

# UNSTEADY FREE CONVECTIVE MHD FLOW AND MASS TRANSFER THROUGH POROUS MEDIUM IN A ROTATING SYSTEM WITH FLUCTUATING HEAT SOURCE/SINK AND CHEMICAL REACTION

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**Abstract** An investigation of unsteady MHD free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium in the presence of heat source, bounded by an infinite vertical porous surface, in a rotating system is presented. The porous plane surface and the porous medium are assumed to rotate in a solid body rotation. The vertical surface is subject to uniform constant suction perpendicular to it and the temperature at this surface fluctuates in time about a non-zero constant mean. Analytical expressions for the velocity, temperature and concentration fields are obtained using perturbation technique.

**Keywords** MHD flow, mass transfer, free convection, heat transfer, chemical reaction, porous medium, rotation.

**MSC(2000)** 76D, 76S, 76W, 76N.

## 1. Introduction

The problems on fluid flow and mass transfer through porous medium in rotating environment have significant role in the application of geophysics, petrochemical engineering, meteorology, oceanography and aeronautics. The stimulus for scientific research on rotating fluid system is basically originated from geophysical and fluid engineering applications. Many aspects of the motion of terrestrial and planetary atmospheres are highly influenced by the effect of rotation. Rotation flow theory is utilized in determining the viscosity of the fluid, in the construction of the turbine and other centrifugal machines.

The study of effects of magnetic field on free convection flow is important in liquid metals, electrolytes, and ionized gases. The thermal physics of hydromagnetic flow problems with mass transfer is of interest in power engineering and metallurgy. Free convection flows are of great interest in a number of industrial applications such

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as fiber and granular insulation, geothermal systems etc. Magnetohydrodynamic (MHD) flows have attracted the attention of a large number of scholars due to their diverse applications. In astrophysics and geophysics, they are applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. In engineering, MHD flows find their application in MHD pumps, MHD bearings, etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomenon of mass transfer is also very common in the theory of stellar structure and observable effects are detectable, at least on the solar surface.

The phenomenon of hydromagnetic flow with heat and mass transfer in an electrically conducting fluid past a porous plate embedded in a porous medium has attracted the attention of a good number of investigators because of its varied applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and in the boundary layer control in the field of aerodynamics. Heat transfer in laminar flow is important in problems dealing with chemical reactions and in dissociating fluids. Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions, homogeneous reaction and heterogeneous reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. Reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer.

The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. The flow of a fluid past a wedge is of fundamental importance since this type of flow constitutes a general and wide class of flows in which the free stream velocity is proportional to a power of the length coordinate measured from the stagnation point. All industrial chemical processes are designed to transform cheaper raw materials to high value products (usually via chemical reaction). A 'reactor', in which such chemical transformations take place, has to carry out several functions like bringing reactants into intimate contact, providing an appropriate environment (temperature and concentration fields) for adequate time and allowing for removal of products. Fluid dynamics plays a pivotal role in establishing relationship between reactor hardware and reactor performance. For a specific chemistry catalyst, the reactor performance is a complex function of the underlying transport processes. The first step in any reaction engineering analysis is formulating a mathematical framework to describe the rate (and mechanisms) by which one chemical species is converted into another in the absence of any transport limitations (chemical kinetics). Once the intrinsic kinetics is available, the production rate and composition of the products can be related, in

principle, to reactor volume, reactor configuration and mode of operation by solving mass, momentum and energy balances over the reactor. This is the central task of a reaction and reactor engineering activity. Analysis of the transport processes and their interaction with chemical reactions can be quite difficult and is intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Recent advances in understanding the physics of flows and computational flow modeling (CFM) can make tremendous contributions in chemical engineering.

Several researchers have analyzed a bewildering variety of flows connected to MHD free convective flow through porous media of a rotating/ non-rotating fluid with heat and mass transfer. Das et al. [3] discussed mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Muthucumaraswamy and Ganesh [7] studied unsteady flow of an incompressible fluid past an impulsively started vertical plate with heat and mass transfer. Unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid in the vicinity of a semi-infinite vertical porous moving plate was discussed by Kim [6]. Acharya et al. [1] discussed magnetic field effect on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/stretching plates has been studied by Rath et al. [8]. MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction has been studied by Dash et al. [4]. Non-parallel vortex instability of natural convection flow over a non-isothermal inclined flat plate with simultaneous thermal and mass diffusion has been studied by Acharya et al. [2]. Dinarvand and Rashidi [5] studied a reliable treatment of homotopy analysis method for two-dimensional viscous flow in a rectangular domain bounded by two moving porous walls. Singh et al. [13] analyzed the heat transfer effects on three dimensional fluctuating flow through a porous medium with a variable permeability. Rashidi and Erfani [3] discussed a new analytical study of MHD stagnation-point flow in porous media with heat transfer. Singh et al. [14] studied the effects of permeability and rotation parameters on oscillatory couette flow through a porous medium in a rotating system.

Saxena and Dubey [15] studied the effects of MHD free convection heat and mass transfer flow of visco-elastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime. Unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion was analysed by Saxena and Dubey [3].

Recently, chemical reaction effects on an unsteady MHD free convective flow past an infinite vertical porous plate with constant suction and heat source have studied by Anand Rao and Shivaiah [10]. Reddy et al. [11] studied radiation and chemical reaction effects on free convection MHD flow through a porous medium bounded by vertical surface.

The main objective of the present problem is to study the effect of unsteady free convective viscous flow through porous media in the presence of heat source and chemical reacting species in a rotating system with fluctuating temperature and concentration. The objective of the present study is to extend the work of Rath et al. [12] by incorporating the heat source as well as chemical reaction on the flow,

heat and mass transfer phenomena.

## 2. Formulation of problem

We consider the unsteady free convective flow of a viscous incompressible fluid bounded by a vertical infinite porous surface in a rotating system in the presence of heat source and chemical reaction. The temperature on the surface varies with the time about a non-zero constant mean while the temperature of the free stream is taken to be constant. We consider that the vertical infinite porous plate rotates with the constant angular velocity  $\Omega$  about an axis which is perpendicular to the vertical plane surface. The cartesian co-ordinate system is chosen such that x,y axes, respectively, are in the vertical upward and perpendicular directions on the plane of the vertical porous surface  $z = 0$ , while z-axis is normal to it.

The interaction of Coriolis force with the free convection sets up a secondary flow in addition to primary flow and hence the flow becomes three dimensional. With the above frame of reference and assumptions, the physical variables are functions of and time only. Consequently the equations expressing the conservation of mass, momentum, energy and the equation of mass transfer, under usual Boussinesq approximation, are given by

$$\frac{\partial w^*}{\partial z^*} = 0, \quad (2.1)$$

$$\frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega v^* = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\nu u^*}{K_p^*}, \quad (2.2)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega u^* = \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\sigma B_0^2 v^*}{\rho} - \frac{\nu v^*}{K_p^*}, \quad (2.3)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z^*} - \frac{\nu w^*}{K_p^*}, \quad (2.4)$$

$$\frac{\partial T}{\partial t^*} + w^* \frac{\partial T}{\partial z^*} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial z^{*2}} + S^*(T - T_\infty), \quad (2.5)$$

$$\frac{\partial C}{\partial t^*} + w^* \frac{\partial C}{\partial z^*} = D \frac{\partial^2 C}{\partial z^{*2}} - K_c^*(C - C_\infty). \quad (2.6)$$

Here  $\rho$  is the density of the fluid,  $\sigma$  is the electrical conductivity of the fluid,  $B_0$  is the uniform magnetic field strength,  $\nu$  is the coefficient of the kinematic viscosity,  $k$  is the thermal conductivity of the fluid,  $C_p$  is the specific heat of the fluid,  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of thermal expansion with concentration,  $g$  is the acceleration due to gravity,  $D$  is the chemical diffusivity of the fluid,  $S^*$  is the heat source/sink parameter,  $K_c^*$  is the chemical reaction parameter and  $K_p^*$  is the permeability of the medium.

The corresponding boundary conditions are

$$\begin{aligned} u^* = 0, v^* = 0, T = T_\omega + \varepsilon(T_\omega - T_\infty)e^{i\omega t}, \\ C = C_\omega + \varepsilon(C_\omega - C_\infty)e^{i\omega t} \text{ at } z^* \rightarrow 0, \\ u^*, v^* \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } z^* \rightarrow \infty, \end{aligned} \quad (2.7)$$

where  $\varepsilon \ll 1$  and  $\omega$  is the frequency of oscillation.

There will be always some fluctuation in the temperature. The plate temperature is assumed to vary harmonically with time. It varies from  $T_\omega \pm \epsilon(T_\omega - T_\infty)$  as 't' varies from 0 to  $\frac{\pi}{2\omega}$ . Now there may also occur some variation in suction at the plate due to the variation of the temperature. Here we assume that the frequency of suction and temperature variation are same.

Integrating equation 2.1 we get

$$w(t) = -w_0(1 + \epsilon A e^{i\omega t}), \quad (2.8)$$

where 'A' is the suction parameter,  $w_0$  is the constant suction velocity and  $\epsilon$  is the small positive number such that  $\epsilon A \leq 1$ . Equation (2.4) determines the pressure distribution along the axis of rotation and the absence of  $\frac{\partial p}{\partial y}$  in equation (2.3) implies that there is a net cross flow in Y direction.

Considering  $U = u^* + iv^*$ , and taking into consideration equation (2.8), the momentum equation (2.2) and (2.3) can be written as

$$\begin{aligned} \frac{\partial U}{\partial t^*} - w_0(1 + \epsilon A e^{i\omega t}) \frac{\partial U}{\partial z^*} + 2\Omega i U \\ = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu \frac{\partial^2 U}{\partial z^{*2}} - \frac{\sigma B_0^2 U}{\rho} - \frac{\nu U}{K_p^*}. \end{aligned} \quad (2.9)$$

Now, we introduce the following non-dimensional quantities

$$\begin{aligned} z' = \frac{w_0 z^*}{\nu}, \quad U' = \frac{U}{w_0}, \quad t' = \frac{t^* w_0^2}{\nu}, \quad w' = \frac{\nu w^*}{w_0^2}, \quad T' = \frac{T - T_\infty}{T_\omega - T_\infty}, \quad C' = \frac{C - C_\infty}{C_\omega - C_\infty}, \\ S_c = \frac{\nu}{D}, \quad P_r = \frac{\rho \nu C_p}{k}, \quad G_r = \frac{g\beta\nu(T_\omega - T_\infty)}{w_0^3}, \quad G_c = \frac{g\beta^*\nu(C_\omega - C_\infty)}{w_0^3}, \\ R_0 = \frac{\Omega\nu}{w_0^2}, \quad M = \sqrt{\frac{\sigma B_0^2 \nu}{\rho w_0^2}}, \quad K_p = \frac{K_p^* w_0^2}{\nu^2}, \quad S = \frac{S^* \nu}{w_0^2}, \quad K_c = \frac{\nu K_c^*}{w_0^2}. \end{aligned}$$

With the above non-dimensional quantities (dropping the dashes) equation (2.9), (2.5) and (2.6) can be written as

$$\frac{\partial U}{\partial t} - (1 + \epsilon A e^{i\omega t}) \frac{\partial U}{\partial z} + 2iR_0 U = G_r T + G_c C + \frac{\partial^2 U}{\partial z^2} - (M^2 + \frac{1}{K_p}) U, \quad (2.10)$$

$$\frac{\partial T}{\partial t} - (1 + \epsilon A e^{i\omega t}) \frac{\partial T}{\partial z} = \frac{1}{P_r} \frac{\partial^2 T}{\partial z^2} + S T, \quad (2.11)$$

$$\frac{\partial C}{\partial t} - (1 + \epsilon A e^{i\omega t}) \frac{\partial C}{\partial z} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2} + K_c C. \quad (2.12)$$

Hence the boundary conditions are

$$\begin{aligned} \text{at } z = 0 : \quad & U = 0, \quad T = 1 + \epsilon e^{i\omega t}, \quad C = 1 + \epsilon e^{i\omega t}, \\ \text{at } z \rightarrow \infty : \quad & U \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0. \end{aligned} \quad (2.13)$$

### 3. Method of solution

In order to reduce the system of partial differential equations (2.10)-(2.12) under their boundary conditions (2.13), to a system of ordinary differential equations in the

non-dimensional form, in view of equation (2.8) and oscillating plate temperature  $T$ , the solution form of equations (2.10), (2.11) and (2.12) are

$$\begin{aligned} U(z, t) &= U_0(z) + \varepsilon e^{i\omega t} U_1(z), \\ T(z, t) &= T_0(z) + \varepsilon e^{i\omega t} T_1(z), \\ C(z, t) &= C_0(z) + \varepsilon e^{i\omega t} C_1(z), \end{aligned} \quad (3.1)$$

which are valid for small amplitude of oscillation. Substituting equation (3.1) into the system of equations (2.10)-(2.12) and equating the harmonic and non-harmonic terms we get

$$\frac{d^2 U_0}{dz^2} + \frac{dU_0}{dz} - (2iR_0 + M^2 + \frac{1}{K_p})U_0 = -G_r T_0 - G_c C_0, \quad (3.2)$$

$$\frac{d^2 U_1}{dz^2} + \frac{dU_1}{dz} - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}U_1 = -G_r T_1 - G_c C_1 - A \frac{dU_0}{dz}, \quad (3.3)$$

$$\frac{d^2 T_0}{dz^2} + P_r \frac{dT_0}{dz} + SP_r T_0 = 0, \quad (3.4)$$

$$\frac{d^2 T_1}{dz^2} + P_r \frac{dT_1}{dz} - (i\omega P_r - SP_r)T_1 = -AP_r \frac{dT_0}{dz}, \quad (3.5)$$

$$\frac{d^2 C_0}{dz^2} + S_c \frac{dC_0}{dz} - K_c S_c C_0 = 0, \quad (3.6)$$

$$\frac{d^2 C_1}{dz^2} + S_c \frac{dC_1}{dz} - (i\omega S_c + K_c S_c)C_1 = -AS_c \frac{dC_0}{dz}. \quad (3.7)$$

The boundary conditions (2.13) using (3.1) becomes

$$\begin{aligned} \text{at } z = 0 : \quad & U_0 = 0, \quad T_0 = 1, \quad C_0 = 1, \quad U_1 = 0, \quad T_1 = 1, \quad C_1 = 1, \\ \text{at } z \rightarrow \infty : \quad & U_0 \rightarrow 0, \quad T_0 \rightarrow 0, \quad C_0 \rightarrow 0, \quad U_1 \rightarrow 0, \quad T_1 \rightarrow 0, \quad C_1 \rightarrow 0. \end{aligned} \quad (3.8)$$

The solution of equations (3.4) and (3.5) using the boundary condition (3.8) are given by

$$T_0(z) = e^{N_5 z}, \quad T_1(z) = e^{M_2 z} + \frac{AP_r N_5}{N_6} (e^{M_2 z} - e^{N_5 z})$$

and we get

$$T(z, t) = T_0(z) + \varepsilon e^{i\omega t} T_1(z) = e^{N_5 z} + \varepsilon e^{i\omega t} \left\{ e^{M_2 z} + \frac{AP_r N_5}{N_6} (e^{M_2 z} - e^{N_5 z}) \right\}. \quad (3.9)$$

The solutions of equations (3.6) and (3.7) using the boundary conditions (3.8) are given by

$$C_0(z) = e^{N_2 z}, \quad C_1(z) = e^{M_4 z} + \frac{AS_c N_2}{N_3} (e^{M_4 z} - e^{N_2 z})$$

and we get

$$C(z, t) = C_0(z) + \varepsilon e^{i\omega t} C_1(z) = e^{N_2 z} + \varepsilon e^{i\omega t} \left\{ e^{M_4 z} + \frac{AS_c N_2}{N_3} (e^{M_4 z} - e^{N_2 z}) \right\}. \quad (3.10)$$

The solution of equations (3.2) and (3.3) using the boundary conditions (3.8) are given by

$$\begin{aligned} U_0(z) &= L_1 e^{N_5 z} + L_2 e^{N_2 z} + L_3 e^{M_6 z}, \\ U_1(z) &= N_{17} e^{M_8 z} + N_{12} e^{M_2 z} + N_{13} e^{N_5 z} + N_{14} e^{M_4 z} + N_{15} e^{N_2 z} + N_{16} e^{M_6 z} \end{aligned}$$

and we get

$$\begin{aligned} U(z, t) &= U_0(z) + \varepsilon e^{i\omega t} U_1(z) = L_1 e^{N_5 z} + L_2 e^{N_2 z} + L_3 e^{M_6 z} \\ &+ \varepsilon e^{i\omega t} (N_{17} e^{M_8 z} + N_{12} e^{M_2 z} + N_{13} e^{N_5 z} + N_{14} e^{M_4 z} + N_{15} e^{N_2 z} + N_{16} e^{M_6 z}). \end{aligned} \quad (3.11)$$

Equation (3.11) reveals that, the steady part of the velocity field has three-layer character, while the oscillating part of the fluid field exhibits a multilayer character.

From equations (3.9) and (3.10), we observe that in case of considerably slow motion of the fluid i.e. when viscous dissipation term is neglected, the temperature profiles are mainly affected by Prandtl number ( $P_r$ ) and source parameter ( $S$ ) and concentration profiles are affected by Schmidt number ( $S_c$ ) and chemical reaction parameter ( $K_c$ ) of the fluid respectively. Considering

$$U_0 = u_0 + iv_0, \quad U_1 = u_1 + iv_1.$$

Now, it is convenient to write the primary and secondary velocity fields, in terms of the fluctuating parts, separating the real and imaginary part from equation (3.11) and taking only the real parts as they have physical significance. The velocity distribution of the flow field can be expressed in fluctuating parts as

$$\begin{aligned} U(z, t) &= U_0(z) + \varepsilon e^{i\omega t} U_1(z) \\ u + iv &= u_0 + iv_0 + \varepsilon (\cos\omega t + i\sin\omega t)(u_1 + iv_1) \\ &= u_0 + iv_0 + \varepsilon u_1 \cos\omega t + i\varepsilon u_1 \sin\omega t + i\varepsilon v_1 \cos\omega t - \varepsilon v_1 \sin\omega t. \end{aligned}$$

Comparing both sides we get

$$\begin{aligned} u(z, t) &= w_0 [u_0(z) + \varepsilon (u_1 \cos\omega t - v_1 \sin\omega t)], \\ v(z, t) &= w_0 [v_0(z) + \varepsilon (u_1 \sin\omega t + v_1 \cos\omega t)]. \end{aligned} \quad (3.12)$$

Hence, the expression for the transient velocity profiles, for  $\omega t = \frac{\pi}{2}$ , are given by

$$\begin{aligned} u(z, \frac{\pi}{2\omega}) &= w_0 [u_0(z) - \varepsilon v_1(z)], \\ v(z, \frac{\pi}{2\omega}) &= w_0 [v_0(z) + \varepsilon u_1(z)]. \end{aligned} \quad (3.13)$$

### Skin friction:

The skin friction at the plate  $z=0$  in terms of amplitude and phase is given by

$$\begin{aligned} \frac{dU}{dz} \Big|_{z_0} &= \frac{dU_0}{dz} \Big|_{z_0} + \varepsilon e^{i\omega t} \frac{dU_1}{dz} \Big|_{z_0} = N_5 L_1 + N_2 L_2 + N_3 L_3 \\ &+ \varepsilon e^{i\omega t} (M_8 N_{17} + M_2 N_{12} + N_5 N_{13} + M_4 N_{14} + N_2 N_{15} + M_6 N_{16}). \end{aligned} \quad (3.14)$$

The  $\tau_{xz}$  and  $\tau_{yz}$  components of the skin friction at the plate are given by

$$\tau_{xz} = \frac{\partial u_0}{\partial z} \Big|_{z_0} - \varepsilon \frac{\partial v_1}{\partial z} \Big|_{z_0} \text{ and } \tau_{yz} = \frac{\partial v_0}{\partial z} \Big|_{z_0} + \varepsilon \frac{\partial u_1}{\partial z} \Big|_{z_0}.$$

**Rate of heat transfer:**

The rate of heat transfer coefficient at the plate  $z = 0$  in terms of amplitude and phase is given by

$$\frac{dT}{dz} \Big|_{z_0} = \frac{dT_0}{dz} \Big|_{z_0} + \varepsilon e^{i\omega t} \frac{dT_1}{dz} \Big|_{z_0} = N_5 + \varepsilon e^{i\omega t} \left\{ M_2 + \frac{AP_r N_5}{N_6} (M_2 - N_5) \right\}. \quad (3.15)$$

**Rate of mass transfer:**

The rate of mass transfer coefficient

$$\frac{dC}{dz} \Big|_{z_0} = \frac{dC_0}{dz} \Big|_{z_0} + \varepsilon e^{i\omega t} \frac{dC_1}{dz} \Big|_{z_0} = N_2 + \varepsilon e^{i\omega t} \left\{ M_4 + \frac{AS_c N_2}{N_3} (M_4 - N_2) \right\}. \quad (3.16)$$

**4. Results and discussion**

The problem of unsteady MHD free convective flow with heat and mass transfer effects in a rotating porous medium in the presence of heat source and chemical reaction has been considered. The solution for velocity field, temperature field and concentration profiles are obtained using the perturbation technique.

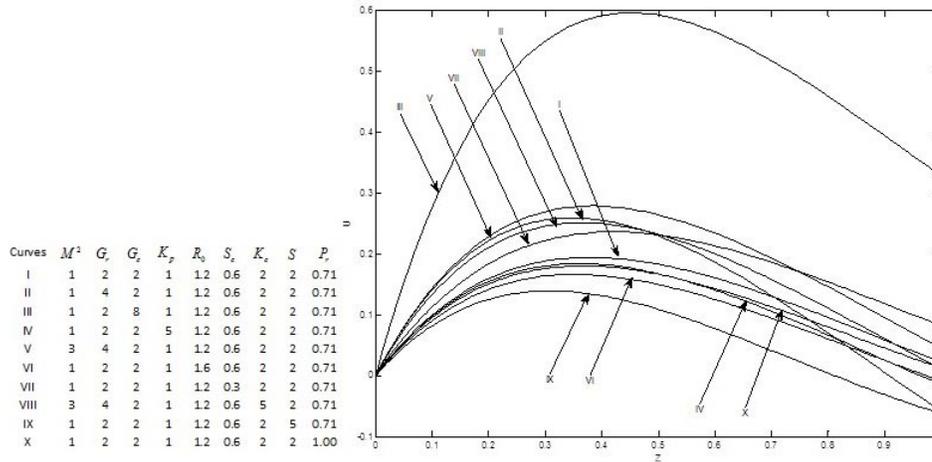


Figure 1. Velocity Profiles

Fig.1 shows the effects of flow parameters such as the magnetic parameter ( $M^2$ ), Grashof numbers for heat and mass transfer ( $G_r$  and  $G_c$ ), porosity parameter ( $K_p$ ), the rotation parameter ( $R_0$ ), Schmidt number ( $S_c$ ), chemical reaction parameter ( $K_c$ ), source parameter ( $S$ ), Prandtl number ( $P_r$ ) on the velocity field. It is observed that the velocity profiles increases whenever there is either an increase in the Grashof number or the modified Grashof number or the Hartmann number whereas the profiles show the reverse trend whenever there is an increase in either of the rotation

parameter or the permeability of the porous medium or heat source. It is concluded that the rotation, permeability of the porous medium and the heat source exert retarding influence on the velocity profiles. From fig.1 it is noted that all the velocity profiles increase steadily near the lower plate and thereafter they show a constant decrease at the other plate. The magnetic parameter is found to accelerate the velocity of the flow field due to the magnetic pull of the Lorentz force acting on the flow field which coincides with the result of Singh [17].

Further, it is to remark that high value of thermal buoyancy parameter ( $G_r = 8$ ) enhances the velocity sharply near the plate and attains the maximum magnitude. For high value of rotation parameter, heat source parameter, permeability of the medium and low value of magnetic parameter flow reversal occurs. Another point to note that the flow reversal is prevented due to low speed of rotation, high value of chemical reacting species and high value of magnetic parameter. One remarkable observation is that for high value of source parameter the flow reversal occurs and attains the lowest magnitude whereas the opposite effect is marked in case of high value of chemical reacting species.

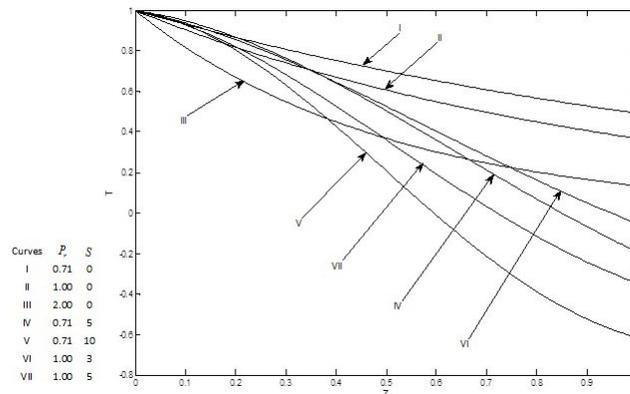


Figure 2. (Temperature Profiles) Effects of  $P_r$  and  $S$  on temperature profile with  $\varepsilon = 0.001$ ,  $A = 0.05$ ,  $\omega = \frac{5\pi}{2}$  and  $\omega t = \frac{\pi}{2}$ .

Fig.2 exhibits the temperature profiles for the different values of Prandtl number and source parameter. It is observed that high value of Prandtl number leads to the decrease the temperature uniformly in all the layers in the absence or presence of source parameter ( $S = 0$  and  $S \neq 0$ ). The same observation was made by Rath et al. [12] in the absence of source parameter. Further, it is found that the temperature decreases in all the layers with an increase in source parameter. It is concluded that the heat source and Prandtl number reduce the temperature in all the layers.

Fig.3 shows the concentration profiles for various values of the Schmidt number and chemical reaction parameter. It reveals the concentration variation for different diffusing species such as  $CO_2$ ,  $H_2O$  and  $NH_3$  having Schmidt number 0.30, 0.60 and 0.78 in air respectively as reported by Rath et al. [12]. It is concluded that for heavier species the concentration decreases at all the layers of the flow domain. It is noted that for heavier diffusing foreign species i.e increasing the Schmidt number reduces the velocity in both magnitude and extent and thinning of thermal boundary layer occurs. Substantial increase in the velocity profiles is observed near the plate with decreasing values of the Schmidt number (lighter diffusing species). It is observed

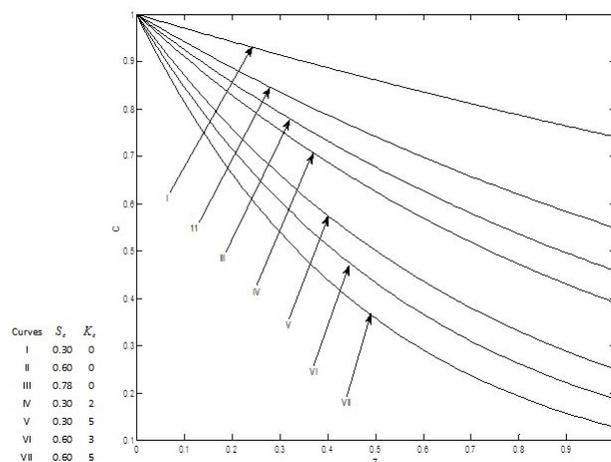


Figure 3. (Concentration Profiles) Effects of  $S_c$  and  $K_c$  on concentration profile with  $\varepsilon = 0.001$ ,  $A = 0.05$ ,  $\omega = \frac{5\pi}{2}$  and  $\omega t = \frac{\pi}{2}$ .

Table 1. (Skin Friction) Variation of Skin friction when  $\varepsilon = 0.001$ ,  $A = 0.05$ ,  $\omega = \frac{5\pi}{2}$  and  $\omega t = \frac{\pi}{2}$ .

SL.NO	$M^2$	$G_r$	$G_c$	$K_p$	$R_0$	$S_c$	$K_c$	$S$	$P_r$	$\tau_{xz}$	$\tau_{yz}$
1	1	2	2	1	1.2	0.6	2	2	0.71	3.29846	3.96063
2	1	8	2	1	1.2	0.6	2	2	0.71	5.86925	8.0583
3	1	2	8	1	1.2	0.6	2	2	0.71	10.6231	11.7449
4	1	2	2	5	1.2	0.6	2	2	0.71	4.62678	3.46093
5	3	2	2	1	1.2	0.6	2	2	0.71	1.16547	3.34871
6	1	2	2	1	1.6	0.6	2	2	0.71	3.07584	2.93958
7	1	2	2	1	1.2	0.3	2	2	0.71	0.95474	1.95169
8	3	2	2	1	1.2	0.6	5	2	0.71	2.07538	4.07094
9	1	2	2	1	1.2	0.6	2	5	0.71	2.75712	3.44373
10	1	2	2	1	1.2	0.6	2	2	7.00	2.39604	1.28955

that the heavier diffusing species have greater retarding effects on the concentration profiles of the flow field which agrees well with the result of Das et al. [3]. It is clear from fig.3 that the concentration profiles decrease with an increase in the chemical reaction parameter.

It is noted from table 1 that both the skin friction components increase due to an increase in either of the Grashof number or the modified Grashof number or for heavier species. Further, it is observed that both the skin friction components decrease due to an increase in either of the rotation parameter or the Hartmann number or the heat source parameter or Prandtl number. It is to note that the permeability of the porous medium increases the skin friction component  $\tau_{xz}$  and decreases the skin friction component  $\tau_{yz}$  whereas the reverse effect is marked in case of chemical reaction parameter.

The magnitude of the heat transfer coefficient for various values of  $\omega$ ,  $t$  and  $S$  are given in table 2. It shows that the magnitude of the heat transfer coefficient in case of both air and water increase whenever there is an increase in either the source

Table 2. The rate of heat transfer ( $N_u$ ) and the phase of rate heat transfer ( $\tan \gamma$ ):  
When  $\varepsilon = 0.001$ ,  $A = 0.05$

$P_r = 0.71(\text{Air})$						$P_r = 7.0(\text{Water})$					
SL.NO	$\omega$	$t$	$S$	$N_u$	$\tan \gamma$	SL.NO	$\omega$	$t$	$S$	$N_u$	$\tan \gamma$
1	2	2	2	1.1902	3.1970	1	2	2	2	3.7374	0.3763
2	2	2	3	1.4580	3.9775	2	2	2	3	4.5774	0.8436
3	3	2	3	1.4609	3.9691	3	3	2	3	4.5892	0.8440
4	3	3	3	1.4581	4.0040	4	3	3	3	4.5763	0.8467
5	3	3	4	1.6837	4.6595	5	3	3	4	5.2855	1.1358
6	4	3	4	1.6865	4.6184	6	4	3	4	5.2972	1.1314
7	4	4	4	1.6830	4.6425	7	4	4	4	5.2834	1.1336

Table 3. Rate of mass transfer:  
When  $\varepsilon = 0.001$ ,  $A = 0.05$

SL NO	$\omega$	$t$	$S_c$	$K_c$	Rate of Mass Transfer
1	2	2	0.22	2	-0.78205
2	4	2	0.22	2	-0.78173
3	2	4	0.22	2	-0.78196
4	2	2	0.60	2	-1.43513
5	2	2	0.78	2	-1.69768
6	2	2	0.60	5	-2.05671
7	2	2	0.60	10	-2.76614

parameter ( $S$ ) or the frequency parameter ( $\omega$ ). But the reverse effect is observed in case of time span. Further, it is observed from table 2 that the heat transfer coefficient in case of water for any particular values of  $\omega$ ,  $t$  and  $S$  is significantly higher when compared with that of air.

Table 3 shows the rate of mass transfer for various values of  $S_c$ ,  $K_c$ ,  $\omega$  and  $t$ . It is observed that the rate of mass transfer decreases when either there is an increase in the Schmidt number ( $S_c$ ) or there is an increase in the chemical reaction parameter ( $K_c$ ). But the rate of mass transfer increases when either there is an increase in the frequency parameter ( $\omega$ ) or there is an increase in the time span ( $t$ ).

### 5. Conclusion

- (i) The modified Grashof number for mass transfer and Grashof number have the effect of accelerating the velocity profiles and the skin friction.
- (ii) The Hartmann number has the effect of increasing the flow field at all the points due to the magnetic pull of the Lorentz force whereas decreasing the skin friction components.
- (iii) The rotation parameter, the permeability of the porous medium and the heat source exert retarding influence on the velocity profiles.
- (iv) The velocity of the fluid layer decreases and the thickness of the thermal boundary layer and further reduction occurs with increasing values of the Schmidt number.

- (v) Flow reversal is prevented for low speed of rotation, high value of chemical reacting species and high value of magnetic parameter.
- (vi) The heat source and prandtl number reduce the temperature in all the layers.
- (vii) The concentration profiles decrease with an increase in the chemical reaction parameter and Schmidt number.
- (viii) The magnitude of the heat transfer coefficient in case of both air and water increase whenever there is an increase in either the source parameter ( $S$ ) or the frequency parameter ( $\omega$ ).
- (ix) The rate of mass transfer decreases when either there is an increase in the Schmidt number ( $S_c$ ) or the chemical reaction parameter ( $K_c$ ). But the rate of mass transfer increases when either there is an increase in the frequency parameter ( $\omega$ ) or the time span ( $t$ ).

## Appendix

$$\begin{aligned}
 M_1 &= \frac{-P_r + \sqrt{P_r^2 + 4(i\omega P_r - SP_r)}}{2}, & M_2 &= \frac{-P_r - \sqrt{P_r^2 + 4(i\omega P_r - SP_r)}}{2}, \\
 M_3 &= \frac{-S_c + \sqrt{S_c^2 + 4(i\omega S_c + K_c S_c)}}{2}, & M_4 &= \frac{-S_c - \sqrt{S_c^2 + 4(i\omega S_c + K_c S_c)}}{2}, \\
 M_5 &= \frac{-1 + \sqrt{1 + 4(2iR_0 + M^2 + \frac{1}{K_p})}}{2}, & M_6 &= \frac{-1 - \sqrt{1 + 4(2iR_0 + M^2 + \frac{1}{K_p})}}{2}, \\
 M_7 &= \frac{-1 + \sqrt{1 + 4\{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}}{2}, \\
 M_8 &= \frac{-1 - \sqrt{1 + 4\{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}}{2}, \\
 L_1 &= \frac{-G_r}{N_5^2 + N_5 - (2iR_0 + M^2 + \frac{1}{K_p})}, & L_2 &= \frac{-G_c}{N_2^2 + N_2 - (2iR_0 + M^2 + \frac{1}{K_p})}, \\
 L_3 &= -(L_1 + L_2), \\
 N_1 &= \frac{-S_c + \sqrt{S_c^2 + 4K_c S_c}}{2}, & N_2 &= \frac{-S_c - \sqrt{S_c^2 + 4K_c S_c}}{2}, \\
 N_3 &= N_2^2 + S_c N_2 - (i\omega S_c + K_c S_c), & N_4 &= \frac{-P_r + \sqrt{P_r^2 - 4SP_r}}{2}, \\
 N_5 &= \frac{-P_r - \sqrt{P_r^2 - 4SP_r}}{2}, & N_6 &= N_5^2 + P_r N_5 - (i\omega P_r - SP_r), \\
 N_7 &= -G_r \left(1 + \frac{AP_r N_5}{N_6}\right), & N_8 &= G_r \frac{AP_r N_5}{N_6} - AN_5 L_1, & N_9 &= -G_c \left(1 + \frac{AS_c N_2}{N_3}\right), \\
 N_{10} &= G_c \frac{AS_c N_2}{N_3} - AN_2 L_2, & N_{11} &= AM_6 L_3, \\
 N_{12} &= \frac{N_7}{M_2^2 + M_2 - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}},
 \end{aligned}$$

$$\begin{aligned}
N_{13} &= \frac{N_8}{N_5^2 + N_5 - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}, \\
N_{14} &= \frac{N_9}{M_4^2 + M_4 - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}, \\
N_{15} &= \frac{N_{10}}{N_2^2 + N_2 - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}, \\
N_{16} &= \frac{N_{11}}{M_6^2 + M_6 - \{i(2R_0 + \omega) + M^2 + \frac{1}{K_p}\}}, \\
N_{17} &= -(N_{12} + N_{13} + N_{14} + N_{15} + N_{16}).
\end{aligned}$$

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