferential attachment, which means that the newly added nodes prefer to connect with existed nodes with large degrees, rather than the ones with low degrees. In this section, we suggest a proximity prestige network model, in which the newly added node preferentially connects with the high proximity prestige nodes. In addition, it has been shown that the real-world social networks always possess large clustering coefficients, and a large clustering coefficient indicates more triangular structure in the network. Therefore, to guarantee having a high clustering coefficient during the network generation, the triangular structure is introduced to build the network model.

With the aforementioned feature in mind, the proximity prestige network model is starting from a finite network with m_0 nodes and 0 edges. At each time step t, a node i is introduced and m ($m \leq m_0$) directed edges are formed between i and a set of nodes already existed in the network. Once a new node joins the network, two steps are required to generate its m links, including (i) preferential attachment step and (ii) the triad formation step. Specifically, for each node i added at time t, m edges are formed by the following process:

(1) The preferential attachment (PA) step: an existed node v is selected with the probability proportional to its proximity prestige, i.e., the probability for an existed node v to be attached by i is

$$\Theta_i = \frac{f(i) + \rho_i}{\sum_{j \in V} f(j) + |V| * \rho_i}, \tag{2.2}$$

where $V = \{n_1, n_2, ..., n_N\}$ is a set containing all the nodes in the complex network at time $t-1$, $\rho_i = \frac{a}{|V|}$ ($a > 0$), which is used to guarantee a certain probability of node with proximity prestige 0 to have a chance to receive new edges. If node v is selected according to probability (2.2), a link from node i to node v is attached, i.e. node v is a following node of i, and node i is followed by v.

(2) The triad formation (TF) step: Let set P contain the following nodes of node v selected in the previous PA step. A set Q is defined to store all nodes

followed by v. A random number $\delta \in [0,1]$ is generated in advance. If a randomly selected node v_j from set $P \bigcup Q$ belongs to set P and δ is no larger than a predefined probability p ($0 \leq p \leq 1$), an edge from i to v_j is added; if $v_j \in P$ and $\delta > p$, an edge from v_j to i is added. Moreover, if $v_j \in Q$ and $\delta \leq q$ ($0 \leq q \leq p$), an edge from i to v_j is added. Otherwise, an edge from v_j to i is added. This asymmetry is common in real social networks, such as the weibo network, where the newly added nodes are more likely to pay close attention to the node with high proximity prestige and the ones followed by it. Thus, in this model, the value of q is no larger than the value of p.

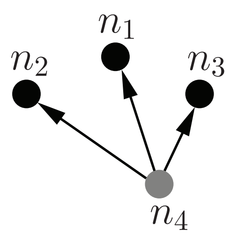
(3) If the neighbored nodes of the selected node v in PA step is no less than $m-1$, the TF step is repeated for $m-1$ times, and m directed edges are generated. Moreover, if the number of nodes neighboring with node v is less than $m-1$, more PA and TF steps are required, until m edges are added into the network.

To illustrate the generation of the proposed network, a network with $m_0 = 3$ nodes and 0 edges is initialized in advance. Suppose $a = 1$. Then, node n_4 is added at time t_0 and $m = 3$ existed nodes are selected from the set $\{n_1, n_2, n_3\}$ with probability $\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$. Three edges from node n_4 to existed nodes are added by repeating 3 times of PA steps as shown in Figure 2(a). The proximity prestige values of the nodes are assigned with $f = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0]$. At time t_1, node n_5 and an edge from n_5 to an existed node with largest proximity prestige, e.g. n_2, are added to the network as illustrated in Figure 2(b). In Figure 2(c), n_4 is selected as a neighbor node of n_2, and with probability q, an edge from n_5 to n_4 is added. As the number of nodes neighboring with n_2 is less than $m-1$, another PA step is required. In Figure 2(d), the mth node of largest proximity prestige in set $\{n_1, n_3\}$ is selected and an edge from n_5 to n_1 is added. The proximity prestige at this time step is $f = [\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0]$. In Figure 2(e), node n_6 is added and an edge from n_6 to n_4 is added at the PA step. An edge from n_6 to n_3 with probability p and an edge from n_5 to n_6 with probability $1-q$ are added to the network at the triad formation step.

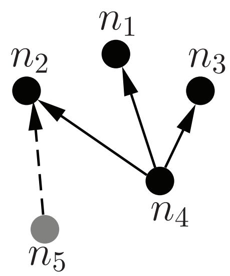
3. Simulation and Analysis

In this section, we set $m_0 = 3$, which means that there are 3 nodes in the initial network without connections. During the generation of the network, once a node is added, $m = 3$ edges are added simultaneously. The first edge is added from the new node to priority selected node with large proximity prestige. Then another two nodes are selected among the neighbors of the first selected node. Suppose $p = 0.9, q = 0.5$, which determine the direction of the edge connecting the newly added node and the aforementioned selected nodes. Repeat the PA and TF steps described in section 2, until the number of nodes reaches 500. The resulting network is shown in Figure 3, and the triangular structure can be apparently observed which indicates that the proximity prestige network has a large clustering coefficient. Moreover, it also indicates that a large number of nodes in this network model have small degrees, and a small number of nodes have high degrees. Thus, the proximity prestige network possesses the scale-free property.

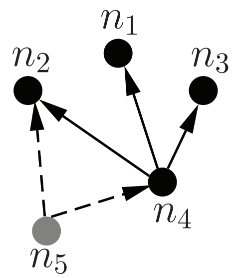
In Table 1, some properties of different networks are listed, including the number N of nodes, average path length l, average in-degree $\bar{d_{in}}$, and clustering coefficient C. Specifically, the networks are



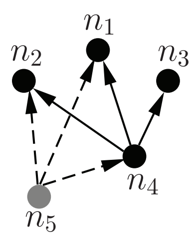
(a) Node n_4 and 3 edges are added by repeating 3 times of PA steps.



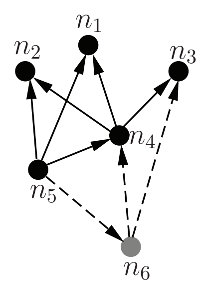
(b) Node n_5 and its first edge are added by a PA step.



(c) The second edge of n_5 is added by a TF step.



(d) The third edge of n_5 is added by another PA step.



(e) Node n_6 and 3 edges are added by 1 PA step and 2 TF steps.

Figure 2. The generation of the proximity prestige network model.

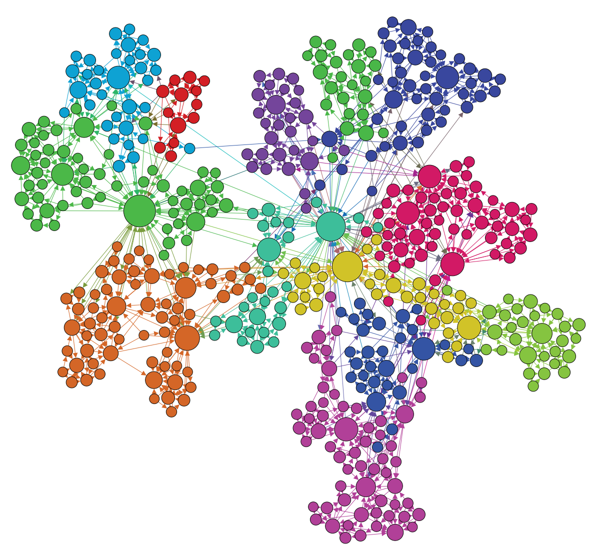


Figure 3. Topology of a proximity prestige network with $N = 500$ nodes.

(1) PPNet11: the proposed proximity prestige network with $m_0 = m = 3$, $a = 1$ and $p = 1.0, q = 1.0$;

(2) PPNet95: the proposed proximity prestige network with $m_0 = m = 3$, $a = 1$ and $p = 0.9, q = 0.5$;

(3) PPNet05: the proposed proximity prestige network with $m_0 = m = 3$, $a = 1$ and $p = 0.5, q = 0.5$; and

(4) BA scale-free network: The network proposed in [3] with $m_0 = m = 3$.

Table 1. Some characteristics of directed proximity prestige networks

Network	N	l	$\bar{d_{in}}$	C
PPNet11	100	1.95	2.91	0.312
	300	2.34	2.97	0.298
	500	3.44	2.98	0.288
	1000	4.23	2.99	0.279
PPNet95	100	3.86	2.91	0.33
	300	4.54	2.97	0.31
	500	5.72	2.98	0.31
	1000	7.11	2.99	0.33
PPNet05	100	4.25	2.91	0.33
	300	4.87	2.97	0.318
	500	5.18	2.98	0.32
	1000	6.12	2.99	0.323
BA scale-free network	100	3.65	2.97	0.082
	300	4.27	2.99	0.047
	500	4.46	2.994	0.029
	1000	4.89	2.997	0.015

Compared with the same scale BA scale-free network, the proximity prestige network has a larger clustering coefficient, indicating that the proximity prestige network has the small-world property. Further, compared with different scales of the proximity prestige network, the average path length and the average in-degree increase when the scale of network becomes larger.

We further remove the direction of edges in the networks generated previously, and construct some undirected networks. Table 2 lists the properties of undirected proximity prestige networks and the BA scale-free network. The same conclusion can be drawn as Table 1. Moreover, with a given number of nodes, the proximity prestige networks listed in Table 2 have the same average degree. This is because the average degree $\bar{d} = E/N$, where E is the number of edges in the network, and m edges are added at each time step.

Figure 4 shows the in-degree and out-degree distributions of the directed proximity prestige networks with $k = 1000$, and the degree distributions of their corresponding undirected networks. In Figure 4(a), $p = 1.0, q = 1.0$, where it can be observed that just $m = 3$ edges are emanating from any node, i.e., the out-degree

Table 2. Some properties of undirected proximity prestige networks

Network	N	l	$\bar{d}$	C
PPNet11	100	2.80	5.82	0.623
	300	3.23	5.94	0.596
	500	3.35	5.964	0.576
	1000	3.71	5.982	0.557
PPNet95	100	2.78	5.82	0.67
	300	3.57	5.94	0.63
	500	3.96	5.964	0.62
	1000	4.28	5.982	0.67
PPNet05	100	2.86	5.82	0.663
	300	3.59	5.94	0.637
	500	3.81	5.964	0.639
	1000	4.55	5.982	0.646
BA scale-free network	100	2.583	5.88	0.159
	300	2.997	5.96	0.092
	500	3.174	5.976	0.054
	1000	3.533	5.988	0.030

d_{out} equals m at any time t. With the values of p and q decrease, the out-degree follows a power-law distribution as shown in Figure 4(b) and Figure 4(c). Moreover, Figure 4 indicates that the in-degree distributions of the directed proximity prestige networks and the degree distributions of their corresponding proximity prestige networks follow the power-law distribution described by

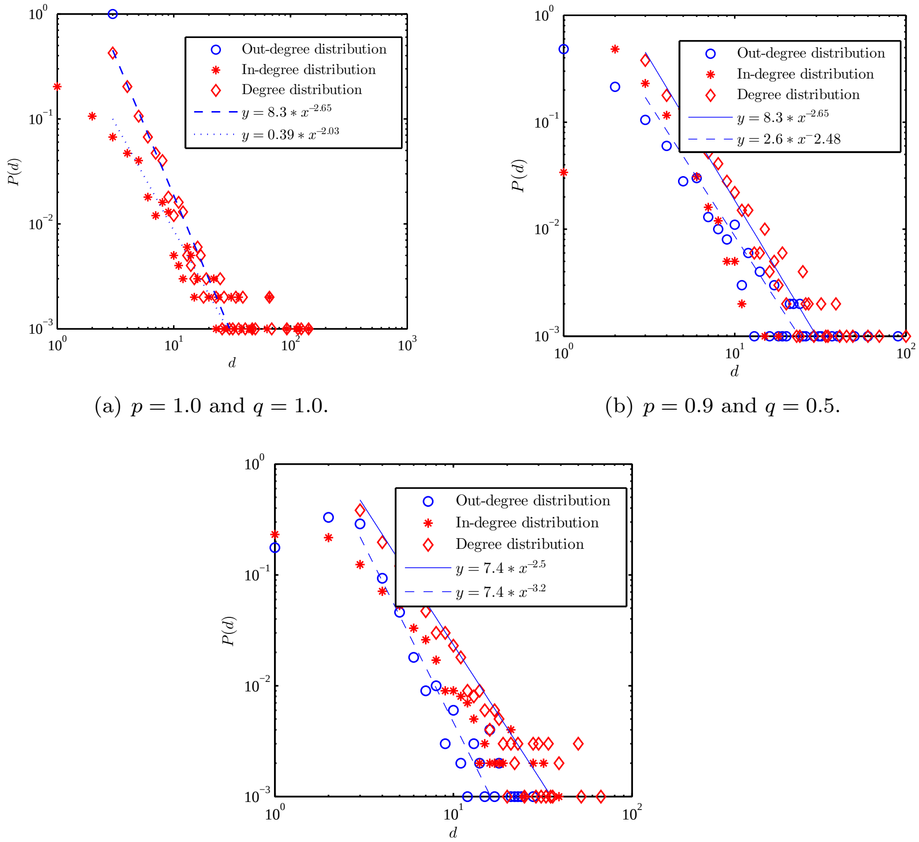
$$P(d) = A \cdot d^{-\gamma}, \tag{3.1}$$

where γ is the characteristic exponent and A is the normalizing coefficient to ensure $\sum_{d=1}^{N} P(d) = 1$. Especially, the degree distributions of the undirected proximity prestige networks conform to the degree distribution property of many real-world networks, i.e. $\gamma \in (2.0, 3.0)$.

3.1. The assortativity coefficients

In undirected networks, the assortativity coefficient describes the connective bias of two nodes with different property values, such as degree, clustering coefficient. It is measured by the Pearson correlation coefficient.

However, the direction property of the edges in directed networks would affect its assortative property. In this section, we will investigate the assortative property of the proposed proximity prestige networks by calculating $r(out, in)$, $r(in, out)$, $r(in, in)$, $r(in, f)$ and $r(f, in)$. Where $r(in, f)$ is quantized as a bias measurement that a large in-degree node connects a large proximity prestige node. Suppose α, $\beta \in \{in, out, f\}$ are one/two type of the properties; $A = (a_{ij})$ is the adjacency matrix of a directed network, in which only if there exists an edge emanating from



(a) $p = 1.0$ and $q = 1.0$.

(b) $p = 0.9$ and $q = 0.5$.

(c) $p = 0.5$ and $q = 0.5$.

Figure 4. Degree distributions of the proximity prestige networks with $N = 1000$.

node i to node j, $a_{ij} = 1$; otherwise, $a_{ij} = 0$. α_i and β_j denote the property values of node i and node j. Then, the assortativity coefficient of a proximity prestige network can be described by equation (3.2).

$$\begin{aligned} r(\alpha,\beta) &= \frac{cov(\alpha,\beta)}{\sigma^2} \\ &= \frac{(\sum_{ij} a_{ij})^{-1}\sum_{ij} a_{ij}(\alpha_i-\mu_\alpha)(\beta_j-\mu_\beta)}{\sqrt{(\sum_{ij} a_{ij})^{-1}\sum_{ij} a_{ij}(\alpha_i-\mu_\alpha)^2}\sqrt{(\sum_{ij} a_{ij})^{-1}\sum_{ij} a_{ij}(\beta_i-\mu_\beta)^2}} \\ &= \frac{\sum_{ij} a_{ij}(\alpha_i-\mu_\alpha)(\beta_j-\mu_\beta)}{\sqrt{\sum_{ij} a_{ij}(\alpha_i-\mu_\alpha)^2\sum_{ij} a_{ij}(\beta_i-\mu_\beta)^2}}. \end{aligned} \tag{3.2}$$

Where, $\mu_\alpha = \sum_{i=1}^{N} \alpha_i$ is the average of the property α. If $r > 0$, the network is assortative according to properties α and β; otherwise, it is a disassortative network due to properties α and β. Table 3 lists the assortativity coefficients of (i) in-degree and out-degree $r(in, out)$, (ii) out-degree and in-degree $r(out, in)$, (iii) in-degree and in-degree $r(in, in)$, (iv) in-degree and proximity prestige $r(in, f)$, (v) proximity prestige and in-degree $r(f, in)$, (vi) proximity prestige and proximity prestige $r(f,f)$. It indicates (i) large in-degree nodes are mostly connected to large proximity prestige nodes; (ii) large proximity prestige nodes are also likely connected to large proximity prestige nodes in the networks.

Table 3. The assortativity coefficients of proximity prestige networks

N	p,q	$r(in,out)$	$r(out,in)$	$r(in,in)$	$r(in,f)$	$r(f,in)$	$r(f,f)$
300	$0.9, 0.5$	-0.0165	-0.0627	0.0235	0.2227	0.0115	0.3083
	$0.5, 0.5$	-0.0314	-0.0766	-0.0157	0.1955	0.0097	0.2255
500	$0.9, 0.5$	-0.0271	-0.0224	0.0400	0.2322	0.0246	0.2806
	$0.5, 0.5$	-0.0403	-0.0525	-0.0457	0.1832	-0.0981	0.1844
1000	$0.9, 0.5$	0.0690	-0.0081	-0.0469	0.2214	-0.0715	0.5270
	$0.5, 0.5$	-0.0244	-0.0456	-0.0274	0.2153	-0.0435	0.2249

3.2. Spreading Feature Analysis

In this section, the information spreading features of proximity prestige networks based on (i) SIR model and (ii) SIS model are analyzed and compared with that of BA scale-free network. All the networks are constructed with 1000 nodes.

3.2.1. Spreading property based on SIR model

Suppose that a proximity prestige network with $p = 0.5, q = 0.5$ is constructed in advance, and 10 of the largest proximity prestige nodes are initially infected. The messages are travelled along the opposite directions of the edges, i.e., messages are transmitted from high proximity prestige nodes to their neighbored nodes. We denote the probability that an infectious node will infect a susceptible neighbor as β. An infectious node becomes a recovered node with probability γ. Each simulation has been repeated for 1000 times and the spreading effect is shown in Figure 5. As usual, $S(t), I(t), R(t)$ represent the proportions of suspected nodes, infected node and recovered nodes at time t, and $S(t) + I(t) + R(t) = 1$.

Figure 5 indicates that the number of infected nodes increases rapidly at the beginning, and decreases sharply after a threshold. Moreover, the number of recovered nodes increases initially, and as the time step approaches 300, the proportion of recovered nodes becomes independent of t.

Further, we set the initially infected nodes as 10 of the largest in-degree nodes, and compare the proportion of infected nodes with that of 10 largest proximity prestige nodes being initially infected. The result is shown in Figure 6. It indicates that the spreading efficiency with 10 largest proximity prestige nodes initially infected is slightly larger than that of initially infected nodes with largest in-degrees.

Figure 7 plots the proportion of infected nodes in PPNet05, PPNet95, PPNet11 and the BA scale-free network, in which the initially infected nodes are set to be 10 largest proximity prestige nodes. It reveals that the number of infected nodes of PPNet11 and PPNet05 are larger than that of the BA scale-free network before the value of $I(t)$ approaching 0. Moreover, the number of infected nodes of the BA scale-free network outperforms the PPNet95 initially, then PPNet95 achieves a higher proportion of infected nodes after a certain time step.

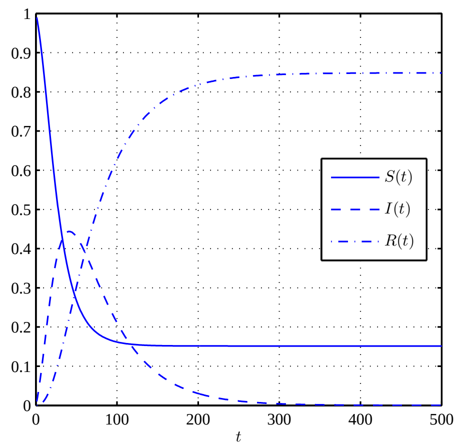


Figure 5. The spreading effect of a proximity prestige network based on SIR model ($\beta = 0.05, \gamma = 0.02$).

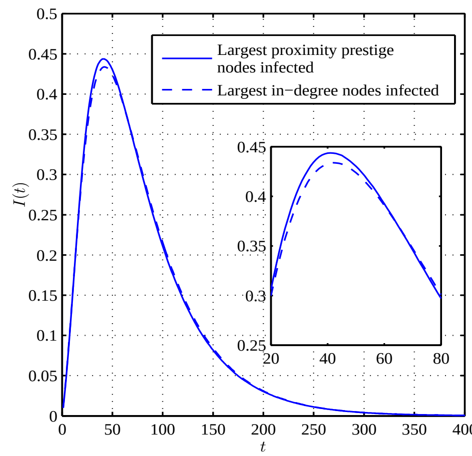


Figure 6. The comparison of spreading effect of a proximity prestige network with different initially infected nodes.

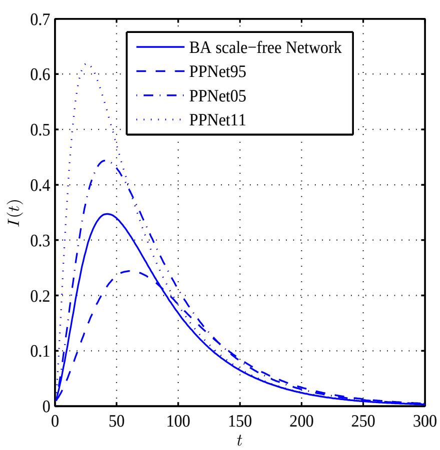


Figure 7. The proportion of infected nodes of the proximity prestige networks and the BA scale-free network on SIR model. ($p = 1.0, q = 1.0$), ($p = 0.9, q = 0.5$), ($p = 0.5, q = 0.5$) and $m_0 = 3, m = 3$ are used for the proximity prestige network while $m_0 = 3$ and $m = 3$ is used for the BA scale-free network.

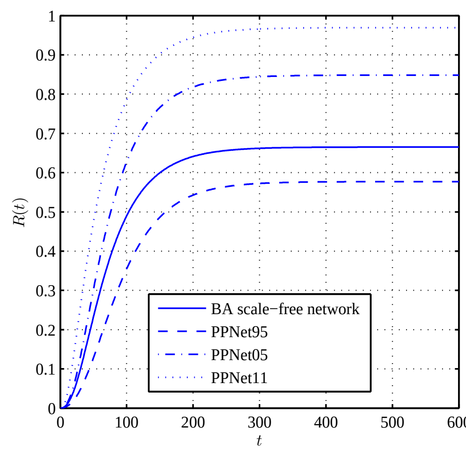


Figure 8. The proportion of recovered nodes of the proximity prestige networks and the BA scale-free network on SIR model.

The proportion of recovered nodes shown in Figure 8 also reveals that the PPNet05 and PPNet11 achieve much better spreading capability than the BA scale-free network.

3.2.2. Spreading property based on SIS model

At each time step, suppose that every susceptible node v is infected along the link in the network with rate $\beta = 0.05$ if it is directed by an infectious node; and an infectious node returns to susceptible state with rate $\gamma = 0.03$. Assume that 10 largest proximity prestige nodes of each network are initially infected. We now further discuss the effective transmission of proximity prestige networks based on the SIS model. Figure 9 plots the infected node number of (i) BA scale-free network, (ii) PPNet95, (iii) PPNet05 and (iv) PPNet11. It indicates that for the same number of nodes and links, the PPNet05 has a larger proportion of infected nodes than the BA scale-free network. Comparing to the number of infected nodes of the PPNet95, it can be seen that the BA scale-free network has a higher proportion of infected nodes when the time steps are relatively small. As the time step reaches a critical point, the PPNet95 outperforms the BA scale-free network and provides a much more effective transmission. Moreover, the PPNet11 needs least time steps to visit all reachable nodes.

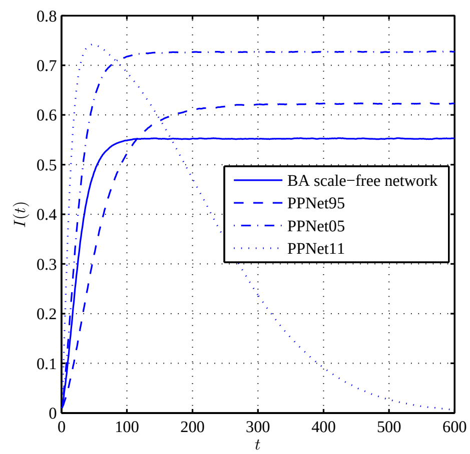


Figure 9. The transmission efficiency of the proximity prestige networks and the BA scale-free network based on SIS model.

4. Conclusion

The topological structure of a complex network and the initially infected nodes are two factors that affect the transmission capacity of epidemic or information over complex networks. Proximity prestige is a structural metric which describes the prestige or importance of a node in a network. In this paper, we construct a scale-free network using preferential attachment based on the proximity prestige. Moreover, triad formation is utilized to ensure that the network model possesses a high clustering coefficient. Simulation results have further shown that the PPNet05 and PPNet11 accomplish higher transmission efficiency than the BA scale-free network based on SIR model. Moreover, the number of reachable nodes in the proximity prestige networks is superior to the same size BA scale-free network based on SIS model.

References

[1] R. Albert and A.-L. Barabási, *Topology of evolving networks: Local events and universality*, Physical Review Letters, 85(2000), 5234-5237.

[2] A.-L. Barabási, R. Albert, and H. Jeong, *Scale-free characteristics of random networks: The topology of the World-Wide Web*, Physica A, 281(2000), 69-77.

[3] A.-L. Barabási, and R. Albert, *Emergence of scaling in random networks*, Science, 286(1999), 509-512.

[4] E. R. Colman, and G. J. Rodgers, *Complex scale-free networks with tunable power-law exponent and clustering*, Physica A, 392(2013), 5501-5510.

[5] K. B. Hajra, and P. Sen, *Modelling aging characteristics in citation networks*, Physica A, 368(2006), 575-582.

[6] P. Holme and B. J. Kim, *Growing scale-free networks with tunable clustering*, Physical Review E, 65(2002), 026107.

[7] L. L. Huang, and Z. Y. Xia, *Measuring user prestige and interaction preference on social network site*, 2009 Eigth IEEE/ACIS International Conference on Computer and Information Science, 2009, 1161-1166.

[8] H. Jeong, B. Tombor, R. Albert, Z. N. Oltvai, and A.-L. Barabási, *The large-scale organization of metabolic networks*, Nature, 407(2000), 651-654.

[9] M. Kitsak, L. K. Gallos, S. Havlin, and F. Liljeros, *Identification of influential spreaders in complex networks*, Nature Physics, 6(2010), 888-893.

[10] Y. Moreno, R. Pastor-Satorras, and A. Vespignani, *Epidemic outbreaks in complex heterogeneous networks*, European Physical Journal B, 26(2002), 521-529.

[11] R. Pastor-Satorras, and A. Vespignani, *Epidemic spreading in scale-free networks*, Physical Review Letters, 86(2001), 3200-3203.

[12] D. D. S. Price, *Networks of scientific papers*, Science, 149(1965), 510-515.

[13] D. D. S. Price, *A general theory of bibliometric and other cumulative advantage processes*, Journal of the American Society for Information Science, 27(1976), 292-306.

[14] F. X. Ren, H. W. Shen, and X. Q Cheng, *Modeling the clustering in citation networks*, Physica A, 391(2012), 3533-3539.

[15] D. H. Shi, G. R. Chen, W. W. K. Thong, and X. Y. Yan, *Searching for optimal network topology with best possible synchronizability*, IEEE Circuits and Systems Magazine, 13(2013), 66-75.

[16] X. F Wang, X. Li, and G. R. Chen, *Network Science: An Introduction*, Beijing: Higher Education Press, 2012.

[17] M. Wang, and G. Yu, *Effect of the age of papers on the preferential attachment in citation networks*, Physica A, 388(2009), 4273.

[18] G. Wilson, and W. Banzhaf, *Discovery of email communication networks from the enron corpus with a genetic algorithm using social network analysis*, 2009 IEEE Congress on Evolutionary Computation, CEC 2009, 2009, Trondheim, Norway, 3256-3263.

[19] F. J. Zhang, Z. Q. You, C. Y. Fan, C. Gao, R. Cohen, Y. Hsueh and Z. K. Zhou, *Friendship quality, social preference, proximity prestige, and self-perceived social competence: Interactive influences on children's loneliness*, Journal of School Psychology, 52(2014), 511-526.