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BI-SOLITONS, BREATHER SOLUTION FAMILY AND ROGUE WAVES FOR THE (2+1)-DIMENSIONAL NONLINEAR SCHRÖDINGER EQUATION*

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Abstract In this paper, bi-solitons, breather solution family and rogue waves for the (2+1)-Dimensional nonlinear Schrödinger equations are obtained by using Exp-function method. These solutions derived from one unified formula which is solution of the standard (1+1) dimension nonlinear Schrödinger equation. Further, based on the solution obtained by other authors, higher-order rational rogue wave solution are obtained by using the similarity transformation. These results greatly enriched the diversity of wave structures for the (2+1)-dimensional nonlinear Schrödinger equations.

Keywords Nonlinear Schrödinger equation, Exp-function method, bi-soliton, breather solution, rogue wave.

MSC(2010) 35Q55, 35A20, 35A25, 35C08.

1. Introduction

The (2+1)-dimensional nonlinear Schrödinger equations are expressed as

$$\begin{aligned} iu_t &= u_{xy} + \gamma^2 uv, \\ v_x &= 2(|u|^2)_y, \end{aligned} \tag{1.1}$$

where $\gamma^2 = \pm 1$, $u(x, y, t)$ is a complex function and $v(x, y, t)$ is a real function, respectively. This system plays important role in nonlinear optical physical field [8, 19]. Some researchers have investigated equations(1.1), derived their solutions [21, 23].

In recent years, rogue wave phenomenon become a hot topic for many researchers. They found that rogue waves appear not only in oceanic conditions [6, 9, 16, 18] but also in plasmon [17], optics [5, 22, 24, 29, 30], superfluids [7], Bose-Einstein condensates [4, 10] and in the form of capillary waves [20].

Rogue wave structure and behavior, have attracted the attentions of a large number of scholars. Recently, they have obtained fruitful results associated with

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the rogue waves [1–7, 9, 10, 12–18, 20, 22, 24–30]. In this work, we continue to investigate the existence of rogue waves and their structures for the (2+1)-dimensional nonlinear Schrödinger equations.

2. Exp-function method to construct solutions for equations(1.1)

Setting $z = k_1x + k_2y$, then equations(1.1) are changed into the following forms

$$\begin{aligned} iu_t &= k_1k_2u_{zz} + \gamma^2 uv, \\ v &= \tfrac{2k_2}{k_1}|u|^2. \end{aligned} \tag{2.1}$$

Therefore, equations(1.1) can be reduced to the standard nonlinear Schrödinger equation(NLSE) [18]

$$iu_t + \alpha u_{zz} + \beta|u|^2u = 0, \tag{2.2}$$

where $\alpha = -k_1k_2$ and $\beta = -\frac{2\gamma^2 k_2}{k_1}$, k_1 and k_2 are arbitrary real constants. In view of the character of its solutions, equation(2.2) is called the" self-focussing" $(\alpha > 0, \beta > 0)$ and "de-focussing" $(\alpha > 0, \beta < 0)$ NLSE, respectively. Here we use NLSE^+ and NLSE^- to denote them.

By using the transformation

$$u(z,t) = re^{(ir^2\beta t)}\left(1 + \tfrac{A(z,t)+iB(z,t)}{F(z,t)}\right), \tag{2.3}$$

equation(2.2) can be transformed into the following trilinear equation

$$\begin{aligned} &2\beta r^2A(z,t)F(z,t)^2 + 2\alpha A(z,t)F_z(z,t)^2 - 2\alpha A_z(z,t)F_z(z,t)F(z,t) \\ &+ \beta r^2A(z,t)^3 + \alpha A_{zz}(z,t)F(z,t)^2 - \alpha A(z,t)F_{zz}(z,t)F(z,t) \\ &- B_t(z,t)F(z,t)^2 + B(z,t)F(z,t)F_t(z,t) + 3\beta r^2A(z,t)^2F(z,t) \\ &+ \beta r^2B(z,t)^2F(z,t) + \beta r^2A(z,t)B(z,t)^2 + i(\beta r^2A(z,t)^2B(z,t) \\ &- A(z,t)F(z,t)F_t(z,t) - \alpha B(z,t)F_{zz}(z,t)F(z,t) + \beta r^2B(z,t)^3 \\ &+ \alpha B_z(z,t)F(z,t)^2 + A_t(z,t)F(z,t)^2 + 2\alpha B(z,t)F_z(z,t)^2 \\ &+ 2\beta r^2A(z,t)B(z,t)F(z,t) - 2\alpha B_z(z,t)F_z(z,t)F(z,t)) = 0, \end{aligned} \tag{2.4}$$

where r is real constant, $A(z,t), B(z,t)$ and $F(z,t)$ are real functions. Separating the real and imaginary parts, we have

$$\begin{aligned} &2\beta r^2A(z,t)F(z,t)^2 + 2\alpha A(z,t)F_z(z,t)^2 + \beta r^2A(z,t)^3 \\ &- 2\alpha A_z(z,t)F_z(z,t)F(z,t) + \alpha A_{zz}(z,t)F(z,t)^2 - \alpha A(z,t)F_{zz}(z,t)F(z,t) \\ &+ B(z,t)F(z,t)F_t(z,t) + 3\beta r^2A(z,t)^2F(z,t) + \beta r^2B(z,t)^2F(z,t) \\ &- B_t(z,t)F(z,t)^2 + \beta r^2A(z,t)B(z,t)^2 = 0, \\ &\beta r^2A(z,t)^2B(z,t) - A(z,t)F(z,t)F_t(z,t) - \alpha B(z,t)F_{zz}(z,t)F(z,t) \\ &+ \beta r^2B(z,t)^3 + \alpha B_z(z,t)F(z,t)^2 + A_t(z,t)F(z,t)^2 \\ &+ 2\alpha B(z,t)F_z(z,t)^2 + 2\beta r^2A(z,t)B(z,t)F(z,t) - 2\alpha B_z(z,t)F_z(z,t)F(z,t) = 0. \end{aligned} \tag{2.5}$$

Suppose $A(z,t), B(z,t)$ and $F(z,t)$ are the following exponential functions [11]

$$\begin{aligned}A(z,t) &= a_1e^{p(Vz+Kt)} + a_2e^{-p(Vz+Kt)} + a_3e^{q(Wz+Lt)} + a_4e^{-q(Wz+Lt)},\\ B(z,t) &= b_1e^{p(Vz+Kt)} + b_2e^{-p(Vz+Kt)} + b_3e^{q(Wz+Lt)} + b_4e^{-q(Wz+Lt)},\\ F(z,t) &= c_1e^{p(Vz+Kt)} + c_2e^{-p(Vz+Kt)} + c_3e^{q(Wz+Lt)} + c_4e^{-q(Wz+Lt)}, \qquad (2.6)\end{aligned}$$

where $a_i, b_i, c_i (i = 1, .., 4), p, q, W, V, K$ and L are constants to be determined. Substituting functions(2.6) into equations(2.5) which yields two algebraic equations with respect to $e^{mp(Vz+Kt)}e^{nq(Wz+Lt)}(m, n = -3, .., 3)$. Equating all coefficients of $e^{mp(Vz+Kt)}e^{nq(Wz+Lt)}(m, n = -3, .., 3)$ to zero yields a set of algebraic equations for $a_i, b_i, c_i(i = 1, .., 4), p, q, W, V, K$ and L. Solving them with the aid of Maple, we can obtain the following results:

$$\begin{aligned}&a_1 = 0, a_2 = 0, a_3 = \frac{b_4^2}{\sqrt{4c_2^2 - b_4^2}}, a_4 = \frac{1}{\sqrt{4c_2^2 - b_4^2}}b_4^2, b_1 = 0, b_2 = 0, b_3 = -b_4,\\ &W = 0, c_1 = c_2, c_3 = -\frac{2c_2^2}{\sqrt{4c_2^2 - b_4^2}}, c_4 = -\frac{2c_2^2}{\sqrt{4c_2^2 - b_4^2}}, L = -\frac{\beta r^2 b_4}{2c_2^2 q}\sqrt{4c_2^2 - b_4^2},\\ &K = 0, V = \frac{rb_4}{2pc_2}\sqrt{-2\frac{\beta}{\alpha}}, \qquad (2.7)\end{aligned}$$

where c_2 and b_4 are arbitrary constants.

Substituting (2.7) with (2.6) into (2.3), solution of equation(2.2) is expressed as

$$u(z,t) = re^{(ir^2\beta t)}\left(1 + \frac{b_4^2\cosh(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t) + ib_4\sqrt{4c_2^2-b_4^2}\sinh(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t)}{c_2\sqrt{4c_2^2-b_4^2}\cosh(\frac{r\sqrt{-2\frac{\beta}{\alpha}}b_4}{2c_2}z) - 2c_2^2\cosh(\frac{\beta r^2 b_4\sqrt{4c_2^2-b_4^2}}{2c_2^2}t)}\right). \quad (2.8)$$

Substituting $z = k_1x + k_2y$, $\alpha = -k_1k_2$ and $\beta = -\frac{2\gamma^2k_2}{k_1}$ into solution(2.8), we obtain solutions of equations(1.1) as follows

$$\begin{aligned}u(x,y,t) &= re^{(-i\frac{2r^2k_2\gamma^2}{k_1}t)}\\ &\left(1 + \frac{b_4^2\cosh(\frac{\gamma^2r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t) - ib_4\sqrt{4c_2^2-b_4^2}\sinh(\frac{\gamma^2r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}{c_2\sqrt{4c_2^2-b_4^2}\cosh(\sqrt{\frac{-\gamma^2}{k_1^2}}\frac{rb_4}{c_2}(k_1x+k_2y)) - 2c_2^2\cosh(\frac{\gamma^2r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}\right), \quad (2.9)\\ v(x,y,t) &= \frac{2k_2}{k_1}|u(x,y,t)|^2.\end{aligned}$$

Solution(2.9) is a unified formula which can produce a series of breather solutions. Obviously, when $b_4 = 0$, solutions (2.9) become plane-wave solutions of equations(1.1) which are written as

$$u(x,y,t) = re^{(-i\frac{2r^2k_2\gamma^2}{k_1}t)}, \quad v(x,y,t) = \frac{2k_2}{k_1}r^2. \qquad (2.10)$$

3. Bi-solitons, breather solution family and rogue waves for equations (1.1)

When suitably selected parameters in solutions (2.9), we can obtain the following solutions of different structures.

Case 1 The bi-soliton solution: when $4c_2^2 - b_4^2 > 0$ and $\gamma^2 = -1$, solutions (2.9) are bi-soliton solutions of Eqs.(1.1) which can be re-written as

$$u(x,y,t) = re^{(i\frac{2r^2k_2}{k_1}t)} \left(1 + \frac{b_4^2\cosh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t) + ib_4\sqrt{4c_2^2-b_4^2}\sinh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}{c_2\sqrt{4c_2^2-b_4^2}\cosh(\frac{rb_4}{k_1c_2}(k_1x+k_2y)) - 2c_2^2\cosh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}\right), \quad (3.1)$$
$$v(x,y,t) = \frac{2k_2}{k_1}|u(x,y,t)|^2.$$

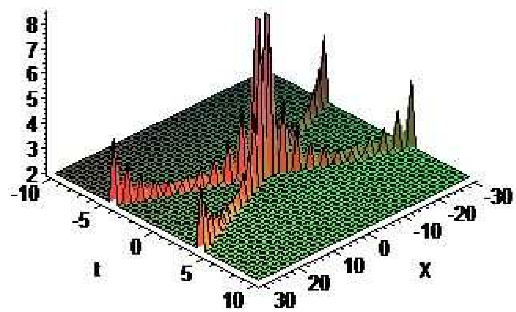

Figure 1. The profile of $|u(x,y,t)|$ in (3.1) with $k_1 = 1, k_2 = 2, r = 2, b_4 = 2, c_2 = 2$ and $X = x + 2y$.

Case 2 The periodic solution: if $\gamma^2 = -1$, setting $b_4 = ib$, then solutions(2.9) become the following periodic solutions

$$u(x,y,t) = re^{(i\frac{2r^2k_2}{k_1}t)}\left(1 - \frac{b^2\cos(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t) - ib\sqrt{4c_2^2+b^2}\sin(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t)}{c_2\sqrt{4c_2^2+b^2}\cos(\frac{rb}{k_1c_2}(k_1x+k_2y)) - 2c_2^2\cos(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t)}\right), \quad (3.2)$$
$$v(x,y,t) = \frac{2k_2}{k_1}|u(x,y,t)|^2.$$

Case 3 Breather solution family: The Akhmediev breather soliton [1,3], the Ma breather soliton [14] and the Peregrine breather soliton [18] have been suggested as models for a class of freak wave events [12]. Thus, the following breather solitons are also known as rogue waves.

I. When $4c_2^2 - b_4^2 > 0$ and $\gamma^2 = 1$, then solutions(2.9) become the following forms which are called **Akhmediev breather solitons**

$$u(x,y,t) = re^{(-i\frac{2r^2k_2}{k_1}t)}\left(1 + \frac{b_4^2\cosh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t) - ib_4\sqrt{4c_2^2-b_4^2}\sinh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}{c_2\sqrt{4c_2^2-b_4^2}\cos(\frac{rb_4}{k_1c_2}(k_1x+k_2y)) - 2c_2^2\cosh(\frac{r^2b_4k_2\sqrt{4c_2^2-b_4^2}}{k_1c_2^2}t)}\right), \quad (3.3)$$
$$v(x,y,t) = \frac{2k_2}{k_1}|u(x,y,t)|^2.$$

II. When $b_4 = ib$ and $\gamma^2 = 1$, then solutions(2.9) become the following forms which are called **Ma breather solitons**

$$
\begin{aligned}
u(x,y,t) &= re^{(-i\frac{2r^2k_2}{k_1}t)} \\
&\qquad (1-\frac{b^2\cos(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t)-ib\sqrt{4c_2^2+b^2}\sin(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t)}{c_2\sqrt{4c_2^2+b^2}\cosh(\frac{rb}{k_1c_2}(k_1x+k_2y))-2c_2^2\cos(\frac{r^2bk_2\sqrt{4c_2^2+b^2}}{k_1c_2^2}t)}), \\
v(x,y,t) &= \frac{2k_2}{k_1}|u(x,y,t)|^2.
\end{aligned} \tag{3.4}
$$

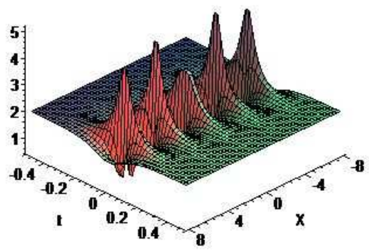

Figure 2. The profile of $|u(x,y,t)|$ in (3.3) with $k_1=1, k_2=2, r=2, b_4=2, c_2=2$ and $X=x+2y$.

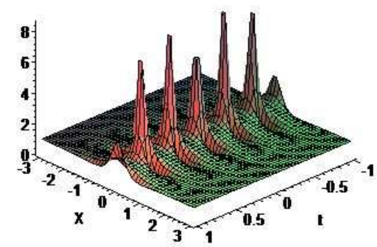

Figure 3. The profile of $|u(x,y,t)|$ in (3.4) with $k_1=1, k_2=2, r=2, b=2, c_2=2$ and $X=x+2y$.

III. **The Peregrine breather soliton(rational solution)** [23]: when $\gamma^2=1$, setting $c_2>0$ and $b_4\to 0$, solutions(2.9) become Peregrine breather forms which are written as

$$
\begin{aligned}
u(x,y,t) &= re^{(-i\frac{2r^2k_2}{k_1}t)}(1-\frac{4k_1(k_1-i4r^2k_2t)}{k_1^2+4r^2(k_1x+k_2y)^2+16k_2^2r^4t^2}), \\
v(x,y,t) &= \frac{2k_2}{k_1}|u(x,y,t)|^2.
\end{aligned} \tag{3.5}
$$

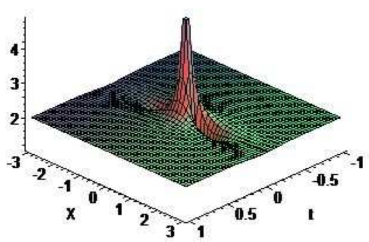

Figure 4. The profile of $|u(x,y,t)|$ in (3.5) with $k_1=1, k_2=2$ and $X=x+2y$.

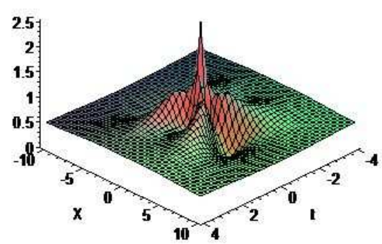

Figure 5. The profile of $|u(x,y,t)|$ in (3.7) with $k_1=1, k_2=-2$ and $X=x-2y$.

Case 4 Higher-order rational rogue wave solution: when $\alpha=\frac{1}{2}$ and $\beta=1$, higher-order rational rogue wave solutions of equation(2.2) are given by Akhmediev [2,3]. Based on Refs. [2,3], we obtained higher-order rational rogue wave solutions of equation(2.2) as follows:

$$
\begin{aligned}
u(z,t) &= \sqrt{\frac{1}{\beta}}\left(1-\frac{G+iH}{D}\right)e^{it},\ \beta>0,\\
G &= \frac{4z^4+12\alpha z^2+72\alpha^2t^2+48\alpha t^2z^2+80\alpha^2t^4-3\alpha^2}{16\alpha^2},\\
H &= \frac{t(4z^4+8\alpha^2t^2+16\alpha t^2z^2+16\alpha^2t^4-15\alpha^2-12\alpha z^2)}{8\alpha^2},\\
D &= \frac{8z^6+\alpha(12+48t^2)z^4+6\alpha^2(4t^2-3)^2z^2+\alpha^3(9+64t^6+432t^4+396t^2)}{192\alpha^3}.
\end{aligned}
\tag{3.6}
$$

When $\gamma^2=1$, substituting $\alpha=-k_1k_2>0$, $\beta=-\frac{2k_2}{k_1}>0$ and $z=k_1x+k_2y$ into (3.6), we have higher-order rational rogue wave solutions of equations(1.1) which are written as

$$
\begin{aligned}
u(x,y,t) &= \frac{\sqrt{2}}{2}\sqrt{-\frac{k_1}{k_2}}\left(1-12\frac{G+iH}{D}\right)e^{it},\\
v(x,y,t) &= \frac{2k_2}{k_1}|u(x,y,t)|^2,
\end{aligned}
\tag{3.7}
$$

where $G=4(k_1x+k_2y)^4+12(-k_1k_2)(1+4t^2)(k_1x+k_2y)^2+(-k_1k_2)^2(80t^4+72t^2-3)$, $H=8t(k_1x+k_2y)^4+(-k_1k_2)(32t^3-24t)(k_1x+k_2y)^2+2(-k_1k_2)^2t(4t^2+5)(4t^2-3)$, $D=(-k_1k_2)^3(9+396t^2+432t^4+64t^6)+6(-k_1k_2)^2(k_1x+k_2y)^2(4t^2-3)^2+12(-k_1k_2)(k_1x+k_2y)^4(1+4t^2)+8(k_1x+k_2y)^6$, k_1 and k_2 are arbitrary constants and satisfy $k_1k_2<0$.

4. Analysis of interactions

From Figure 1, we find that two solitons moving towards each other, they met and elastic collision occurred, then separated reverse movement. In the event of a collision, amplitude increased significantly. Figure 2 and Figure 3 represent two breather soliton, respectively. One of the solitons produces breather effects on spatial direction. However, another soliton produces breather effects on time direction.

Figure 6 represents a rogue waves which is multiple high-amplitude waves gradually together into a wave, as time increases, the amplitude gradually decreases, the wavelength becomes larger, eventually becomes a plane wave. Thus, rogue waves came suddenly and disappeared without a trace.

5. Conclusion

In this paper, the (2+1)-dimensional NLS equations are transformed into the standard (1+1)-dimension NLS equation by using appropriate transformation. One unified formula solution of the standard (1+1)-dimension NLS equation, which yields bi-solitons and a series of breather solitons(rogue waves), is obtained based on Exp-function method. Then, solutions of the (2+1)-dimensional NLS equations, which contain Akhmediev breather soliton, Ma breather soliton and Peregrine breather soliton and so on, are represented. At the same time, based on the solutions of the (1+1)-dimension NLS equation obtained by other authors, higher-order rational

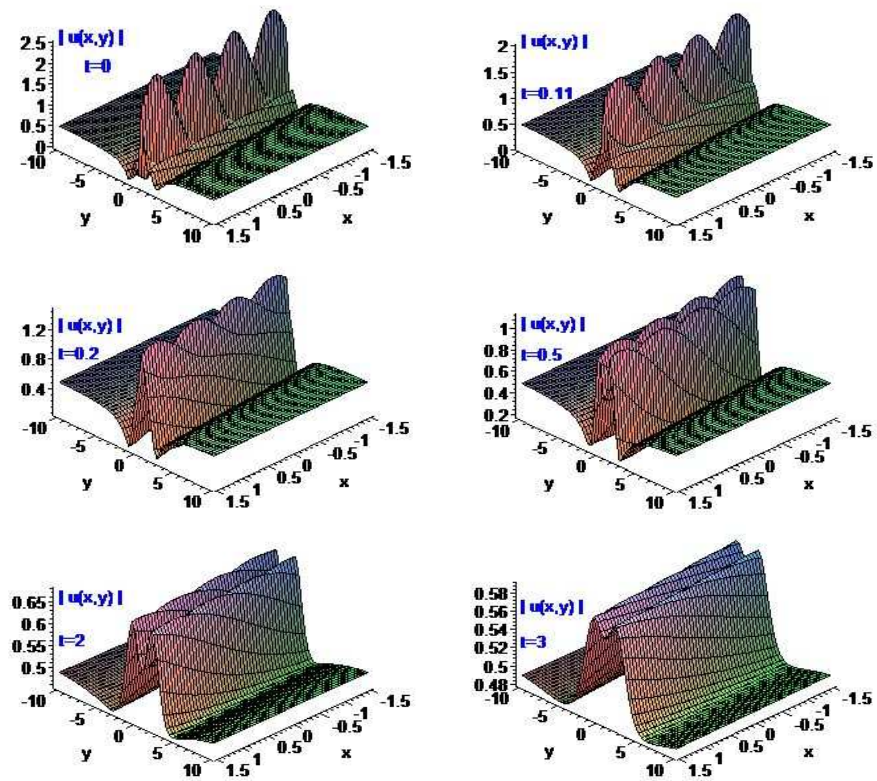

Figure 6. The profiles of $|u(x, y, t)|$ in (3.7) with $k_1 = 1, k_2 = -2$ and $t = 0$, 0.11, 0.2, 0.5, 2, 3.

rogue wave solutions are obtained for (2+1)-dimensional NLS equations by using the similarity transformation. Several arbitrary parameters are involved to generate abundant wave structures which greatly enriched the diversity of wave structures for the (2+1)-dimensional nonlinear Schrödinger equations.

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