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CHEMICAL REACTION EFFECT ON MHD OSCILLATORY FLOW THROUGH A POROUS MEDIUM BOUNDED BY TWO VERTICAL POROUS PLATES WITH HEAT SOURCE AND SORET EFFECT

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Abstract An attempt has been made to study hydromagnetic oscillatory flow of an electrically conducting incompressible viscous fluid through a porous medium bounded by vertical porous plates with heat source and chemical reaction. One plate of the channel is kept stationary and the other is oscillating with uniform velocity. The plates of the channel are subjected to constant injection and suction velocities respectively.The novelty of the present study is to investigate the combine effects of heat source and the Soret effects in the presence of chemical reaction. The closed form solutions of the governing equations are obtained for the velocity, temperature and concentration profiles.Effects of the various parameters on the velocity, temperature, concentration, skin friction, the rate of heat and mass transfer coefficients are nimerically evaluted and discussed with the help of graphs. It is interesting to note that an increase in chemical reaction parameter or Soret number or permeability parameter reduce the fluid velocity and enhance the skin friction coefficients . Further, it is observed that the effect of increasing heat source parameter is to accelerate the temperature and velocity of the flow at all points while decelerate the concentration and skin friction coefficients.

Keywords Hydromagnetic, oscillatory flow, porous medium, chemical reaction, heat source, Soret effect.

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1. Introduction

The influence of magnetic field on viscous incompressible flow of electrically conducting fluid has its importance in many applications such as extrusion of plastics in the manufacture of rayon and nylon, purification of crude oil, pulp, paper industry, textile industry and in different geophysical cases etc. In many process industries, the cooling of threads or sheets of some polymer materials is of importance in the production line. The rate of cooling can be controlled effectively to achieve final

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products of desired characteristics by drawing threads, etc. in the presence of an electrically conducting fluid subject to a magnetic field. MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications are in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants.

Magnetohydrodynamics free convective flows bounded by two vertical plates are studied because of their wide application and hence it has attracted the attention of many research scholars, investigators and scientists. MHD is applied to study the stellar and solar structure, interstellar matter, radio propagation through the ionosphere etc. The ionised gas or plasma can be made to interact with the magnetic field and can frequently alter heat transfer on the bounding surface. Heat transfer by thermal radiation is of great importance when we are concerned with space applications, higher operating temperatures and also power engineering. In processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and the flow in a desert, cooler, heat and mass transfer occurs simultaneously.

To study the underground water resources, seepage of water in river beds, the filtration and water purification processes in chemical engineering, one need the knowledge of the fluid flow through porous medium. The porous medium is in fact a non homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid saturated medium which has dynamical properties equal to those of non homogeneous continuum. Ahmadi and Manvi [1] have derived the general equation of motion and applied the results to some basic flow problems. Ram and Mishra [20] applied these equations to study MHD flow of conducting fluid through porous media. Geindreau and Auriault [8] studied the effect of magnetic field on flow through porous medium. On the other hand, in view of increasing technical applications using magnetohydrodynamics (MHD) effects, it is desirable to extend many of the available hydrodynamic solutions to include the effects of magnetic field for those cases where the viscous fluid is electrically conducting. The various applications of MHD flows in technological fields have been compiled by Moreau [14]. Sharma and Sharma [22] have studied the effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical porous plate bounded by porous medium. Singh and Mathew [23] have studied the injection/suction effect on an oscillatory hydromagnetic flow in a rotating horizontal porous channel. Lighthill [13] initiated an important class of two-dimensional time-dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations of the free stream velocity about a mean value. This was extended by Stuart [24] to the case of oscillatory flow over an infinite plate.

Oscillatory flows are associated with higher rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems such as reciprocating engines, pulse combustors and chemical reactors etc. Cooper et al. [5] have been made a detailed study on fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer. Fusegi [7] has numerically studied the influence of convective heat transfer from periodic open cavities in a channel with oscillatory flow. Muthucumaraswamy [15] has studied the effect of heat and mass transfer on flow past an oscillatory vertical plate with variable temperature.

The Soret effect or thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. The term soret effect most often applies to aerosol mixtures, but may also commonly refer to the phenomenon in all phases of matter. It has been used in commercial precipitators for applications similar to electro static precipitators, manufacturing of optical fibre in vapour deposition processes, facilitating drug discovery by allowing the detection of aptamer binding by comparison of the bound versus unbound motion of the target molecule. It is also used to separate different polymers particles in fluid flow fractionation. Anghel and Takhar [2] studied Dufour effect and Soret effects on free convection boundary layer over a vertical surface embedded in porous medium. Ahmed [3] studied MHD convection with soret and Dufour effect in a three dimensional flow past an infinite vertical porous plate.

Raju et al [21] studied Soret effect due to natural convection between heated inclined plates. Singh and Garg [25] have studied the radiative heat transfer in MHD oscillatory flow through porous medium bounded by two vertical porous plates. Recently, Chand et al. [6] have studied hydro magnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and Soret effect.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have therefore received a considerable amount of attention in recent years. Possible applications of this type of flow can be found in many industries. The rate of reaction depends on the concentration of species itself. Muthucumaraswamy and Janakiraman [16] have studied the mass transfer effect on isothermal vertical oscillating plate in presence of chemical reaction. Senapati and Dhal [26] have studied magnetic effect on mass and heat transfer of hydrodynamic flow past a vertical oscillating plate in presence of chemical reaction. The effects of chemical reaction on MHD free convective oscillatory flow past a porous plate with viscous dissipation and heat sink has been studied by Sekhar and Reddy [27]. The heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat source has been studied by Barik et al. [4].

The objective of this paper is to analyze hydromagnetic oscillatory flow through a porous medium bounded by vertical porous plates with heat source and Soret effect in presence of chemical reaction. Further, the mass transfer phenomena considered in this problem is associated with chemically reacting species. The aim of the present study is to extend the work of Chand et al. [6] to chemical reacting species with mass transfer.

2. Formulation of problem

Consider the flow of an electrically conducting viscous incompressible fluid through saturated porous medium bounded by two insulated vertical porous plates distance d apart in the presence of the heat source. A co-ordinate system is chosen with origin at the stationary plate which is subjected to a constant injection velocity V_0. The other plate is oscillating in its own plane with a velocity $U'(t')$ about a nonzero

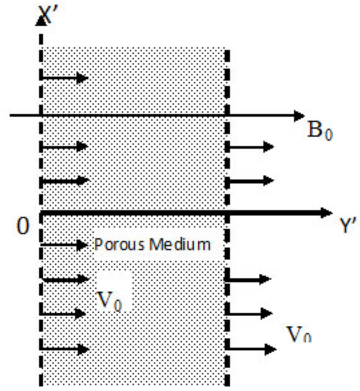

Figure 1. The Physical configuration of the problem.

constant mean velocity U_0 and is subjected to same constant suction velocity V_0. A homogeneous magnetic field of strength B_0 is applied normal to the plane of the plates. The plates of the channel are assumed infinite in extent. Hence all the physical properties of the fluid are functions of y' and t' except the pressure. Under the usual Boussinesq approximations the flow is governed by the following equations

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = V_0, \tag{2.1}$$

$$\begin{aligned} \frac{\partial u'}{\partial t'} + V_0 \frac{\partial u'}{\partial y'} = \quad & -\frac{1}{p}\frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'} \\ & + g\beta(T' - T_d) + g\beta_c(C' - C_d), \end{aligned} \tag{2.2}$$

$$\frac{\partial T'}{\partial t'} + V_0 \frac{\partial T'}{\partial y'} = \frac{K}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{Q}{\rho c_p}(T' - T_d), \tag{2.3}$$

$$\frac{\partial C'}{\partial t'} + V_0 \frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + D_1 \frac{\partial^2 T'}{\partial y'^2} - R'(C' - C_d), \tag{2.4}$$

where (u, v) are velocity components in x and y directions, g is acceleration due to gravity, T' is the temperature of the fluid, C' is the species concentration, β is the coefficient of thermal expansion, β_c is the volumetric expansion coefficient, ν is the kinematic viscosity of the fluid, k is effective thermal conductivity, ρ is the density of the fluid, K' is the permeability, C_p is the specific heat at constant pressure, D is the diffusion coefficient, D' is the thermal diffusivity, B_0 is the electromagnetic induction, σ is the conductivity of the fluid, V_0 constant suction/ injection, d is the distance between two plates and p is non-dimensional pressure.

With the following boundary conditions

$$\begin{cases} u' = 0, T' = T_0 + \varepsilon(T_0 - T_d)cos\omega' t', \ C' = C_0 + \varepsilon(C_0 - C_d)cos\omega' t' \text{ at } y = 0, \\ u' = U'(t') = U_0(1 + cos\omega' t'), \ T' = T_d, \ C' = C_d \text{ at } y = d. \end{cases} \tag{2.5}$$

Eliminating the modified pressure gradient under the usual boundary layer approximation equation (2.2) reduces to

$$\begin{aligned}\frac{\partial u'}{\partial t'}+V_0\frac{\partial u'}{\partial y'} = & \frac{\partial U'}{\partial t'}+\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2(u'-U')}{\rho}-\frac{\nu(u'-U')}{K'} \\ & +g\beta(T'-T_d)+g\beta_c(C'-C_d).\end{aligned} \tag{2.6}$$

Let us introduce the following non dimensional quantities

$$\begin{aligned}& y=\frac{y'}{d}, t=\frac{t'V_0}{d}, u=\frac{u'}{U_0}, \theta=\frac{T'-T_d}{T_0-T_d}, C=\frac{C'-C_d}{C_0-C_d}, Pe=\frac{\rho C_p V_0 d}{k}, \\ & Sc=\frac{\nu}{D}, R=\frac{R'd}{V_0}, K=\frac{K'V_0}{\nu d}, M=B_0 d\sqrt{\frac{\sigma}{\mu}}, Re=\frac{V_0 d}{\nu}, S=\frac{Q'd}{\rho C_p V_0}, U=\frac{U'}{U_0}, \\ & \omega=\frac{\omega' d}{V_0}, Gr=\frac{\nu g\beta(T_0-T_d)}{U_0V_0^2}, Gm=\frac{\nu g\beta_c(C_0-C_d)}{U_0V_0^2}, S_0=\frac{D_1(T_0-T_d)}{dV_0(C_0-C_d)},\end{aligned} \tag{2.7}$$

where G_r is Grashof number, G_m modified Grashof number, M is magnetic number, R_e is the Reynolds number, S_c is the Schmidt number, K permeability parameter porous medium, P_e is the Peclet number, R is the chemical reaction parameter, S is the heat source parameter, S_0 is the Soret number, ω is the frequence of oscillation, t is the non-dimensional time coordinate, u is the non-dimensional velocity, U_0 is the uniform velocity of the plate.

Using equations (2.1), (2.3), (2.4) and (2.6), we get the following non dimensional governing equations:

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+\frac{1}{Re}\frac{\partial^2 u}{\partial y^2}-(\frac{M^2}{Re}+\frac{1}{K})(u-U)+GrRe\theta+GmReC, \tag{2.8}$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial \theta}{\partial y}=\frac{1}{Pe}\frac{\partial^2 \theta}{\partial y^2}+S\theta, \tag{2.9}$$

$$\frac{\partial C}{\partial t}+\frac{\partial C}{\partial y}=\frac{1}{ScRe}\frac{\partial^2 C}{\partial y^2}+S_0\frac{\partial^2 \theta}{\partial y^2}-RC. \tag{2.10}$$

With the following boundary conditions

$$\begin{cases} u=0,\ \theta=1+\frac{\epsilon}{2}(e^{i\omega t}+e^{-i\omega t}),\ C=1+\frac{\epsilon}{2}(e^{i\omega t}+e^{-i\omega t}) \text{ at } y=0, \\ u=1+\frac{\epsilon}{2}(e^{i\omega t}+e^{-i\omega t}),\ \theta=0,\ C=0 \text{ at } y=1. \end{cases} \tag{2.11}$$

3. Method of solution

In order to solve equations (2.8), (2.9) and (2.10) for purely oscillatory flow, we assume the solution following Hamza et al. [10], Seth et al. [28], Kumar and Singh [11] and Singh and Mathew [29]

$$\begin{aligned}\theta &= \theta_0(y)+\frac{\epsilon}{2}\theta_1(y)e^{i\omega t}+\frac{\epsilon}{2}\theta_2(y)e^{-i\omega t}, \\ C &= C_0(y)+\frac{\epsilon}{2}C_1(y)e^{i\omega t}+\frac{\epsilon}{2}C_2(y)e^{-i\omega t}, \\ u &= u_0(y)+\frac{\epsilon}{2}u_1(y)e^{i\omega t}+\frac{\epsilon}{2}u_2(y)e^{-i\omega t}.\end{aligned} \tag{3.1}$$

Moreover, the present form of solution is a general one i.e

(i) $\theta_1(y)$ and $\theta_2(y)$ both are zero, then this corresponds to steady solution.

(ii) $\theta_2(y) = 0$, then this corresponds to the work of Mustafa et al. [17], Gbadeyan et al. [9] and Girish Kumar and Satyanarayana [12].

(iii) $\theta_0(y)$ and $\theta_2(y)$ both are zero, then this corresponds to the work of Mehmood and Ali [18], Makinde and Mhone [19] and Senapati and Dhal [30].

(iv) $\theta_1(y) \neq 0$ and $\theta_2(y) \neq 0$, then this corresponds the present form of the solution Hamza et al. [10], Seth et al. [28], Kumar and Singh [11] and Singh and Mathew [29].

Using equations (3.1) in (2.8) to (2.10) we get the following set of equations

$$\frac{d^2\theta_0}{dy^2} - Pe\frac{d\theta_0}{dy} + SPe\theta_0 = 0, \tag{3.2}$$

$$\frac{d^2\theta_1}{dy^2} - Pe\frac{d\theta_1}{dy} + (S - i\omega)Pe\theta_1 = 0, \tag{3.3}$$

$$\frac{d^2\theta_2}{dy^2} - Pe\frac{d\theta_2}{dy} + (S + i\omega)Pe\theta_2 = 0, \tag{3.4}$$

$$\frac{d^2C_0}{dy^2} - ScPe\frac{dC_0}{dy} - ScReRC_0 = -S_0ScRe\frac{d^2\theta_0}{dy^2}, \tag{3.5}$$

$$\frac{d^2C_1}{dy^2} - ScPe\frac{dC_1}{dy} - ScRe(R + i\omega)C_1 = -S_0ScRe\frac{d^2\theta_1}{dy^2}, \tag{3.6}$$

$$\frac{d^2C_2}{dy^2} - ScPe\frac{dC_2}{dy} - ScRe(R - i\omega)C_2 = -S_0ScRe\frac{d^2\theta_2}{dy^2}, \tag{3.7}$$

$$\begin{aligned} \frac{d^2u_0}{dy^2} - Re\frac{du_0}{dy} - (M^2 + \frac{Re}{K})u_0 = \quad & -GrRe^2\theta_0 - GmRe^2C_0 \\ & -(M^2 + \frac{Re}{K})U_0, \end{aligned} \tag{3.8}$$

$$\begin{aligned} \frac{d^2u_1}{dy^2} - Re\frac{du_1}{dy} - (M^2 + \frac{Re}{K} + i\omega Re)u_1 = \quad & -GrRe^2\theta_1 - GmRe^2C_1 \\ & -(M^2 + \frac{Re}{K} + i\omega Re)U_0, \end{aligned} \tag{3.9}$$

$$\begin{aligned} \frac{d^2u_2}{dy^2} - Re\frac{du_1}{dy} - (M^2 + \frac{Re}{K} + i\omega Re)u_2 = \quad & -GrRe^2\theta_2 - GmRe^2C_2 \\ & -(M^2 + \frac{Re}{K} + i\omega Re)U_0. \end{aligned} \tag{3.10}$$

Solving the equations (3.2) to (3.10), we get

$$\begin{aligned} \theta = \quad & A_1e^{m_1y} + A_2e^{m_2y} + \frac{\epsilon}{2}(A_3e^{m_3y} + A_4e^{m_4y})e^{i\omega t} \\ & + \frac{\epsilon}{2}(A_5e^{m_5y} + A_6e^{m_6y})e^{-i\omega t} \end{aligned} \tag{3.11}$$

$$\begin{aligned} C = \quad & A_9e^{m_7y} + A_{10}e^{m_8y} + A_7e^{m_1y} + A_8e^{m_2y} \\ & + \frac{\epsilon}{2}(A_{13}e^{m_9y} + A_{14}e^{m_{10}y} + A_{11}e^{m_3y} + A_{12}e^{m_4y})e^{i\omega t} \\ & + \frac{\epsilon}{2}(A_{17}e^{m_{11}y} + A_{18}e^{m_{12}y} + A_{15}e^{m_5y} + A_{16}e^{m_6y})e^{-i\omega t} \end{aligned} \tag{3.12}$$

$$\begin{aligned} u = \quad & A_{27}e^{m_{13}y} + A_{28}e^{m_{14}y} + A_{19}e^{m_1y} + A_{20}e^{m_2y} \\ & + A_{21}e^{m_7y} + A_{22}e^{m_8y} + A_{23}e^{m_1y} + A_{24}e^{m_2y} + U_0 \end{aligned} \tag{3.13}$$

$$
\begin{aligned}
&+\frac{\epsilon}{2}(A_{37}e^{m_{15}y} + A_{38}e^{m_{16}y} + A_{29}e^{m_3y} + A_{30}e^{m_4y} + A_{31}e^{m_9y}\\
&+A_{32}e^{m_{10}y} + A_{33}e^{m_3y} + A_{34}e^{m_4y} + U_0)e^{i\omega t}\\
&+\frac{\epsilon}{2}(A_{47}e^{m_{17}y} + A_{48}e^{m_{18}y} + A_{39}e^{m_5y} + A_{40}e^{m_6y} + A_{41}e^{m_{11}y}\\
&+A_{42}e^{m_{12}y} + A_{43}e^{m_5y} + A_{44}e^{m_6y} + U_0)e^{-i\omega t}.
\end{aligned}
$$

The non dimensional skin friction at the moving plate of the channel is given by

$$
\begin{aligned}
\tau &= -\mu(\frac{\partial u}{\partial y})_{y=1}\\
&= A_{27}m_{13}e^{m_{13}} + A_{28}m_{14}e^{m_{14}} + A_{19}m_1e^{m_1} + A_{20}m_2e^{m_2} + A_{21}\\
&\quad m_7e^{m_7} + A_{22}m_8e^{m_8} + A_{23}m_1e^{m_1} + A_{24}m_2e^{m_2}\\
&\quad +\frac{\epsilon}{2}(A_{37}m_{15}e^{m_{15}} + A_{38}m_{16}e^{m_{16}} + A_{29}m_3e^{m_3} + A_{30}m_4e^{m_4} + A_{31}m_9e^{m_9}\\
&\quad +A_{32}m_{10}e^{m_{10}} + A_{33}m_3e^{m_3} + A_{34}m_4e^{m_4})e^{i\omega t}\\
&\quad +\frac{\epsilon}{2}(A_{47}m_{17}e^{m_{17}} + A_{48}m_{18}e^{m_{18}} + A_{39}m_5e^{m_5} + A_{40}m_6e^{m_6} + A_{41}m_{11}e^{m_{11}}\\
&\quad +A_{42}m_{12}e^{m_{12}} + A_{43}m_5e^{m_5} + A_{44}m_6e^{m_6})e^{-i\omega t}.
\end{aligned} \tag{3.14}
$$

The rate of heat transfer at the moving plate of the channel in terms of non dimensional nusselt number is given by

$$
\begin{aligned}
Nu &= -(\frac{\partial \theta}{\partial y})_{y=1}\\
&= A_1m_1e^{m_1} + A_2m_2e^{m_2} + \frac{\epsilon}{2}(A_3m_3e^{m_3} + A_4m_4e^{m_4})e^{i\omega t}\\
&\quad +\frac{\epsilon}{2}(A_5m_5e^{m_5} + A_6m_6e^{m_6})e^{-i\omega t}.
\end{aligned} \tag{3.15}
$$

The rate of mass transfer coefficient at the moving plate of the channel in terms of non dimensional Sherwood number is given by

$$
\begin{aligned}
Sh &= -(\frac{\partial C}{\partial y})_{y=1} = A_9m_7e^{m_7} + A_{10}m_8e^{m_8} + A_7m_1e^{m_1} + A_8m_2e^{m_2}\\
&\quad +\frac{\epsilon}{2}(A_{13}m_9e^{m_9} + A_{14}m_{10}e^{10} + A_{11}m_3e^{m_3} + A_{12}m_4e^{m_4})e^{i\omega t}\\
&\quad +\frac{\epsilon}{2}(A_{17}m_{11}e^{m_{11}} + A_{18}m_{12}e^{m_{12}} + A_{15}m_5e^{m_5} + A_{16}m_6e^{m_6})e^{-i\omega t}.
\end{aligned} \tag{3.16}
$$

Here

$$m_1 = \frac{Pe + \sqrt{Pe^2 - 4SPe}}{2}, \qquad m_2 = \frac{Pe - \sqrt{Pe^2 - 4SPe}}{2},$$

$$m_3 = \frac{Pe + \sqrt{Pe^2 - 4(S - i\omega)Pe}}{2}, \qquad m_4 = \frac{Pe - \sqrt{Pe^2 - 4(S - i\omega)Pe}}{2},$$

$$m_5 = \frac{Pe + \sqrt{Pe^2 - 4(S + i\omega)Pe}}{2}, \qquad m_6 = \frac{Pe - \sqrt{Pe^2 - 4(S + i\omega)Pe}}{2},$$

$$m_7 = \frac{ScRe + \sqrt{Sc^2Re^2 + 4ScReR}}{2}, \qquad m_8 = \frac{ScRe - \sqrt{Sc^2Re^2 + 4ScReR}}{2},$$

$$m_9 = \frac{ScRe + \sqrt{Sc^2Re^2 + 4ScRe(R + i\omega)}}{2},$$

$$m_{10}=\frac{ScRe-\sqrt{Sc^2Re^2+4ScRe(R+i\omega)}}{2},$$
$$m_{11}=\frac{ScRe+\sqrt{Sc^2Re^2+4ScRe(R-i\omega)}}{2},$$
$$m_{12}=\frac{ScRe-\sqrt{Sc^2Re^2+4ScRe(R-i\omega)}}{2},$$
$$m_{13}=\frac{Re+\sqrt{Re^2+4(M^2+\frac{Re}{K})}}{2},\qquad m_{14}=\frac{Re-\sqrt{Re^2+4(M^2+\frac{Re}{K})}}{2},$$
$$m_{15}=\frac{Re+\sqrt{Re^2+4(M^2+\frac{Re}{K}+i\omega Re)}}{2},$$
$$m_{16}=\frac{Re-\sqrt{Re^2+4(M^2+\frac{Re}{K}+i\omega Re)}}{2},$$
$$m_{17}=\frac{Re+\sqrt{Re^2+4(M^2+\frac{Re}{K}-i\omega Re)}}{2},$$
$$m_{18}=\frac{Re-\sqrt{Re^2+4(M^2+\frac{Re}{K}-i\omega Re)}}{2},$$
$$A_1=\frac{-e^{m_2}}{e^{m_1}-e^{m_2}},\qquad A_2=\frac{e^{m_1}}{e^{m_1}-e^{m_2}},\qquad A_3=\frac{-e^{m_4}}{e^{m_3}-e^{m_4}},$$
$$A_4=\frac{e^{m_3}}{e^{m_3}-e^{m_4}},\qquad A_5=\frac{-e^{m_6}}{e^{m_5}-e^{m_6}},\qquad A_6=\frac{e^{m_5}}{e^{m_3}-e^{m_4}},$$
$$A_7=\frac{-S_0ScReA_1m_1^2}{m_1^2-ScRem_1-ScReR},\qquad A_8=\frac{-S_0ScReA_2m_2^2}{m_2^2-ScRem_2-ScReR},$$
$$A_9=\frac{-e^{m_8}+A_7(e^{m_8}-e^{m_1})+A_8(e^{m_8}-e^{m_2})}{e^{m_7}-e^{m_8}},$$
$$A_{10}=\frac{e^{m_7}-A_7(e^{m_7}-e^{m_1})+A_8(e^{m_7}-e^{m_2})}{e^{m_7}-e^{m_8}},$$
$$A_{11}=\frac{-S_0ScReA_3m_3^2}{m_3^2-ScRem_3-ScRe(R+i\omega)},\quad A_{12}=\frac{-S_0ScReA_4m_4^2}{m_4^2-ScRem_4-ScRe(R+i\omega)},$$
$$A_{13}=\frac{-e^{m_{10}}+A_{11}(e^{m_{10}}-e^{m_3})+A_{12}(e^{m_{10}}-e^{m_4})}{e^{m_9}-e^{m_{10}}},$$
$$A_{14}=\frac{e^{m_{10}}-A_{11}(e^{m_9}-e^{m_3})-A_{12}(e^{m_9}-e^{m_4})}{e^{m_9}-e^{m_{10}}},$$
$$A_{15}=\frac{-S_0ScReA_5m_5^2}{m_5^2-ScRem_5-ScRe(R-i\omega)},\quad A_{16}=\frac{-S_0ScReA_6m_6^2}{m_6^2-ScRem_6-ScRe(R-i\omega)},$$
$$A_{17}=\frac{-e^{m_{12}}+A_{14}(e^{m_{12}}-e^{m_5})+A_{15}(e^{m_{12}}-e^{m_6})}{e^{m_{11}}-e^{m_{12}}},$$
$$A_{18}=\frac{e^{m_{11}}-A_{14}(e^{m_{11}}-e^{m_5})-A_{15}(e^{m_{11}}-e^{m_6})}{e^{m_{11}}-e^{m_{12}}},$$
$$A_{19}=\frac{-GrRe^2A_1}{m_1^2-Rem_1-(M^2+\frac{Re}{K})},\qquad A_{20}=\frac{-GrRe^2A_2}{m_2^2-Rem_2-(M^2+\frac{Re}{K})},$$
$$A_{21}=\frac{-GmRe^2A_9}{m_7^2-Rem_7-(M^2+\frac{Re}{K})},\qquad A_{22}=\frac{-GmRe^2A_{10}}{m_8^2-Rem_8-(M^2+\frac{Re}{K})},$$

$$A_{23} = \frac{-GmRe^2A_7}{m_1^2 - Rem_1 - (M^2 + \frac{Re}{K})}, \qquad A_{24} = \frac{-GmRe^2A_8}{m_2^2 - Rem_2 - (M^2 + \frac{Re}{K})},$$

$$A_{25} = A_{19} + A_{20} + A_{21} + A_{22} + A_{23} + A_{24} + U_0,$$

$$A_{26} = A_{19}e^{m_1} + A_{20}e^{m_2} + A_{21}e^{m_7} + A_{22}e^{m_8} + A_{23}e^{m_1} + A_{24}e^{m_2} + U_0,$$

$$A_{27} = \frac{-1 + A_{26} - A_{25}e^{m_{14}}}{e^{m_{14} - e^{m_{13}}}}, \qquad A_{28} = \frac{1 - A_{26} + A_{25}e^{m_{13}}}{e^{m_{14} - e^{m_{13}}}},$$

$$A_{29} = \frac{-GrRe^2A_3}{m_3^2 - Rem_3 - (M^2 + \frac{Re}{K} + i\omega Re)}, \; A_{30} = \frac{-GrRe^2A_4}{m_4^2 - Rem_4 - (M^2 + \frac{Re}{K} + i\omega Re)},$$

$$A_{31} = \frac{-GmRe^2A_{13}}{m_9^2 - Rem_9 - (M^2 + \frac{Re}{K} + i\omega Re)},$$

$$A_{32} = \frac{-GmRe^2A_{14}}{m_{10}^2 - Rem_{10} - (M^2 + \frac{Re}{K} + i\omega Re)},$$

$$A_{33} = \frac{-GmRe^2A_{11}}{m_3^2 - Rem_3 - (M^2 + \frac{Re}{K} + i\omega Re)}, \; A_{34} = \frac{-GmRe^2A_{12}}{m_4^2 - Rem_4 - (M^2 + \frac{Re}{K} + i\omega Re)},$$

$$A_{35} = A_{29} + A_{30} + A_{31} + A_{32} + A_{33} + A_{34} + U_0,$$

$$A_{36} = A_{29}e^{m_3} + A_{30}e^{m_4} + A_{31}e^{m_9} + A_{32}e^{m_{10}} + A_{33}e^{m_3} + A_{34}e^{m_4} + U_0,$$

$$A_{37} = \frac{-1 + A_{36} - A_{35}e^{m_{16}}}{e^{m_{16} - e^{m_{15}}}}, \qquad A_{38} = \frac{1 - A_{36} + A_{35}e^{m_{15}}}{e^{m_{16} - e^{m_{15}}}},$$

$$A_{39} = \frac{-GrRe^2A_5}{m_5^2 - Rem_5 - (M^2 + \frac{Re}{K} - i\omega Re)}, \; A_{40} = \frac{-GrRe^2A_6}{m_6^2 - Rem_6 - (M^2 + \frac{Re}{K} - i\omega Re)},$$

$$A_{41} = \frac{-GmRe^2A_{17}}{m_{11}^2 - Rem_{11} - (M^2 + \frac{Re}{K} - i\omega Re)},$$

$$A_{42} = \frac{-GmRe^2A_{18}}{m_{12}^2 - Rem_{12} - (M^2 + \frac{Re}{K} - i\omega Re)},$$

$$A_{43} = \frac{-GmRe^2A_{15}}{m_5^2 - Rem_5 - (M^2 + \frac{Re}{K} - i\omega Re)}, A_{44} = \frac{-GmRe^2A_{16}}{m_6^2 - Rem_6 - (M^2 + \frac{Re}{K} - i\omega Re)},$$

$$A_{45} = A_{39} + A_{40} + A_{41} + A_{42} + A_{43} + A_{44} + U_0,$$

$$A_{46} = A_{39}e^{m_5} + A_{40}e^{m_6} + A_{41}e^{m_{11}} + A_{42}e^{m_{12}} + A_{43}e^{m_5} + A_{44}e^{m_6} + U_0,$$

$$A_{47} = \frac{-1 + A_{46} - A_{45}e^{m_{18}}}{e^{m_{18} - e^{m_{17}}}}, \qquad A_{48} = \frac{1 - A_{46} + A_{45}e^{m_{17}}}{e^{m_{18} - e^{m_{17}}}}.$$

4. Results and discussion

To discuss the physical significance of various parameters, the numerical calculations have been carried out and only the real parts of the solutions are considered. Our results are found to be in good agreement with the results of Singh and Garg [25] in the absence of heat source, the mass transfer and chemical reaction. Further, the result is also in good agreement with the result of Chand et al. [6] in the absence of chemical reaction. The general nature of the velocity, temperature and concentration are parabolic.

Fig.2 exhibits the velocity profile relating to diffusing species, Hydrogen ($S_c = 0.22$) and Oxygen ($S_c = 0.66$) in the air with. An increase in Reynolds number (R_e), magnetic parameter (M), heat source parameter (S), Grashoff number (G_r) and modified Grashoff number (G_m) leads to increase the velocity throughout the flow field. This implies that the present study is an buoyancy assisting flow with thermal buoyancy (G_r) and mass buoyancy (G_m). It is interesting to note that an increase in suction Reynolds number increases the velocity distribution significantly (curves I and II). This indicates that an increasing inertia force with constant viscosity enhances the velocity of the fluid whereas increasing Schmidt number $S_c = 0.66$ (Oxygen) i.e in case of heavier species leads to decrease the velocity.

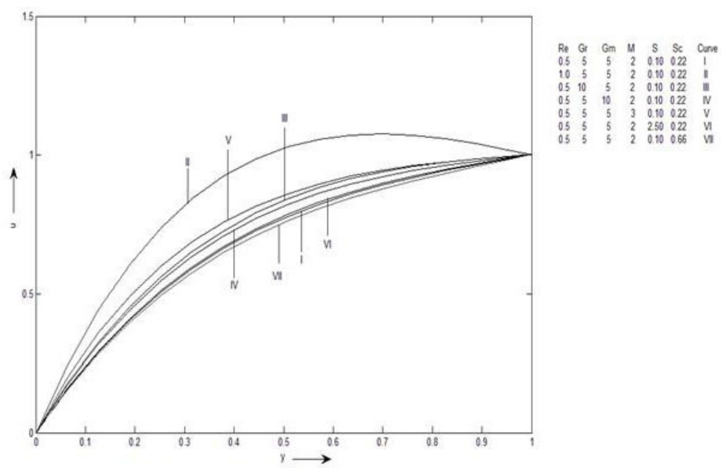

Figure 2. Velocity profile for $S_0 = 6.89,\ K = 0.5,\ P_e = 1,\ w = 5,\ U_0 = 1,\ R = 5.$

Further, it is revealed that the effect of magnetic field is to increase the velocity at all points of the flow domain. Usually application of magnetic field gives rise to a resistive force which opposes the primary flow but in the present case, the magnetic parameter enhances the velocity. This indicates that the flow is overpowered by the buoyancy effects resulting an increase in velocity even in the presence of magnetic field.

The role of Peclet number (P_e) which is the product of two important parameters controlling the velocity boundary layer as well as thermal boundary layer i.e product of Prandtl number and Reynolds number is shown in fig.3 and fig.4. It is remarked that an increase in Peclet number (P_e) increases both velocity and temperature distribution.

Prandtl number is the ratio of kinematic viscosity and thermal diffusivity. If other things remain same, an increase in kinematic viscosity vis-a-vis momentum diffusivity leads to enhance the velocity boundary layer thickness. Similar discussion may be carried out with thermal diffusivity due to heat conduction contributing to growth of thermal boundary layer.

From fig.3, it is further observed that an increase in permeability parameter (K), Soret number (S_0) and Chemical reaction parameter (R) leads to decrease the velocity whereas frequency of oscillation (ω) and parameter U_0 enhance it. Then it is concluded that presence of porous matrix causes a slow flow whereas presence of heat source and suction at the plate enhance it. Further, it is observed in fig.3 and fig.4 an increase in heat source parameter (S) increases both velocity and temperature distribution.

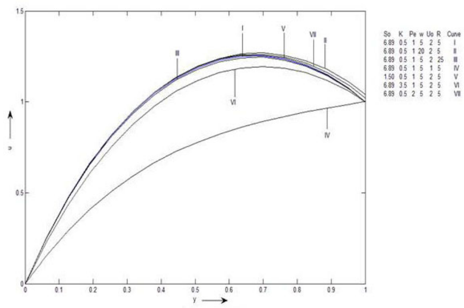

Figure 3. Velocity profile for $G_r = 5,\ G_m = 5,\ M = 2,\ S = 0.10,\ S_c = 0.22,\ R_e = 0.5,\ t = 0$.

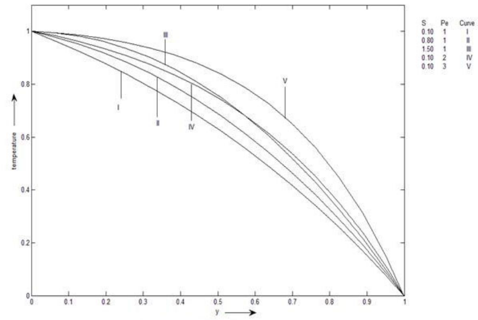

Figure 4. The temperature profiles.

From fig.5 it is evident that an increase in $R_e,\ P_e,\ S,\ S_c,\ R$ and S_0 is found to be counterproductive in obtaining higher concentration except the frequency of oscillation parameter (ω) which has a positive effect on concentration distribution.

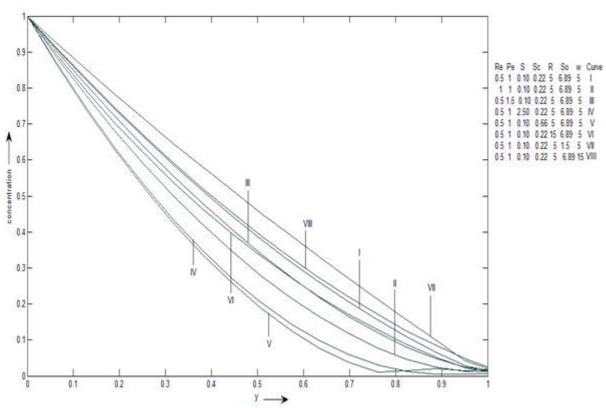

Figure 5. The concentration profiles.

Fig.6 and fig.7 depict the periodic variation in skin friction profiles with time. The variation of skin friction is an oscillatory type with the lapse of time. Moreover, the buoyancy parameters, magnetic parameter and heat source is favorable for reducing the skin friction, which is desirable, whereas presence of Schmidt number (S_c), Reynolds number (R_e), suction parameter (U_0), chemical reaction parameter (R), Soret number (S_0) and porosity parameter (K) leads to experience greater skin friction.

Fig.8 displays the variation of Sherwood number (S_h). It is seen that the rate of mass transfer is favoured by chemical reaction parameter (R) and Soret number (S_0) whereas reverse effect is observed in case of $R_e,\ P_e,\ S_c$ and S.

Variations of Nusselt number (N_u) are illustrated in fig.9. The rate of heat transfer is affected by peclet number (P_e), heat source parameter (S) and frequency

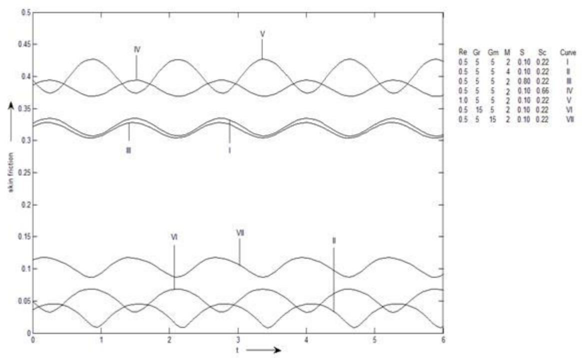

Figure 6. Variation of skin friction.

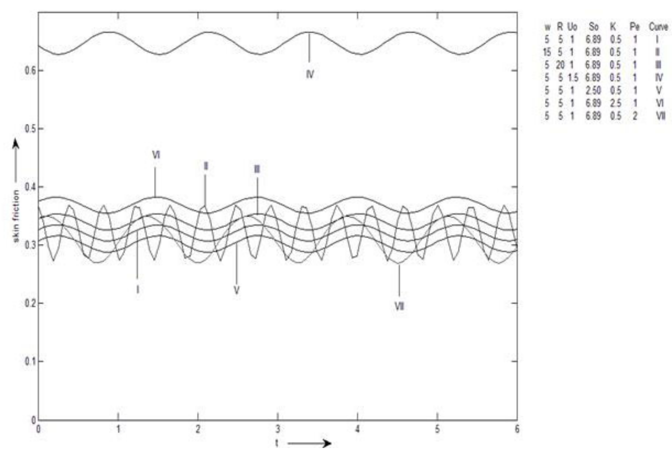

Figure 7. Variation of skin friction.

of oscillation (ω). Those parameters have a positive contribution in enhancing the higher rate of heat transfer in the flow domain.

5. Conclusion

(i) Buoyancy forces as well as suction play a significant role in enhancing the boundary layer thickness.

(ii) Presence of chemical reaction and coupling of mass transfer with thermal energy (Soret effect) has a thinning effect on boundary layer.

(iii) Skin friction and rate of mass transfer at the plate exhibit fluctuative character whereas rate of heat transfer presents a uniformity of variation with time.

(iv) It is noticed that an increase in heat source parameter (S) and Peclet number (P_e) increases both velocity and temperature distribution in the flow domain.

(v) The effect of the permeability parameter is just opposite to that of Lorentz

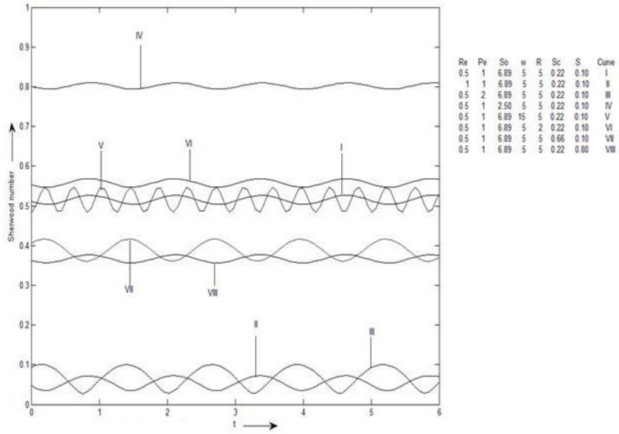

Figure 8. Variation of Sherwood number.

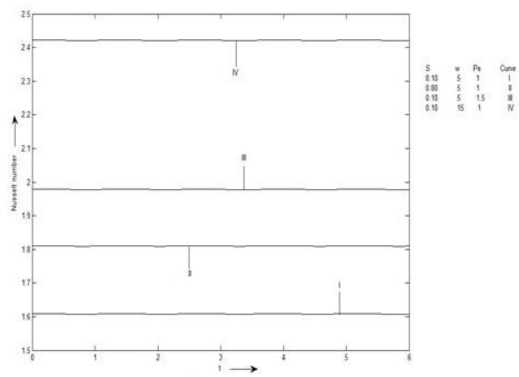

Figure 9. Variation of Nusselt number.

force parameter.

(vi) The effect of increasing Soret number is to accelerate the skin friction and decelerates the velocity.

(vii) The heat transfer is reduced with heat generation parameter.

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