THE EFFECT OF MASS TRANSFER ON MHD FREE CONVECTIVE RADIATING FLOW OVER AN IMPULSIVELY STARTED VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM

Bharat Keshari Swain 1,†, and Nityananda Senapati 1

Abstract The laminar convective heat and mass transfer flow of an incompressible, viscous, electrically conducting fluid over an impulsively started vertical plate with conduction-radiation embedded in a porous medium in presence of transverse magnetic field has been studied. An exact solution is derived by solving the dimensionless governing coupled partial differential equations. As the equations are nonlinear, so Laplace transform technique is used to solve it. The effects of important physical parameters on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number have been analyzed through graphs. The results of the present study agree well with the previous solutions obtained without mass transfer. After the consideration of mass transfer, some different results are noticed. Applications of the present study arise in material processing systems and different industries.

Keywords MHD, Porosity, Thermal radiation, shear stress, Nusselt number, Sherwood number.

MSC(2010) 76D, 76W, 76S.

1. Introduction

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$C^*$</td>
<td>Species concentration ($kgm^{-3}$),</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure ($Jkg^{-1}K$),</td>
</tr>
<tr>
<td>$C_{\infty}$</td>
<td>Species concentration in the free stream ($kgm^{-3}$),</td>
</tr>
<tr>
<td>$C_w$</td>
<td>Species concentration at the surface ($kgm^{-3}$),</td>
</tr>
<tr>
<td>$D$</td>
<td>Chemical molecular diffusivity ($m^2s^{-1}$),</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity ($ms^{-2}$),</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Radiative heat flux,</td>
</tr>
<tr>
<td>$G_r$</td>
<td>Thermal Grashof number,</td>
</tr>
<tr>
<td>$G_m$</td>
<td>Mass Grashof number,</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Permeability parameter,</td>
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<tr>
<td>$M$</td>
<td>Hartmann number,</td>
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<tr>
<td>$N_u$</td>
<td>Nusselt number,</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Prandtl number,</td>
</tr>
<tr>
<td>$S_h$</td>
<td>Sherwood number,</td>
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Many transport processes exist in industries and technology where the transfer of heat and mass occurs simultaneously as a result of thermal diffusion and diffusion of chemical species. Natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial applications such as meteorology, chemical industry, cooling of nuclear reactors, cosmic fluid dynamics, magneto hydrodynamics power generators and in the earths core. Many studies that considered combined heat and mass transfer in natural convection boundary layer flows over heated surfaces with various geometries can be found in the monograph by Gebhart et.al [10].

The subject of magneto hydrodynamics has attracted the attention of a large number of scholars due to its diverse applications in several problems of technological importance. Heat transfer by thermal radiation is becoming of great importance when we are concerned with space applications, higher operating temperatures and also power engineering. In astrophysics and geophysics, it is mainly applied to study the stellar and solar structures, radio propagation through the ionosphere etc. Recently it is of great interest to study the effects of magnetic field and other participating parameters on heat and mass transfer in porous medium when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. Ganeswara Reddy M and Bhaskar Reddy N [13] studied mass transfer and heat generation effects on MHD free convection flow in a porous medium. B Shankar et.al [15] investigated radiation and mass transfer effects on unsteady MHD free convective fluid flow in a porous medium with heat generation. Ahmed et.al [3] studied the finite difference approach in porous media transport modelling for Magneto hydrodynamic unsteady flow over a vertical plate. Ahmed et.al [4] gave an exact solution to the Hartmann newtonian radiating MHD flow for a rotating vertical porous channel immersed in a Darcian porous regime.

Extensive research work has been published on an impulsively started vertical plate with of course under different boundary conditions depending upon the physics of the problem. An exact solution to the Navier-Stokes equation of the flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate moving in its own plane was first examined by Stokes [1] which is being often referred as Rayleigh’s problem in the literature and has inspired several investigators. For the first time, the exact solution of the Stokes problem for the case of infinite vertical

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$T_w^*$</td>
<td>Fluid temperature at the surface (K),</td>
</tr>
<tr>
<td>$T_\infty^*$</td>
<td>Fluid temperature in the free stream (K),</td>
</tr>
<tr>
<td>$u$</td>
<td>Dimensionless velocity component (ms$^{-1}$),</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Schmidt number</td>
</tr>
<tr>
<td>$T^*$</td>
<td>Temperature (K),</td>
</tr>
<tr>
<td>$u_0$</td>
<td>plate velocity (ms$^{-1}$),</td>
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</table>

Greek symbols:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Coefficient of volume expansion for heat transfer(K$^{-1}$),</td>
</tr>
<tr>
<td>$B_c$</td>
<td>Coefficient of volume expansion for mass transfer(K$^{-1}$),</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Dimensionless fluid temperature (K),</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity (Wm$^{-1}$K$^{-1}$),</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity (m$^2$s$^{-1}$),</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density (kgm$^{-3}$),</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity,</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shearing stress (Nm$^{-2}$)</td>
</tr>
<tr>
<td>$C$</td>
<td>Dimensionless species concentration (kgm$^{-3}$),</td>
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</table>

Subscripts:

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<thead>
<tr>
<th>Subscript</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>Conditions on the wall,</td>
</tr>
<tr>
<td>$\infty$</td>
<td>Free stream condition</td>
</tr>
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</table>

Here it is proposed to study the effect of mass transfer on MHD convective laminar radiating flow over an impulsively started vertical plate embedded in a porous medium. The magnetic field is imposed transversely to the plate. The whole system is solved by Laplace transform method to obtain an exact solution.

2. Mathematical Formulation

The laminar convective heat and mass transfer flow of an incompressible, viscous, electrically conducting fluid over an impulsively started vertical plate with conduction-radiation embedded in a porous medium in presence of transverse magnetic field has been studied. The \( x^{*} \) axis is taken along the plate in the vertical upward direction and the \( y^{*} \) axis is taken normal to the plate. A transverse magnetic field of uniform strength \( B_{0} \) is assumed to be applied normal to the plate. It is also assumed that the thermal radiation along the plate is negligible as compared to that in the normal direction. The induced magnetic field and viscous dissipation is assumed to be negligible. Initially it is assumed that the plate and fluid are at the same temperature \( T_{\infty}^{*} \) in the stationary condition with concentration level \( C_{\infty}^{*} \) at all the points. At time, \( t^{*} > 0 \) the plate is given an impulsive motion in its own plane with velocity \( u_{0} \). The temperature of the plate and the concentration level are also raised to \( T_{w}^{*} \) and \( C_{w}^{*} \). They are maintained at the same level for all time \( t^{*} > 0 \). Then under the above assumption the unsteady flow with usual Boussinesq’s approximation is governed by the following equations

\[
\frac{\partial u^{*}}{\partial t^{*}} = g\beta (T^{*} - T_{\infty}^{*}) + g\beta_{c} (C^{*} - C_{\infty}^{*}) + \nu \frac{\partial^{2} u^{*}}{\partial y^{*2}} - \left( \frac{\sigma B_{0}^{2}}{\rho} + \frac{\nu}{K^{*}} \right) u^{*}, \tag{2.1}
\]

\[
\rho C_{p} \frac{\partial T^{*}}{\partial t^{*}} = k \frac{\partial^{2} T^{*}}{\partial y^{*2}} - \frac{\partial q_{r}}{\partial y^{*}}, \tag{2.2}
\]
\[
\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}}. \tag{2.3}
\]

The initial and boundary conditions are
\[
t^* \leq 0: \ u^* = 0, T^* = T^*_\infty, C^* = C^*_\infty \quad \text{for every } y,
\]
\[
t^* > 0: \ u^* = 0, T^* = T^*_W, C^* = C^*_W \quad \text{at } y = 0,
\]
\[
t^* > 0: \ u^* \to 0, T^* \to T^*_\infty, C^* \to C^*_\infty \quad \text{at } y \to \infty. \tag{2.4}
\]

The radiative heat flux term is simplified by making use of the Rosseland approximation \[16\] as
\[
q_r = -\frac{4}{3} \frac{\sigma^*}{a^*} \frac{\partial T^*_4}{\partial y^*}, \tag{2.5}
\]
where \(\sigma^*\) and \(a^*\) are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. It should be noted that by using the Rosseland approximation, we limit our analysis to optically thik fluids. If temperature differences within the flow are sufficiently small, such that \(T^*_4\) may be expressed as a linear function of the temperature, then the Taylor series for \(T^*_4\) about \(T^*_\infty\), after neglecting higher order terms, is given by
\[
T^*_4 \approx 4 \frac{T^*}{T^*_\infty} - 3 \frac{T^*_\infty}{4}.
\tag{2.6}
\]
Substituting (2.5) and (2.6) in (2.2) we have
\[
\rho C_p \frac{\partial T^*}{\partial t^*} = \left[ k + \frac{16}{3} \frac{\sigma^*}{a^*} T^*_\infty \right] \frac{\partial^2 T^*}{\partial y^{*2}}. \tag{2.7}
\]

Let us introduce the following non dimensional terms in (2.1), (2.7) and (2.3).
\[
y = \frac{u_0 y^*}{\nu}, u = \frac{u^*}{u_0}, P_r = \frac{\rho \nu C_p}{\nu^3}, S_c = \frac{\nu}{D}, t = \frac{u_0^2 t^*}{\nu}, K_r = \frac{u_0^2 K^*}{\nu^2},
\]
\[
\theta = \frac{T^* - T^*_\infty}{T^*_W - T^*_\infty}, C = \frac{C^* - C^*_\infty}{C^*_W - C^*_\infty}, M = \frac{\sigma B_2^* \nu}{\rho u_0^2}, N_a = \frac{ka^*}{4\sigma^* T^*_\infty^3},
\]
\[
G_r = \frac{\nu g \beta (T^*_W - T^*_\infty)}{u_0^3}, G_m = \frac{\nu g \beta_c (C^*_W - C^*_\infty)}{u_0^3}. \tag{2.8}
\]

Hence the non dimensional form of (2.1), (2.7) and (2.3) are
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left( M + K_r^{-1} \right) u + G_r \theta + G_m C, \tag{2.9}
\]
\[
3 N_a P_r \frac{\partial \theta}{\partial t} = \left( 3 N_a + 4 \right) \frac{\partial^2 \theta}{\partial y^2}, \tag{2.10}
\]
\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2}. \tag{2.11}
\]

The transformed initial and boundary conditions are
\[
t \leq 0: \ u = 0, \theta = 0, C = 0 \quad \text{for every } y,
\]
\[
t > 0: \ u = 1, \theta = 1, C = 1 \quad \text{at } y = 0,
\]
\[
t > 0: \ u \to 0, \theta \to 0, C \to 0 \quad \text{as } y \to \infty. \tag{2.12}
\]
3. Method of Solution

The equations (2.9) to (2.11) are nonlinear, coupled partial differential equations. So we want to solve them by using Laplace transform technique. Taking Laplace Transform, the equations (2.9), (2.10) and (2.11) reduce to

\[
\frac{\partial^2 \bar{u}}{\partial y^2} - \left( N + s \right) \bar{u} + G_r \bar{\theta} + G_m \bar{C} = 0, \tag{3.1}
\]

\[
\frac{\partial^2 \bar{\theta}}{\partial y^2} - As \bar{\theta} = 0, \tag{3.2}
\]

\[
\frac{\partial^2 \bar{C}}{\partial y^2} - sS_c \bar{C} = 0, \tag{3.3}
\]

where \( s \) is Laplace transform parameter.

The boundary conditions (2.12) reduce to

\[
\bar{u} = \frac{1}{s}, \bar{\theta} = \frac{1}{s}, \bar{C} = \frac{1}{s} \quad \text{when} \quad y = 0,
\]

\[
\bar{u} = 0, \bar{\theta} = 0, \bar{C} = 0 \quad \text{when} \quad y \to \infty. \tag{3.4}
\]

Solving (3.1), (3.2) and (3.3) with boundary conditions (3.4) we get

\[
\bar{\theta} = \frac{1}{s} e^{-\sqrt{s}xy}, \tag{3.5}
\]

\[
\bar{C} = \frac{1}{s} e^{-\sqrt{s}xy}, \tag{3.6}
\]

\[
\bar{u} = \left[ \frac{1}{s} \left( \frac{G_r}{As - (N + s)} + \frac{G_m}{S_c s - (N + s)} \right) \right] e^{-\sqrt{s}xy} - \frac{1}{s} \left( \frac{G_r e^{-\sqrt{s}xy}}{As - (N + s)} + \frac{G_m e^{-\sqrt{s}xy}}{S_c s - (N + s)} \right). \tag{3.7}
\]

Inverting the equations (3.5), (3.6) and (3.7) we get

\[
\theta = \text{erfc}(\eta \sqrt{A}), \tag{3.8}
\]

\[
C = \text{erfc}(\eta \sqrt{S_c}), \tag{3.9}
\]

\[
u = f_1 e^{-2\eta \sqrt{Nt}} \text{erfc}(\eta - \sqrt{Nt}) + e^{2\eta \sqrt{Nt}} \text{erfc}(\eta + \sqrt{Nt}) + f_2 e^{-\eta \sqrt{Mt}} \text{erfc}(\eta - \sqrt{Mt}) + e^{\eta \sqrt{Mt}} \text{erfc}(\eta + \sqrt{Mt}) + f_3 e^{-\eta \sqrt{M \eta}} \text{erfc}(\eta - \sqrt{M \eta}) + e^{\eta \sqrt{M \eta}} \text{erfc}(\eta + \sqrt{M \eta}) \]

\[
+ f_4 \text{erfc}(\eta \sqrt{A}) + f_5 \text{erfc}(\eta \sqrt{S_c}), \tag{3.10}
\]

where \( b = \frac{N}{A^{1/2}}, N = M + \frac{1}{K_r}, A = \frac{3N_a P_r}{3N_a P_r + 1}, \eta = \frac{y}{2\sqrt{t}}, f_1 = \frac{1}{2} - \frac{1}{2} \left( \frac{G_r + G_m}{N} \right), f_2 = \frac{1}{2} \left( \frac{G_r}{N} \right) \right) e^{bt}, f_3 = \frac{1}{2} \left( \frac{G_m}{N} \right) \right) e^{bt}, f_4 = \frac{G_r}{N}, f_5 = \frac{G_m}{N}. \)
The Skinfriction at the surface of the plate is given by

\[ \tau = -\left[ \frac{\partial u(y,t)}{\partial y} \right]_{y=0} = -\frac{1}{2\sqrt{\nu}} \left[ \frac{\partial u(y,t)}{\partial \eta} \right]_{\eta=0} \]

\[ = \frac{1}{2\sqrt{\nu}} \left[ f_1(4\sqrt{N_t}e^{-N_t}) + \frac{4}{\sqrt{\pi}}e^{-N_t} + f_2(4\sqrt{A_b}e^{-t} + 4\sqrt{\nu}e^{-Abt}) \right. \]

\[ + f_3(4\sqrt{S_c}e^{-S_c}) - f_2(4\sqrt{Ab}e^{-b} + 4\sqrt{\nu}e^{-bt}) \]

\[ - f_3(4\sqrt{S_c}e^{-S_c}) + \frac{4}{\sqrt{\pi}}(S_c) + f_4(2\sqrt{A}) + f_5(2\sqrt{S_c}) \]. \quad (3.11)\]

The Nusselt number and the Sherwood number at the plate are respectively

\[ N_u = -\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = \sqrt{\frac{A}{\pi t}}, \]

\[ \text{and} \quad S_h = -\left( \frac{\partial C}{\partial y} \right)_{y=0} = \sqrt{\frac{S_c}{\pi t}}. \quad (3.12)\]

4. Results and Discussion

To discuss the physical significance of various parameters involved in the results (3.8) to (3.12), the numerical calculations have been carried out. Our results are found in agreement with the results of Ahmed and Batin [14] in the absence of the mass transfer parameter. The effects of the various parameters entering in the governing equations on the velocity, temperature, skin friction, Nusselt number and Sherwood number are shown through graphs.

Figure 1: In this figure, the effects of \( M, G_r \) and \( G_m \) on the velocity are noticed, while other parameters are kept constant. The Grashof number \( G_r \) signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force. It is noticed that when thermal buoyancy force dominates over viscous force then velocity increases. This result agrees with the result of Ahmed and Batin [14]. Again Modified Grashof number \( G_m \) defines the ratio of the species buoyancy force to the viscous hydrodynamic force. The fluid velocity increases due to increase in the species buoyancy force relative to viscous hydrodynamic force. Also it is observed that as the magnetic parameter \( M \) increases velocity decreases.

Figure 2: The velocity profiles for different values of \( Pr, t \) and \( K_r \) are plotted in this figure. It is found that the velocity increases with increasing values of \( K_r \). This signifies that the increasing values of \( K_r \) reduce the drag force which assists the fluid considerably to move fast. Figure shows that the increasing values of \( Pr \) result in a decreasing velocity. Again for increasing values of time parameter \( t \), velocity increases.

Figure 3: Here it is noticed that velocity decreases with increase in radiation parameter \( N_a \). This is also in accordance with the previous result of Ahmed and Batin [14] (i.e. flow without mass transfer). Again the effect of \( Sc \) on velocity is same as \( N_a \).

Figure 4: Here it is noticed that both of \( S_c \) and \( t \) have the same effect on the concentration. As these parameters increase, concentration increases.
Figure 5. This figure illustrates that the temperature rises for decreasing values of $P_r$. On the other hand the parameters $t$ and $K_r$ are directly proportional to the temperature.

Figure 6. It reveals that the effect of parameters $N_a$ and $S_c$ on skin friction show quite opposite to that on the velocity of the fluid. But parameter $K_r$ has same effect on velocity and skin friction.

Figure 7. This figure shows that the skin friction increases as parameters $G_r$ and $G_m$ are increased. But parameter $M$ has opposite effect on skin friction.

Figure 8. The Nusselt number is found to increase with the increasing value of the Prandtl number. The same trend is also noticed for the parameter $N_a$.

Figure 9. Interestingly this figure shows that the Schmidt number has same effect on Sherwood number as it shows for concentration i.e. the Sherwood number increases with the increasing values of $S_c$. 

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure1.png}
\caption{Velocity profiles for $M,G_r$ and $G_m$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure2.png}
\caption{Velocity profiles for $P_r,t$ and $K_r$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure3.png}
\caption{Velocity profiles for $N_a$ and $S_c$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure4.png}
\caption{Concentration profiles for $S_c$ and $t$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure5.png}
\caption{Temperature profiles for $P_r,N_a$ and $t$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{Figure6.png}
\caption{Skin friction profiles for $N_a,K_r$ and $S_c$.}
\end{figure}
5. Conclusion

In this paper, the problem of convective laminar radiating flow with mass transfer over an impulsively started vertical plate embedded in a porous medium in the presence of magnetic field was analyzed. The governing system of equations was solved analytically with the help of Laplace transform method. The effects of the parameters on the velocity, concentration, temperature skin friction, Sherwood number were studied in details.

Some important conclusions are summarized as follows.

- The flow is generally accelerated with the increase of porosity parameter ($K_r$).
- Velocity and temperature are decreased with increasing values of Prandtl number ($Pr$) and radiation parameter $Na$.
- Schmidt number and time parameter have same effects on concentration. When these parameters increase, concentration decreases.
- With increasing values of convection parameter $Gr$ and modified Grashof number ($G_m$), the flow velocity is accelerated. But $M$ shows opposite effects.
- Increasing radiation parameter and Schmidt number serves to depress shear stress.

References


