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THE GEOMETRICAL ANALYSIS OF A PREDATOR-PREY MODEL WITH MULTI-STATE DEPENDENT IMPULSES*

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Abstract Starting from the practical problems of integrated pest management, we establish a predator-prey model for pest control with multi-state dependent impulsive, which adopts two different control methods for two different thresholds. By applying geometry theory of impulsive differential equations and the successor function, we obtain the existence of order one periodic solution. Then the stability of the order one periodic solution is studied by analogue of the Poincaré criterion. Finally, some numerical simulations are exerted to show the feasibility of the results.

Keywords Semi-continuous dynamic system, successor function, order one periodic solution, stability.

MSC(2010) 34D20, 34C25, 34A37, 92B05.

1. Introduction

Plutella xylostella as a worldwide leading pest brings a great impact on the production of vegetables including the yield and quality. Its prevention often applies chemical pesticides while the abuse of pesticides shall kill natural enemies of plutella xylostella, thereby causing the pest to reproduce in great numbers. The increase of the number of plutella xylostella requires more chemical use, which shall lead to its resistance to chemicals. Then to keep ecological balance in a low cost, how to set up a practical mathematical model for plutella xylostella control is an interesting problem.

Impulsive differential equation is often used to describe such changes in an instant or a short time as cancer radiotherapy, impulsive injection of drugs, fish fry putting and breakout of locusts, which is much preciser than the common differential equation. Many scholars have studied the systems with impulsive differential equations including periodic impulse system [1, 6, 10, 13, 14, 16, 20–28] and state-dependent impulse system [2, 7, 17, 19, 29–33], and obtained some good results. For

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integrated pest management (IPM), Tang and Cheke [18] first presented the state-dependent impulsive "Volterra" model concerning the existence and stability of order-one and order-two periodic solutions. Recently, Liu et al. [11] investigated a Holling I pest management model with time pulse, and an asymptotic stability of periodic solution was proved when the impulsive period is less than some critical value. Jiang and Jiao et al. [8, 9] proposed a stage-structured pest control model with state impulse and phase structure, which obtained the existence and attractivity of periodic solution. Nie et al. [15] established the pest management system with two state-dependencies

$$\begin{cases} \left.\begin{aligned} &x'(t) = x(t)(r - by(t)), \\ &y'(t) = y(t)\left(\frac{\lambda b x(t)}{1 + bcx(t)} - p\right), \end{aligned}\right\} & x \neq h_1, h_2, \\ \left.\begin{aligned} &\triangle x(t) = 0, \\ &\triangle y(t) = \kappa, \end{aligned}\right\} & x = h_1, \\ \left.\begin{aligned} &\triangle x(t) = -\alpha x(t), \\ &\triangle y(t) = -\beta y(t) + \delta, \end{aligned}\right\} & x = h_2, \end{cases} \tag{1.1}$$

and investigated the existence and the stability of periodic solution for the model by Poincaré map and the properties of the Lamber M function.

The advantage of system (1.1) is that it assumes two economic injury levels, which gives a new idea to control pests. However, it can be seen from system (1.1), when $x = h_1$, the natural enemy $y(t)$ is released with κ, and then its density reaches the level $y(t) + \kappa$ and the amount of pests keeps at h_1 by the third and fourth equations of (1.1). The number of natural enemy is released with κ again, and its density reaches the level $y(t) + 2\kappa$. Similarly, the above process is repeated n times, then $y(t)$ comes to $y(t) + n\kappa$. If $n \to \infty$, then $y(t) + nk \to \infty$. Actually, this situation is unreasonable in reality.

When the density of plutella xylostella population $x(t)$ reaches the minor economic injury level h_1, namely, plutella xylostella can not constitute a serious harm, only to release its natural enemy population $y(t)$ to control as much as possible to reduce the damage to the ecological environment. The above process is repeated until the quantity of the natural enemy population of the plutella xylostella is greater than a certain level, that is, the number of natural enemy is enough to maintain the ecological balance, the first strategy should be cancelled, the natural enemy of the plutella xylostella and the plutella xylostella will propagate on the basis of the natural order. When the density of plutella xylostella population $x(t)$ reaches the greater level h_2, that is plutella xylostella number can cause devastating damage, only the release of natural enemy can not significantly reduce the damage, so we have to spray a certain pesticide at the same time.

Based on the above analysis and the integrated pest management, the predator-prey system (1.1) with state-dependent impulse can be written as

$$\begin{cases} \left.\begin{aligned} &x'(t) = x(t)(r - by(t)), \\ &y'(t) = y(t)\left(\frac{\lambda bx(t)}{1+bcx(t)} - p\right), \end{aligned}\right\} x \neq h_2 \ or \ x = h_1, \ y > y^*, \\ \left.\begin{aligned} &\triangle x(t) = 0, \\ &\triangle y(t) = \kappa, \end{aligned}\right\} \quad x = h_1, \ y \leqslant y^*, \\ \left.\begin{aligned} &\triangle x(t) = -\alpha x(t), \\ &\triangle y(t) = -\beta y(t) + \delta, \end{aligned}\right\} \quad x = h_2, \end{cases} \tag{1.2}$$

where $x(t)$ represents the density of the plutella xylostella at time t; $y(t)$ represents the density of the natural enemies of the plutella xylostella at time t. r, b, $\lambda, h_1, h_2, \delta$ and p are all positive constants and $h_1 < h_2$, $y^* = \frac{r}{b}$. The numbers $\alpha, \beta \in (0, 1)$ refer to the proportion of plutella xylostella and its natural enemies killed by the pesticide, δ is the release quantity of natural enemies population of the plutella xylostella. $\frac{\lambda bx(t)}{1+bcx(t)}$ is the per capita functional response of natural enemies of the plutella xylostella. When the number of the plutella xylostella reaches the smaller threshold h_1 at time t_{h_1}, natural enemies of the plutella xylostella need to be released and the quantity of natural enemies abruptly reaches $y(t_{h_1}) + \kappa$. When the number of the plutella xylostella reaches the larger threshold h_2 at time t_{h_2}, pesticide is sprayed and natural enemies are released at the same time, the number of plutella xylostella and natural enemies of the plutella xylostella suddenly turn to $(1-\alpha)h_2$ and $(1-\beta)y(t_{h_2}) + \delta$, respectively.

The paper is structured as follows. In Section 2, some basic concepts and important lemmas as preliminaries are introduced. In the next section, the existence of order one periodic solution of system (1.2) is proved by successor function. In Section 4, by using analogue of the Poincaré criterion, we get the stable conditions of periodic solution of (1.2) under impulse. Finally, we make a summary and the feasibility of our results are illustrated by numerical simulations.

2. Preliminaries

Definition 2.1 ([3]). A triple (X, Π, R^+) is called a semi-dynamical system if X is a metric space, R^+ is the set of all non-negative real and $\Pi(Q, t) : X \times R^+ \to X$ is a continuous map such that:

(i) $\Pi(Q, 0) = Q$ for all $P \in X$;

(ii) $\Pi(\Pi(Q, t), s) = \Pi(Q, t + s)$ for all $Q \in X$ and $t, s \in R^+$.

Also a semi-dynamical system (X, Π, R^+) is denoted as (X, Π).

Definition 2.2 ([4]). $(X, \Pi; E, I)$ is called an impulsive semi-dynamical system if the following conditions are satisfied:

(i) (X, Π) is a semi-dynamical system;

(ii) E is a nonempty subset of X;

(iii) function $I : E \to X$ is continuous and for any $Q \in E$, there exists a $\varepsilon > 0$ such that for any $0 < |t| < \varepsilon$, $\Pi(Q, t) \notin E$.

For any $Q \in X$, the map $\Pi_Q : R^+ \to X$ defined as $\Pi_Q(t) = \Pi(Q,t)$ is continuous and we call $\Pi_Q(t)$ the orbit passing through point Q. The set $C^+(Q) = \{\Pi(Q,t) \mid 0 \le t < +\infty\}$ and the set $C^-(Q) = \{\Pi(Q,t) \mid -\infty < t \le 0\}$ is called positive semi-orbit and the negative semi-orbit of point Q, respectively.

Definition 2.3 ([5])**.** We consider the following state-dependent impulsive differential equations

$$\begin{cases} \left.\begin{aligned} x'(t) &= \Phi(x,y), \\ y'(t) &= \Psi(x,y), \end{aligned}\right\} & (x,y) \notin M\{x,y\}, \\ \left.\begin{aligned} \triangle x(t) &= U(x,y), \\ \triangle y(t) &= V(x,y), \end{aligned}\right\} & (x,y) \in M\{x,y\}, \end{cases} \tag{2.1}$$

there exists a continuous impulse function $I : I(M) = N$, here M is the impulsive set, N is the phase set. M and N are the straight line or curve line on the plane. We define the dynamical system as a semi-continuous dynamical system, which is composed of the solution mapping defined by the state impulsive differential equations (2.1) and it is denoted as (Ω, f, I, M).

Definition 2.4 ([34])**.** Assuming that the pulse set M and the phase set N are both straight lines, as shown in Figure 1. For any point $A \in N$, then $\Pi(A,t) = C \in M$, $I(C) = B \in N$, we denote the ordinates of point A and B are y_A and y_B, respectively. Then B is defined as the successor point of A, and $f(A) = y_B - y_A$ is the successor function of point A.

Definition 2.5 ([5])**.** A trajectory $\Pi(Q_0,t)$ is called an order one periodic solution with period t if there exists a point $Q_0 \in N$ and $t > 0$ such that $Q = \Pi(Q_0,t) \in M$ and $Q^+ = I(Q) = Q_0 \in N$.

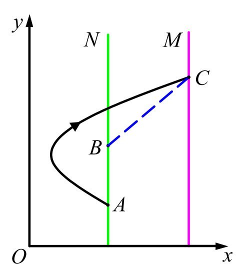

Figure 1. The geometric diagram of the successor function.

We get the following Lemmas from the continuity of composite function and the property of continuous function:

Lemma 2.1 (Lemma 2.6, [3])**.** *Successor function defined in Definition 2.4 is continuous.*

Lemma 2.2 (Lemma 2.8, [12])**.** *In system* (1.2)*, if there exist* $A \in N$*,* $B \in N$ *satisfying successor function* $f(A)f(B) < 0$*, then there must exist a point* $Q(Q \in N)$

satisfying Q between point A and point B such that $f(Q) = 0$, then system (1.2) *has an order one periodic solution.*

Lemma 2.3 (Theorem 2.3, [34])**.** *(Analogue of the Poincaré criterion) The τ-periodic solution $x = \xi(t), y = \eta(t)$ of the system*

$$\begin{cases} \left.\begin{array}{l} x'(t) = \Phi(x, y), \\ y'(t) = \Psi(x, y), \end{array}\right\} & if\ \Gamma(x, y) \neq 0, \\ \left.\begin{array}{l} \triangle x(t) = U(x, y), \\ \triangle y(t) = V(x, y), \end{array}\right\} & if\ \Gamma(x, y) = 0 \end{cases}$$

is orbital asymptotic stability, if the multiplier μ_2 satisfies the condition $|\mu_2| < 1$, where

$$\mu_2 = \Pi_{i=1}^{q} \Delta_i \exp \int_0^{\tau} [\frac{\partial \Phi}{\partial x}(\xi(t), \eta(t)) + \frac{\partial \Psi}{\partial y}(\xi(t), \eta(t))]dt,$$

$$\Delta_i = \frac{\Phi_+(\frac{\partial V}{\partial y}\frac{\partial \Gamma}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial \Gamma}{\partial y} + \frac{\partial \Gamma}{\partial x}) + \Psi_+(\frac{\partial U}{\partial x}\frac{\partial \Gamma}{\partial y} - \frac{\partial U}{\partial y}\frac{\partial \Gamma}{\partial x} + \frac{\partial \Gamma}{\partial y})}{\Phi \frac{\partial \Gamma}{\partial x} + \Psi \frac{\partial \Gamma}{\partial y}},$$

and $\Phi, \Psi, \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial \Gamma}{\partial x}, \frac{\partial \Gamma}{\partial y}$ are calculated at the point $(\xi(\tau_i), \eta(\tau_i))$ and $\Phi_+ = \Phi(\xi(\tau_i^+), \eta(\tau_i^+)),\ \Psi_+ = \Psi(\xi(\tau_i^+), \eta(\tau_i^+))$.

In Section 3 and Section 4, we use these concepts and lemmas to geometrically discuss the existence and the stability of periodic solution of system (1.2).

Next we only consider the system (1.2) with no impulsive effects:

$$\begin{cases} x'(t) = x(t)(r - by(t)), \\ y'(t) = y(t)\left(\dfrac{\lambda b x(t)}{1 + bcx(t)} - p\right). \end{cases} \tag{2.2}$$

As is known to all, the system (2.2) has two equilibrium points $O(0, 0)$ and $R(\frac{p}{b(\lambda - pc)}, \frac{r}{b}) = R(x^*, y^*)(\lambda > pc)$, where O is a saddle point and R is a stable centre. There is an unique closed orbit which through any point in the first quadrant including the point R.

In present paper, we assume that $\lambda > pc$ holds. To be the biological meaningful of system (1.2), we restrict the region $D = \{(x, y) : x \geq 0, y \geq 0\}$. Vector graph of system (2.2) as shown in the following figure (see Figure 2).

3. Existence of the Periodic Solution

In this section, we show the existence of order one periodic solution of system (1.2) by using the successor function defined in Section 2 and qualitative theory of

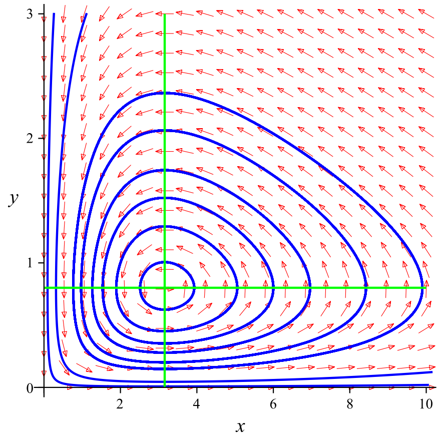

Figure 2. Illustration of vector graph of system (2.2).

differential equation. Now, we denote that

$$\begin{aligned} M_1 &= \left\{(x,y) \mid x = h_1, 0 \le y \le \frac{r}{b}\right\}, \\ M_2 &= \{(x,y) \mid x = h_2, y \ge 0\}, \\ N_1 &= I(M_1) = \left\{(x,y) \mid x = h_1, \frac{r}{b} < y \le \frac{r}{b} + \kappa\right\}, \\ N_2 &= I(M_2) = \{(x,y) \mid x = (1-\alpha)h_2, y \ge \delta\}, \end{aligned}$$

where the line M_1 and the line N_1 are the first impulsive set and of (1.2) and the corresponding phase set, respectively; M_2 and N_2 are the second impulsive set and of (1.2) and the corresponding phase set, respectively.

In system (1.2), the isoclinic line $x'(t) = 0$ and the isoclinic line $y'(t) = 0$ are denoted by L_1 and L_2, respectively, i.e.,

$$\begin{aligned} L_1 &= \left\{(x,y) \mid y = \frac{r}{b}, x \ge 0\right\}, \\ L_2 &= \left\{(x,y) \mid x = \frac{p}{b(\lambda - pc)}, y \ge 0\right\}. \end{aligned}$$

If $I \in \Omega - M$, $F(I)$ is referred as the first intersection of $C^+(I)$ and M, namely, there exists a $t_I \in R_+$ such that $F(I) = \Pi(I, t_I) \in M$, and for $0 < t < t_I$, such that $\Pi(P,t) \cap M = \emptyset$. If $J \in \Omega - N$, $R(J)$ is the first intersection of $C^-(J)$ and N, namely, there exists a $t_J \in R_+$ such that $R(J) = \Pi(J, -t_J) \in N$, and for $-t_J < t < 0$, such that $\Pi(J,t) \cap N = \emptyset$.

In order to facilitate the following narrative, we make some assumptions. For any point Q, we set x_Q as its abscissa and y_Q as its ordinate. If the point $Q(h, y_Q) \in M$, then the point $Q^+ \in N$ is the corresponding phase point of Q after the pulse. Because of the actual meaning, in present paper we assume the impulsive set always lies in the left side of the point R, i.e. $h_1 < \frac{p}{b(\lambda - pc)}$ and $h_2 < \frac{p}{b(\lambda - pc)}$.

From Figure 2, the orbit with any initiating point of $D = \{(x,y) \mid x \ge 0, y \ge 0\}$ intersect at set N_1 or N_2 with time increasing. Hence, the following two cases are considered.

3.1. The orbit starting from the phase set N_1

Let the point $A(h_1, \frac{r}{b})$ is the intersection of L_1 and N_1, and the intersection of L_1 and L_2 be $R(\frac{p}{b(\lambda-pc)}, \frac{r}{b})$. Take a point $B_1(h_1, \frac{r}{b}+\varepsilon) \in N_1$ above A, where $\varepsilon > 0$ is small enough, the orbit starting from B_1 hits the point $Q_1(h_1, y_{Q_1}) \in M_1$, pulse occurs at the point Q_1, then we obtain the successor point $Q_1^+(h_1, y_{Q_1}+\kappa)$ of B_1. Because B_1 is next to A, Q_1 is next to A and Q_1^+ must lies above A, i.e., $\frac{r}{b} < y_{Q_1}+\kappa$ holds, so the successor function $f(B_1) = y_{Q_1}+\kappa-(\frac{r}{b}+\varepsilon) > 0$.

By regulating κ, the position of Q_2^+ has the following three cases:

Case I $y_{Q_1}+\kappa = \frac{r}{b}+\varepsilon$

For this case, the successor point Q_1^+ and B_1 are completely coincident, so the successor function $f(B_1) = y_{Q_1}+\kappa-(\frac{r}{b}+\varepsilon) = 0$. thus the curve $\widehat{B_1Q_1Q_1^+}$ forms a periodic solution of (1.2). (As shown in Figure 3(a))

On the other hand, the orbit Γ_2 passing through the point Q_1^+ intersects with M_1 at $Q_2(h_1, y_{Q_2})$, because any two orbits are disjoint, so we have $y_{Q_2} < y_{Q_1} < \frac{r}{b}$. The point Q_2 is influenced by pulse to $Q_2^+(h_1, y_{Q_2}+\kappa)$.

Case II $\frac{r}{b} < y_{Q_2}+\kappa < y_{Q_1}+\kappa$

If the point Q_1^+ lies above the point B_1, thus the successor function $f(B_1) = y_{Q_1}+\kappa-(\frac{r}{b}+\varepsilon) > 0$. In this case, the point Q_2^+ is located above the point A and under Q_1^+, then the successor function of Q_1^+ is $f(Q_1^+) = y_{Q_2}+\kappa-(y_{Q_1}+\kappa) < 0$. Therefor, $f(B_1)f(Q_1^+) < 0$. By Lemma 2.2, system(1.2) has an order one periodic solution, whose initial point Q is between B_1 and Q_1^+ in set N_1.(As shown in Figure 3(b))

Case III $\frac{r}{b} \geq y_{Q_2}+\kappa$

If point Q_2^+ is below point A, i.e., $Q_2^+ \in M_1$, thus Q_2^+ jumps to $Q_2^{++}(h_1, y_{Q_2}+2\kappa)$ after the effect of impulse.

If $\frac{r}{b} < y_{Q_2}+2\kappa < y_{Q_1}+\kappa$, i.e., Q_2^{++} is above point A, like the argument of Case II, system (1.2) has an order one periodic solution.

If $\frac{r}{b} > y_{Q_2}+2\kappa$, i.e., Q_2^{++} is below point A, the above process is repeated until there exists $n \in Z_+$ such that Q_2^{++} jumps to $Q_2^{n+}((h_1, y_{Q_2}+n\kappa)$ after $n-2$ times' impulsive effects which satisfies $\frac{r}{b} < y_{Q_2}+n\kappa < y_{Q_1}+\kappa$. Similar to the Case II, system (1.2) has an order one periodic solution (see Figure 3(c)).

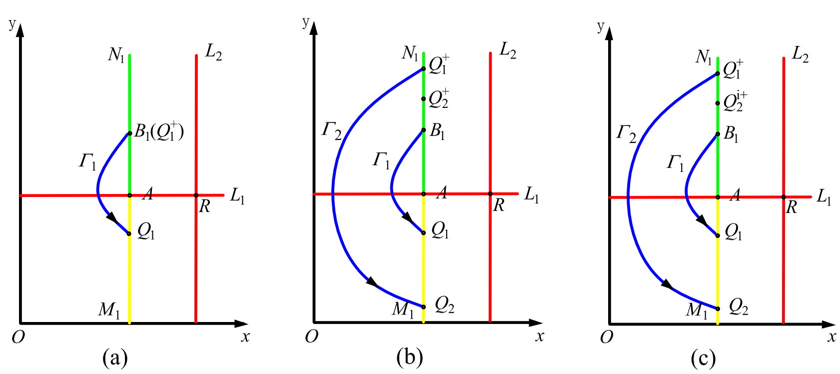

Figure 3. The orbit starting from the phase set N_1 (Case I, Case II and Case III in Section 3.1).

According to the above analysis results, we get the following theorem.

Theorem 3.1. *If* $\lambda > pc, 0 < h_1 < \frac{p}{b(\lambda - pc)}$, *then the system* (1.2) *has an order one periodic solution.*

3.2. The orbit starting point from the phase set N_2

Suppose point B is the intersection of L_1 and N_1 and point A is the intersection of L_1 and N_2. On the one hand, take a point $S \in N_2$ which is above A. The trajectory starting from S of (1.2) becomes vertical only as it crosses B, and then it goes through N_2 from the left to the right, after reaching the point $Q_2 \in M_2$. The orbit passes through point A which tangents to N_2 at point A and intersects with M_2 at a point $Q_0(h_2, y_{Q_0})$. Since point $Q_0 \in M_2$, then pulses to point Q_0, denote Q_0^+ as the phase point of Q_0 after the effect of impulse.

According to the third and fourth equations of system (1.2), the following is got

$$\begin{cases} x_{Q_0^+} = (1-\alpha)h_2, \\ y_{Q_0^+} = (1-\beta)y_{Q_0} + \delta. \end{cases}$$

By regulating δ, there are the following cases:

Case I $y_{Q_0^+} = \frac{r}{b} = y_A$

This moment, the successor point Q_0^+ of A coincides with A, thus the curve $\widehat{AQ_0A}$ forms a periodic orbit of (1.2) (see Figure 4(a)).

Case II $\frac{r}{b} < (1-\beta)y_{Q_0} + \delta < y_S$

If point Q_0^+ is below point S and above point A, take a point $B_1((1-\alpha)h_2, \varepsilon + \frac{r}{b}) \in N_2$ above A, where $\varepsilon > 0$ is small enough. Let $F(B_1) = Q_1(h_2, y_{Q_1}) \in M_2$, then Q_1 pulses to Q_1^+. Because of continuous dependence of the solution on time and initial value, we can see $y_{Q_1} < y_{Q_0}$ and point Q_1 is close to Q_0 enough, so point Q_1^+ is close to Q_0^+ enough and $y_{Q_1^+} < y_{Q_0^+}$, then $f(B_1) = y_{Q_1^+} - y_{B_1} > 0$.

On the other hand, since $Q_2(h_2, y_{Q_2}) \in M_2$, then the phase point $Q_2^+((1-\alpha)h_2, y_{Q_2^+})$ is obtained. Due to the field and the disjointness of any two orbits, we can see, Q_2^+ must be below S, so the successor function $f(S) = y_{Q_2^+} - y_S < 0$.

According to Lemma 2.2, an order one periodic solution of system (1.2) is existent, which the initial point is between B_1 and S in set N_2. (As shown in Figure 4(b))

Case III $(1-\beta)y_{Q_0} + \delta < \frac{r}{b}$

If point Q_0^+ is below A, that is $(1-\beta)y_{Q_0} + \delta < \frac{r}{b}$, then the successor function $f(A) = (1-\beta)y_{Q_0} + \delta - \frac{r}{b} < 0$.

On the other hand, take another point $B_1((1-\alpha)h_2, \varepsilon) \in N_2$, where $\varepsilon > 0$ is small enough. The orbit passes through point B_1 hits point $Q_1(h_2, y_{Q_1}) \in M_2$, and then jumps onto the point $Q_1^+((1-\alpha)h_2, y_{Q_1^+}) \in N_2$, because $\varepsilon > 0$ is small enough, we have $y_{Q_1^+} > \varepsilon$. Thus we have $f(B_1) = y_{Q_1^+} - \varepsilon > 0$.

According to Lemma 2.2, the order one periodic solution of system(1.2) is existent, which the initial point is between B_1 and A in set N_2. (As shown in Figure 4(c))

Case IV $y_S < (1-\beta)y_{Q_0} + \delta$

Supposing point Q_0^+ is above S, we consider the following two cases:

(i) If $y_S \geq y_{Q_2^+}$, then point Q_2^+ is below point S, thus we obtain $f(S) = y_{Q_2^+} - y_S < 0$. Thus the order one periodic solution of system (1.2) is existent, which the initial point is between point B_1 and point S in set N_2. (As shown in Figure 4(d))

(ii) If $y_S < y_{Q_2^+}$, then Q_2^+ is above the point S. By the vector field of system (1.2), we can see the orbit of system (1.2) with any initiating point on the N_2 will ultimately stay in $\Omega_1 = \{(x,y)|0 \leq x \leq h_1, y \geq 0\}$ after one effect of impulse. (As shown in Figure 4(e))

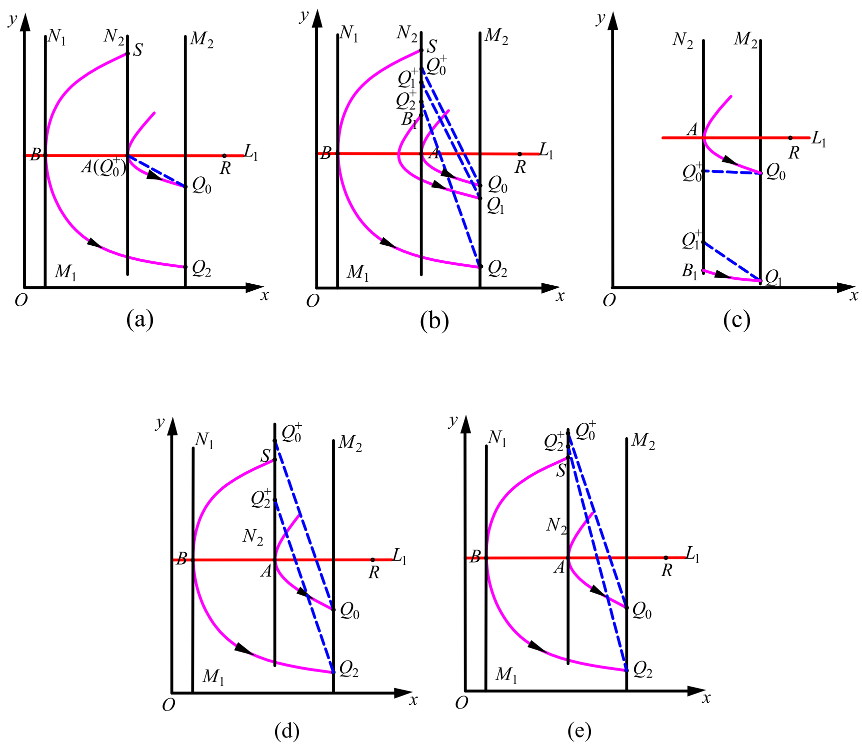

Figure 4. The orbit starting from the phase set N_2 (Case I, Case II, Case III, Case IV(i) and Case IV(ii) in Section 3.2).

According to the above analysis results, we get the following theorem.

Theorem 3.2. *Based on the conditions $\lambda > pc$ and $0 < h_1 < h_2 < \frac{p}{b(\lambda - pc)}$, if $y_{Q_0^+} \leq y_A$, an order one periodic solution of the system (1.2) is existent; if $y_{Q_0^+} > y_A$ and $y_S > y_{Q_2^+}$, an order one periodic solution of the system (1.2) is existent; if $y_{Q_0^+} > y_A$ and $y_S < y_{Q_2^+}$, the order one periodic solution of the system (1.2) is nonexistent. The orbit of system (1.2) with any initiating point on the N_2 will ultimately stay in $\Omega_1 = \{(x,y) \mid 0 \leq x \leq h_1, y \geq 0\}$ after one effect of impulse.*

4. The stability analysis of periodic solutions

By analysis on Section 3, we discuss the stability of order one periodic solutions by the analogue of the Poincaré criterion.

4.1. The orbit starting from the phase set N_1

We assume $x = \xi(t), y = \eta(t)$ be a τ-periodic solution of system (1.2) and $\xi_1 = \xi(\tau), \eta_1 = \eta(\tau); \xi_0 = \xi(0), \eta_0 = \eta(0); \xi_1^+ = \xi(\tau^+), \eta_1^+ = \eta(\tau^+)$, then we get

$$\xi_1^+ = \xi_0 = h_1, \eta_1^+ = \eta_0 = \eta_1 + \kappa.$$

According to Lemma 2.3, let $\Phi(x,y) = x(t)(r - by(t)), \Psi(x,y) = y(t)\left(\frac{\lambda bx(t)}{1+bcx(t)} - p\right)$, $U(x,y) = 0, V(x,y) = \kappa, \Gamma(x,y) = x - h_1$. Then

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial x} = 0,\ \frac{\partial U}{\partial y} = \frac{\partial V}{\partial y} = 0,\ \frac{\partial \Gamma}{\partial x} = 1,\ \frac{\partial \Gamma}{\partial y} = 0,$$

$$\begin{aligned}\Delta_1 =& \frac{\Phi_+\left(\frac{\partial V}{\partial y}\frac{\partial \Gamma}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial \Gamma}{\partial y} + \frac{\partial \Gamma}{\partial x}\right) + \Psi_+\left(\frac{\partial U}{\partial x}\frac{\partial \Gamma}{\partial y} - \frac{\partial U}{\partial y}\frac{\partial \Gamma}{\partial x} + \frac{\partial \Gamma}{\partial y}\right)}{\Phi\frac{\partial \Gamma}{\partial x} + \Psi\frac{\partial \Gamma}{\partial y}}\\ =& \frac{\Phi(\xi_1^+, \eta_1^+)(0\times 1 - 0\times 0 + 1) + \Psi(\xi_1^+, \eta_1^+)(0\times 0 - 0\times 1 + 0)}{\Phi(\xi_1, \eta_1)\times 1 + \Psi(\xi_1, \eta_1)\times 0}\\ =& \frac{\xi_0(r - b\eta_0)}{\xi_1(r - b\eta_1)}\end{aligned}$$

and

$$\begin{aligned}\int_0^\tau \left(\frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y}\right) dt =& \int_0^\tau \left[r - by(t) + \frac{\lambda bx(t)}{1 + bcx(t)} - p\right] dt\\ =& \int_0^\tau \left[\frac{\dot{x}}{x(t)} + \frac{\dot{y}}{y(t)}\right] dt\\ =& \int_0^\tau d\ln x(t)y(t)\\ =& \ln\frac{\xi_1\eta_1}{\xi_0\eta_0}.\end{aligned}$$

Therefore,

$$\begin{aligned}\mu_2 =& \Delta_1 \exp\int_0^\tau \left[\frac{\partial \Phi}{\partial x}(\xi(t), \eta(t)) + \frac{\partial \Psi}{\partial y}(\xi(t), \eta(t))\right] dt\\ =& \frac{\xi_0(r - b\eta_0)}{\xi_1(r - b\eta_1)} \times \exp\left(\ln\frac{\xi_1\eta_1}{\xi_0\eta_0}\right)\\ =& \frac{(r - b\eta_0)(\eta_0 - \kappa)}{\eta_0[r - b(\eta_0 - \kappa)]}.\end{aligned}$$

The following theorem is obtained.

Theorem 4.1. *If*

$$\lambda > pc, h_1 < \frac{p}{b(\lambda - pc)}$$

and

$$\frac{r + b\kappa - \sqrt{r^2 + b^2\kappa^2}}{2b} < \eta_0 < \frac{r + b\kappa + \sqrt{r^2 + b^2\kappa^2}}{2b},$$

then the periodic solution of system (1.2) *is stable.*

4.2. The orbit starting from the phase set N_2

We assume $x = x(t), y = y(t)$ be a τ-periodic solution to system (1.2) and $x_1 = x(\tau), y_1 = y(\tau); x_0 = x(0), y_0 = y(0); x_1^+ = x(\tau^+), y_1^+ = y(\tau^+)$, then we get

$$x_1^+ = x_0 = (1-\alpha)h_2, y_1^+ = y_0 = (1-\beta)y_1 + \delta.$$

According to Lemma 2.3, let $\Phi(x,y) = x(t)(r - by(t)), \Psi(x,y) = y(t)\left(\frac{\lambda bx(t)}{1+bcx(t)} - p\right)$, $U(x,y) = -\alpha x, V(x,y) = -\beta y + \delta, \Gamma(x,y) = x - h_2$.

Then

$$\frac{\partial U}{\partial x} = -\alpha, \frac{\partial U}{\partial y} = 0, \ \frac{\partial V}{\partial x} = 0, \ \frac{\partial V}{\partial y} = -\beta, \ \frac{\partial \Gamma}{\partial x} = 1, \ \frac{\partial \Gamma}{\partial y} = 0,$$

$$\begin{aligned}\Delta_1 =& \frac{\Phi_+\left(\frac{\partial V}{\partial y}\frac{\partial \Gamma}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial \Gamma}{\partial y} + \frac{\partial \Gamma}{\partial x}\right) + \Psi_+\left(\frac{\partial U}{\partial x}\frac{\partial \Gamma}{\partial y} - \frac{\partial U}{\partial y}\frac{\partial \Gamma}{\partial x} + \frac{\partial \Gamma}{\partial y}\right)}{\Phi\frac{\partial \Gamma}{\partial x} + \Psi\frac{\partial \Gamma}{\partial y}}\\ =& \frac{\Phi(x_1^+, y_1^+)(-\beta \times 1 + 0 \times 0 + 1) + \Psi(x_1^+, y_1^+)(-\alpha \times 0 + 0 \times 1 + 0)}{\Phi(x_1, y_1) \times 1 + \Psi(x_1, y_1) \times 0}\\ =& \frac{(1-\beta)x_0(r - by_0)}{x_1(r - by_1)},\end{aligned}$$

and

$$\begin{aligned}\int_0^\tau \left(\frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial y}\right) dt =& \int_0^\tau \left[r - by(t) + \frac{\lambda bx(t)}{1 + bcx(t)} - p\right] dt\\ =& \int_0^\tau \left[\frac{\dot{x}}{x(t)} + \frac{\dot{y}}{y(t)}\right] dt\\ =& \int_0^\tau d\ln x(t)y(t)\\ =& \ln\frac{x_1y_1}{x_0y_0}.\end{aligned}$$

Thus,

$$\begin{aligned}\mu_2 =& \Delta_1 \exp\int_0^\tau \left[\frac{\partial \Phi}{\partial x}(\xi(t), \eta(t)) + \frac{\partial \Psi}{\partial y}(\xi(t), \eta(t))\right] dt\\ =& \frac{(1-\beta)x_0(r - by_0)}{x_1(r - by_1)} \times \exp\left(\ln\frac{x_1y_1}{x_0y_0}\right)\\ =& \frac{y_1(1-\beta)(r - by_0)}{y_0(r - by_1)}.\end{aligned}$$

The following theorem is obtained.

Theorem 4.2. *If*

$$\lambda > pc, h_2 < \frac{p}{b(\lambda - pc)}$$

and

$$\frac{\omega - \sqrt{\omega^2 - 4br\delta(1-\beta)(2-\beta)}}{2b(2-\beta)} < y_0 < \frac{\omega + \sqrt{\omega^2 - 4br\delta(1-\beta)(2-\beta)}}{2b(2-\beta)},$$

where

$$\omega = b\delta(2-\beta) + 2r(1-\beta),$$

then the periodic solution of system (1.2) *is stable.*

5. Simulations and Conclusion

We enumerate the following two examples to verify the merit of our results.

$$\begin{cases} \left.\begin{aligned} & x'(t) = x(t)(0.4 - 0.5y(t)), \\ & y'(t) = y(t)\left(\frac{0.25x(t)}{1+0.1x(t)} - 0.6\right), \end{aligned}\right\} x \neq h_1,\ h_2 \text{ or } x = h_1,\ y > y^*, \\ \left.\begin{aligned} & \triangle x(t) = 0, \\ & \triangle y(t) = \kappa, \end{aligned}\right\} \qquad x = h_1,\ y \leqslant y^*, \\ \left.\begin{aligned} & \triangle x(t) = -\alpha x(t), \\ & \triangle y(t) = -\beta y(t) + \delta, \end{aligned}\right\} x = h_2, \end{cases} \tag{5.1}$$

where $\alpha, \beta \in (0,1),\ \kappa > 0,\ \delta > 0,\ 0 < h_1 < h_2$. Next the impulsive effect is considered on the dynamics of system (5.1).

Example 5.1. Existence and stability of order one periodic solution with the orbits starting from the phase set N_1. We set $h_1 = 1,\ \kappa = 0.8$, Figure 5(a) illustrates that Theorem 3.1 hold, the order one periodic solution of system (5.1) is existent. Figures 5(b) and 5(c) are the time series of $x(t), y(t)$, respectively. This shows that system (5.1) has an stable periodic solution when the amount of the plutella xylostella population reaches the level h_1, then the conditions of Theorem 4.1 hold.

Example 5.2. Existence and stability of positive periodic solution with the orbits starting from the phase set N_2. We set $h_1 = 0.7,\ \alpha = 0.6,\ \beta = 0.8,\ \delta = 0.8,\ h_2 = 3.5$, Figure 6(a) illustrates that Theorem 3.2 hold, the order one periodic solution of system (5.1) is existent. Figures 6(b) and 6(c) are the time series of $x(t), y(t)$, respectively. This illustrates that system (5.1) has an stable periodic solution when the amount of the plutella xylostella population reaches the level h_2. Therefore the conditions of Theorem 4.2 hold.

In this paper, using the method of successive function and geometric analysis theory, there exists order one periodic solution for system (1.2) under impulsive effects, and further using the analogue of Poincaré criterion to prove that the periodic solution is stable. The system (1.2) has a wider range of application than the

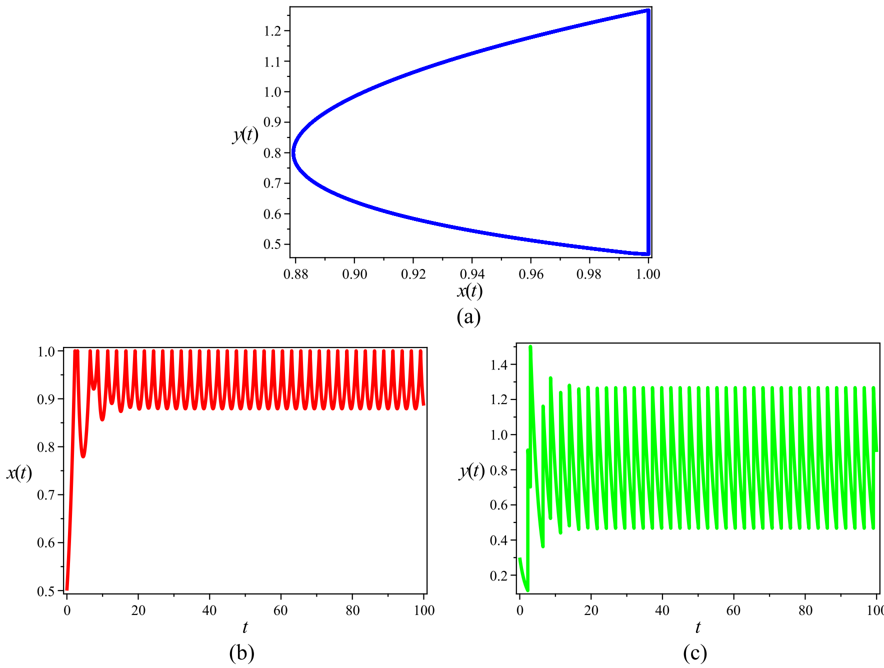

Figure 5. Description of behavior of periodic solutions of the system (5.1). (a) Existence of order one periodic solution corresponding to Theorem 3.1. (b) Time series of $x(t)$. (c) Time series of $y(t)$.

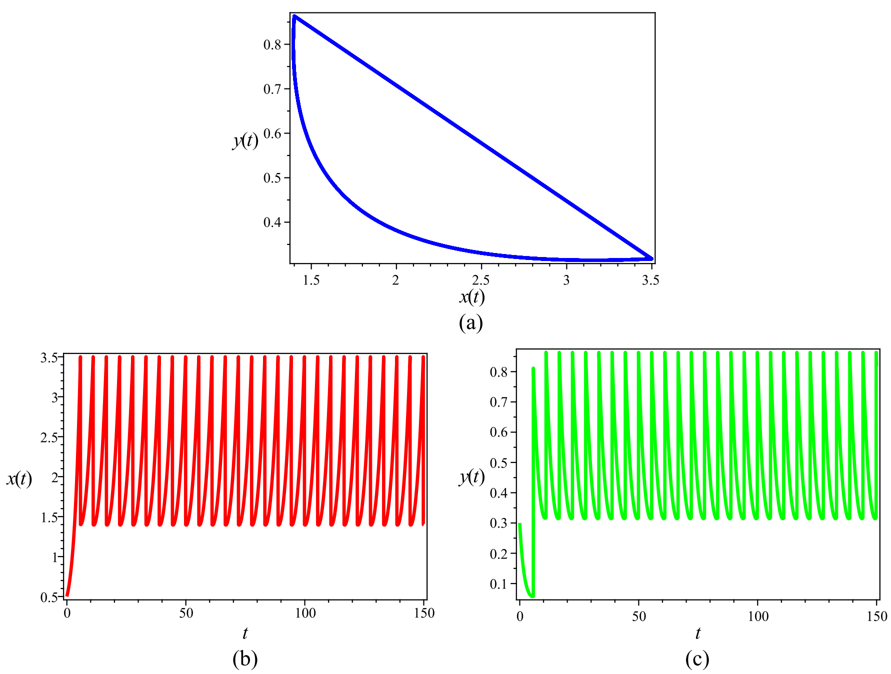

Figure 6. Description of behavior of periodic solutions of the system (5.1). (a) Existence of order one periodic solution corresponding to Theorem 3.2. (b) Time series of $x(t)$. (c) Time series of $y(t)$.

conditions given by the [15]. From the research results and numerical simulation, we can control the number of plutella xylostella is lower than its economic threshold by applying impulsive effects once, twice, or a finite number of times. By using the biological and chemical comprehensive control method, it greatly improves the crop yield. And the method of theorems is more effective and easier to operate than [8, 9, 11, 15, 18], so they are worthy of further promotion.

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