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THE EXTENDED RICCATI EQUATION METHOD FOR TRAVELLING WAVE SOLUTIONS OF ZK EQUATION*

Xiaofeng Li [1,†], Jiahua Han[1] and Fei Wang[1]

Abstract In this article, the extended Riccati equation method is applied to seeking more general exact travelling wave solutions of the ZK equation. The traveling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and the rational functions. When the parameters are taken as special values, the solitary wave solutions are obtained from the hyperbolic function solutions. Similarly, the periodic wave solutions are also obtained from the trigonometric function solutions. The approach developed in this paper is effective and it may also be used for solving many other nonlinear evolution equations in mathematical physics.

Keywords The extended Riccati equation method, ZK equation, hyperbolic function solutions, trigonometric function solutions, rational solutions.

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1. Introduction

The nonlinear complex physical phenomena arise in many fields of physics, mechanics, biology, chemistry and engineering, and nonlinear equations are related to it. It plays an important role in seeking the exact travelling wave solutions of nonlinear equation in the study of the nonlinear equations. Many powerful methods have been developed, such as the hyperbolic tangent method [2, 3], Backlund transformation method [11–13], Darboux transformation method [4], homogeneous balance method [10, 14], the Jacobi elliptic function expansion method [5, 9, 17], Adomian decomposition method [6–8], the (G'/G)-expansion method [1, 18], and so on. A search of directly seeking for exact solutions of nonlinear has been more interesting in recent years because of the availability of symbolic computation Mathematica or Maple. These computation systems are adequately utilized to perform some complicated and tediously algebraic and differential calculations on a computer. By using these methods and tools, a large number of nonlinear differential equations have been solved and their abundant exact solutions have been obtained.

Zakharov-Kuznetsov(ZK) equation is used to describe the wave form of exercise (2+1)-dimensional space, which is the promotion of famous KdV equation model and the application of progressive multi-scale found in a magnetic field of a magnetic plasma wave. Unlike the Kadomtsev-Petviashvili(KP) equation, the ZK

[†]the corresponding author. Email address:szxylxf@163.com(X. Li)
[1]College of Physics and Material Science, Anhui University, 230039, Hefei, Anhui, China
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equation is not integrable by the inverse scattering transform method. It was found that the solitary-wave solutions of the ZK equation are inelastic. The Zakharov-Kuznetsov(ZK) equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field [15]. In [15], the ZK equation is solved by the sine-cosine and the tanh-function methods. In [16], the numbers of solitary waves and periodic waves of the modified Zakharov-Kuznetsov equation are obtained.

The (G'/G)-expansion method was proposed originally by Wang et al., which is one of the most effective direct methods to obtain travelling wave solutions of a large number of nonlinear evolution equations. This useful method is widely employed by many authors [1, 18]. The key ideas of the G'/G-expansion method are that the travelling wave solutions of nonlinear evolution equations can be expressed by polynomials in G'/G, where G satisfies a second order linear differential equation, the degree of the polynomials can be determined by considering the homogeneous balance between the highest order partial derivatives and nonlinear terms appearing in nonlinear evolution equations considered, the coefficients of the polynomials can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the proposed method.

In this paper, a new method named the extended Riccati equation method was proposed to find the exact travelling wave solutions to ZK equations. We used the extended Riccati equation method that we combined Riccati equation with (G'/G)-expansion method to obtain exact solutions to nonlinear evolution. We will get two group values of coefficients regarding Riccati equation and nonlinear evolution. By contrast to both Riccati equation method and (G'/G)-expansion method, at this point, it is surely a meaningful improvement and innovation we have made to obtain much more abundant solutions. Following the description of the extended Riccati equation method, one can have access to exact solutions to nonlinear evolution smoothly.

2. Description of the extended Riccati equation method

Step 1: We consider the nonlinear evolution equations, in three independent variables x, y, t and dependent variable u:

$$N(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, u_y, u_{xy} \cdots) = 0, \tag{2.1}$$

seeking their travelling wave solutions of the following form:

$$u(x, y, t) = u(\xi), \quad \xi = kx + cy + dt, \tag{2.2}$$

where k, c and d are arbitrary constants. Equation (2.1) can be converted to an ordinary differential equation:

$$N_1(u, u', u'', \cdots) = 0. \tag{2.3}$$

Step 2: In order to construct travelling wave solutions of nonlinear equations, it is reasonable to introduce the following ansatz:

$$u(x, y, t) = u(\xi) = \sum_{i=-n}^{n} a_i f^i(\xi), \tag{2.4}$$

where a_i are constants to be determined later, the balancing number n is a positive integer which can be determined by balancing the highest order derivative terms with the highest power nonlinear terms in (2.3) and $f(\xi)$ satisfies the following elliptic equations:

$$f(\xi)' = p + qf^2(\xi), \tag{2.5}$$

where p, q are real parameters. And $f(\xi)$ can also be expanded to the following ansatz:

$$f(\xi) = \sum_{i=-m}^{m} b_i \left(\frac{G'}{G}\right)^m, \quad b_m \neq 0 \tag{2.6}$$

and $G(\xi)$ satisfies the following elliptic equations:

$$(G(\xi))'' + \lambda(G(\xi))' + \mu G(\xi) = 0, \tag{2.7}$$

where b_i are constants to be determined later, λ, μ are real parameters. m is a positive integer which can be determined by balancing the highest order derivative terms with the highest power nonlinear terms in (2.5), and so we can get $m = 1$.

Step 3: We substitute (2.6) and (2.7) into (2.5), equate the coefficients of all powers of (G'/G) to zero, and we can get solutions of $f(\xi)$ with computerized symbolic computation.

Step 4: Then we substitute (2.4) and (2.5) into (2.3), equating the coefficients of all powers of $f(\xi)$ to zero, solve this set of algebraic equations with computerized symbolic computation, insert these results and solutions of $f(\xi)$ into (2.4). Finally, set $\xi = kx + cy + dt$,we obtain the exact travelling wave solutions of (2.1).

3. Applications

We consider the Zakharov-Kuznetsov (ZK) equations in the following form:

$$u_t + uu_x + u_{xxx} + u_{xyy} = 0. \tag{3.1}$$

To seek travelling wave solutions of(3.1), we make the transformation $\xi = kx + sy - \omega t$, where ω, k, s are constants to be determined later. Then (3.1) reduce to

$$-\omega u + \frac{ku^2}{2} + (k^3 + ks^2)u'' = 0. \tag{3.2}$$

By balancing the highest order derivative terms and nonlinear terms in (3.2), we get $n = 2$. Then we can suppose that (3.2) has the solutions in the form:

$$u(\xi) = a_{-2}f^{-2}(\xi) + a_{-1}f^{-1}(\xi) + a_0 + a_1 f(\xi) + a_2 f^2(\xi). \tag{3.3}$$

By substituting (2.5) and (3.3) into (3.2), collecting all terms with the same powers of $f^i(\xi)$ and setting each coefficient of the polynomials to zero, and solving the over-determined algebraic equations by Mathematica, we can obtain the following results:

Set 1

$$\begin{aligned}&kp(k^2 + s^2) \neq 0,\ a_0 = 8pq(k^2 + s^2),\ a_1 = 0,\ a_2 = -12q^2(k^2 + s^2),\\ &a_{-1} = 0,\ a_{-2} = -12p^2(k^2 + s^2),\ \omega = 8kpq(k^2 + s^2) + ka_0.\end{aligned} \tag{3.4}$$

Set 2

$$kp(k^2+s^2)\neq 0,\ a_0=-4pq(k^2+s^2),\ a_1=0,\ a_2=0,$$
$$a_{-1}=0,\ a_{-2}=-12p^2(k^2+s^2),\ \omega=8kpq(k^2+s^2)+ka_0. \tag{3.5}$$

Set 3

$$kq(k^2+s^2)\neq 0,\ a_0=-4pq(k^2+s^2),\ a_1=0,\ a_2=-12q^2(k^2+s^2),$$
$$a_{-1}=0,\ a_{-2}=0,\ \omega=8kpq(k^2+s^2)+ka_0. \tag{3.6}$$

Similarly, we can also get the following results:

Case 1

$$\lambda=0,\quad q\neq 0,\quad b_0=\pm\frac{\sqrt{\mu-pq}}{q},\quad b_1=0,\quad b_{-1}=\frac{\mu}{q}. \tag{3.7}$$

Case 2

$$q\neq 0,\quad p=\frac{4\mu-\lambda^2}{4q},\quad b_0=-\frac{\lambda}{2q}, b_1=-\frac{1}{q},\quad b_{-1}=0. \tag{3.8}$$

In the following discussion, If we choose Set 1 , we will get

$$\xi=kx+sy-16kpq(k^2+s^2)t.$$

If we choose Set 2 and Set 3, we will get

$$\xi=kx+sy-4kpq(k^2+s^2)t.$$

Using Case 1, substituting Set 1, Set 2, Set 3 and the general solutions of (2.6) into (3.3), we have three types of travelling wave solutions of the ZK equations as follows (c_1 and c_2 are arbitrary constants).

When $\mu<0$, we obtain the hyperbolic function solutions of (3.1).

$$\begin{aligned}
u_1(\xi)= \;& 8pq(k^2+s^2)\\
&-12q^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q}\left(\sqrt{-\mu}\tfrac{c_1\sinh\sqrt{-\mu}\xi+c_2\cosh\sqrt{-\mu}\xi}{c_2\sinh\sqrt{-\mu}\xi+c_1\cosh\sqrt{-\mu}\xi}\right)^{-1}\right)^2\\
&-12p^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q}\left(\sqrt{-\mu}\tfrac{c_1\sinh\sqrt{-\mu}\xi+c_2\cosh\sqrt{-\mu}\xi}{c_2\sinh\sqrt{-\mu}\xi+c_1\cosh\sqrt{-\mu}\xi}\right)^{-1}\right)^{-2},\\
u_2(\xi)= \;& -4pq(k^2+s^2)\\
&-12p^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q}\left(\sqrt{-\mu}\tfrac{c_1\sinh\sqrt{-\mu}\xi+c_2\cosh\sqrt{-\mu}\xi}{c_2\sinh\sqrt{-\mu}\xi+c_1\cosh\sqrt{-\mu}\xi}\right)^{-1}\right)^{-2},\\
u_3(\xi)= \;& -4pq(k^2+s^2)\\
&-12q^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q}\left(\sqrt{-\mu}\tfrac{c_1\sinh\sqrt{-\mu}\xi+c_2\cosh\sqrt{-\mu}\xi}{c_2\sinh\sqrt{-\mu}\xi+c_1\cosh\sqrt{-\mu}\xi}\right)^{-1}\right)^2.
\end{aligned}$$

If $c_1\neq 0,\ (c_1)^2>(c_2)^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi)= \;& 8pq(k^2+s^2)-12q^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q\sqrt{-\mu}}\coth(\sqrt{-\mu}\xi+\xi_0)\right)^2\\
&-12p^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q\sqrt{-\mu}}\coth(\sqrt{-\mu}\xi+\xi_0)\right)^{-2},\\
u_2(\xi)= \;& -4pq(k^2+s^2)-12p^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q\sqrt{-\mu}}\coth(\sqrt{-\mu}\xi+\xi_0)\right)^{-2},\\
u_3(\xi)= \;& -4pq(k^2+s^2)-12q^2(k^2+s^2)\left(\pm\tfrac{\sqrt{\mu-pq}}{q}+\tfrac{\mu}{q\sqrt{-\mu}}\coth(\sqrt{-\mu}\xi+\xi_0)\right)^2,
\end{aligned}$$

where $\xi_0 = \tanh^{-1}\frac{c_2}{c_1}$.

If $c_2 \neq 0,\ (c_2)^2 > (c_1)^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi) = \;& 8pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q\sqrt{-\mu}}\tanh(\sqrt{-\mu}\xi+\xi_0)\right)^2 \\
& -12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q\sqrt{-\mu}}\tanh(\sqrt{-\mu}\xi+\xi_0)\right)^{-2}, \\
u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q\sqrt{-\mu}}\tanh(\sqrt{-\mu}\xi+\xi_0)\right)^{-2}, \\
u_3(\xi) = \;& -4pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q\sqrt{-\mu}}\tanh(\sqrt{-\mu}\xi+\xi_0)\right)^2,
\end{aligned}$$

where $\xi_0 = \tanh^{-1}\frac{c_1}{c_2}$.

When $\mu > 0$, we get the trigonometric function solutions of (3.1).

$$\begin{aligned}
u_1(\xi) = \;& 8pq(k^2+s^2) \\
& -12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\frac{-c_1\sin\sqrt{\mu}\xi + c_2\cos\sqrt{\mu}\xi}{c_2\sin\sqrt{\mu}\xi + c_1\cos\sqrt{\mu}\xi}\right)^{-1}\right)^2 \\
& -12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\frac{-c_1\sin\sqrt{\mu}\xi + c_2\cos\sqrt{\mu}\xi}{c_2\sin\sqrt{\mu}\xi + c_1\cos\sqrt{\mu}\xi}\right)^{-1}\right)^{-2}, \\
u_2(\xi) = \;& -4pq(k^2+s^2) \\
& -12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\frac{-c_1\sin\sqrt{\mu}\xi + c_2\cos\sqrt{\mu}\xi}{c_2\sin\sqrt{\mu}\xi + c_1\cos\sqrt{\mu}\xi}\right)^{-1}\right)^{-2}, \\
u_3(\xi) = \;& -4pq(k^2+s^2) \\
& -12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\frac{-c_1\sin\sqrt{\mu}\xi + c_2\cos\sqrt{\mu}\xi}{c_2\sin\sqrt{\mu}\xi + c_1\cos\sqrt{\mu}\xi}\right)^{-1}\right)^2.
\end{aligned}$$

If $c_1 \neq 0,\ (c_1)^2 > (c_2)^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi) = \;& 8pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)\right)^{-1}\right)^2 \\
& -12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)\right)^{-1}\right)^{-2}, \\
u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)\right)^{-1}\right)^{-2}, \\
u_3(\xi) = \;& -4pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\tan(\xi_0-\sqrt{\mu}\xi)\right)^{-1}\right)^2,
\end{aligned}$$

where $\xi_0 = \tan^{-1}\frac{c_2}{c_1}$.

If $c_2 \neq 0,\ (c_2)^2 > (c_1)^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$\begin{aligned}
u_1(\xi) = \;& 8pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\cot(\sqrt{\mu}\xi+\xi_0)\right)^{-1}\right)^2 \\
& -12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\cot(\sqrt{\mu}\xi+\xi_0)\right)^{-1}\right)^{-2}, \\
u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\cot(\sqrt{\mu}\xi+\xi_0)\right)^{-1}\right)^{-2}, \\
u_3(\xi) = \;& -4pq(k^2+s^2) - 12q^2(k^2+s^2)\left(\pm\frac{\sqrt{\mu-pq}}{q} + \frac{\mu}{q}\left(\sqrt{\mu}\cot(\sqrt{\mu}\xi+\xi_0)\right)^{-1}\right)^2,
\end{aligned}$$

where $\xi_0 = \tan^{-1}\frac{c_1}{c_2}$.

Using Case 2, substituting Set 1, Set 2, Set 3 and the general solutions of (2.6) into (3.3), we have three types of travelling wave solutions of the ZK equations as follows (c_1 and c_2 are arbitrary constants).

When $pq < 0$, we obtain the hyperbolic function solutions of (3.1).

$$\begin{aligned} u_1(\xi) = \;& 8pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tfrac{c_1 \sinh\sqrt{-pq}\xi + c_2\cosh\sqrt{-pq}\xi}{c_2\sinh\sqrt{-pq}\xi + c_1\cosh\sqrt{-pq}\xi}\right)^2 \\ & -12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\tfrac{c_1 \sinh\sqrt{-pq}\xi + c_2\cosh\sqrt{-pq}\xi}{c_2\sinh\sqrt{-pq}\xi + c_1\cosh\sqrt{-pq}\xi}\right)^{-2}, \\ u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\tfrac{c_1 \sinh\sqrt{-pq}\xi + c_2\cosh\sqrt{-pq}\xi}{c_2\sinh\sqrt{-pq}\xi + c_1\cosh\sqrt{-pq}\xi}\right)^{-2}, \\ u_3(\xi) = \;& -4pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tfrac{c_1 \sinh\sqrt{-pq}\xi + c_2\cosh\sqrt{-pq}\xi}{c_2\sinh\sqrt{-pq}\xi + c_1\cosh\sqrt{-pq}\xi}\right)^2. \end{aligned}$$

If $c_1 \neq 0,\ (c_1)^2 > (c_2)^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned} u_1(\xi) = \;& 8pq(k^2+s^2) + 12pq(k^2+s^2)\tanh^2(\sqrt{-pq}\xi + \xi_0) \\ & -12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\tanh(\sqrt{-pq}\xi+\xi_0)\right)^{-2}, \\ u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\tanh(\sqrt{-pq}\xi+\xi_0)\right)^{-2}, \\ u_3(\xi) = \;& -4pq(k^2+s^2) + 12pq(k^2+s^2)\tanh^2(\sqrt{-pq}\xi+\xi_0), \end{aligned}$$

where $\xi_0 = \tanh^{-1}\frac{c_2}{c_1}$.

If $c_2 \neq 0,\ (c_2)^2 > (c_1)^2$, then $u(\xi)$ becomes the solitary wave solutions of (3.1) as follows:

$$\begin{aligned} u_1(\xi) = \;& 8pq(k^2+s^2) + 12pq(k^2+s^2)\coth^2(\sqrt{-pq}\xi + \xi_0) \\ & -12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\coth(\sqrt{-pq}\xi+\xi_0)\right)^{-2}, \\ u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\tfrac{\sqrt{-pq}}{q}\coth(\sqrt{-pq}\xi+\xi_0)\right)^{-2}, \\ u_3(\xi) = \;& -4pq(k^2+s^2) + 12pq(k^2+s^2)\coth^2(\sqrt{-pq}\xi+\xi_0), \end{aligned}$$

where $\xi_0 = \tanh^{-1}\frac{c_1}{c_2}$.

When $pq = 0$, we get the rational function solutions of (3.1) ($u_2(\xi)$ has no reasonable solutions)

$$\begin{aligned} u_1(\xi) &= -12(k^2+s^2)\left(\tfrac{c_2}{c_1+c_2\xi}\right)^2, \\ u_3(\xi) &= -12(k^2+s^2)\left(\tfrac{c_2}{c_1+c_2\xi}\right)^2. \end{aligned} \tag{3.9}$$

When $pq > 0$, we get the trigonometric function solutions of (3.1).

$$\begin{aligned} u_1(\xi) = \;& 8pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tfrac{-c_1 \sin\sqrt{pq}\xi + c_2\cos\sqrt{pq}\xi}{c_2\sin\sqrt{pq}\xi + c_1\cos\sqrt{pq}\xi}\right)^2 \\ & -12p^2(k^2+s^2)\left(-\tfrac{\sqrt{pq}}{q}\tfrac{-c_1 \sin\sqrt{pq}\xi + c_2\cos\sqrt{pq}\xi}{c_2\sin\sqrt{pq}\xi + c_1\cos\sqrt{pq}\xi}\right)^{-2}, \\ u_2(\xi) = \;& -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\tfrac{\sqrt{pq}}{q}\tfrac{-c_1 \sin\sqrt{pq}\xi + c_2\cos\sqrt{pq}\xi}{c_2\sin\sqrt{pq}\xi + c_1\cos\sqrt{pq}\xi}\right)^{-2}, \\ u_3(\xi) = \;& -4pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tfrac{-c_1 \sin\sqrt{pq}\xi + c_2\cos\sqrt{pq}\xi}{c_2\sin\sqrt{pq}\xi + c_1\cos\sqrt{pq}\xi}\right)^2. \end{aligned}$$

If $c_1 \neq 0,\ (c_1)^2 > (c_2)^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$\begin{aligned} u_1(\xi) = \quad & 8pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tan(\xi_0 - \sqrt{pq}\xi)\right)^2 \\ & -12p^2(k^2+s^2)\left(-\frac{\sqrt{pq}}{q}\tan(\xi_0 - \sqrt{pq}\xi)\right)^{-2}, \\ u_2(\xi) = \quad & -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\frac{\sqrt{pq}}{q}\tan(\xi_0 - \sqrt{pq}\xi)\right)^{-2}, \\ u_3(\xi) = \quad & -4pq(k^2+s^2) + 12pq(k^2+s^2)\left(\tan(\xi_0 - \sqrt{pq}\xi)\right)^2, \end{aligned}$$

where $\xi_0 = \tan^{-1}\frac{c_2}{c_1}$.

If $c_2 \neq 0,\ (c_2)^2 > (c_1)^2$, then $u(\xi)$ becomes the periodic wave solutions of (3.1) as follows:

$$\begin{aligned} u_1(\xi) = \quad & 8pq(k^2+s^2) + 12pq(k^2+s^2)\left(\cot(\sqrt{pq}\xi + \xi_0)\right)^2 \\ & -12p^2(k^2+s^2)\left(-\frac{\sqrt{pq}}{q}\cot(\sqrt{pq}\xi + \xi_0)\right)^{-2}, \\ u_2(\xi) = \quad & -4pq(k^2+s^2) - 12p^2(k^2+s^2)\left(-\frac{\sqrt{pq}}{q}\cot(\sqrt{pq}\xi + \xi_0)\right)^{-2}, \\ u_3(\xi) = \quad & -4pq(k^2+s^2) + 12pq(k^2+s^2)\left(\cot(\sqrt{pq}\xi + \xi_0)\right)^2, \end{aligned}$$

where $\xi_0 = \tan^{-1}\frac{c_1}{c_2}$.

4. Conclusion

In short, we have proposed the extended Riccati equation method and utilized it to find the exact solutions of nonlinear equations with the help of mathmatica software. We have successfully obtained some travelling wave solutions of the ZK equations. When the parameters are taken as special values, the solitary wave solutions and the periodic wave solutions are obtained, and solutions obtained are partly new. At last, it is worthwhile to mention that this method is effective for solving other nonlinear evolution equations in mathematical physics. It is truly a promising and powerful method deserves further employing and studying.

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