Journal of Applied Analysis and Computation
Volume 4, Number 4, November 2014
Website:http://jaac-online.com/
pp. 367–381

EFFECT OF CHEMICAL REACTION ON MHD FLOW OF A VISCO-ELASTIC FLUID THROUGH POROUS MEDIUM

M.K. Nayak[1,†], G.C. Dash[2] and L.P. Singh[3]

Abstract The effect of heat and mass transfer on free convective flow of a visco-elastic incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependant oscillatory permeability and suction in the presence of a uniform transverse magnetic field, heat source and chemical reaction has been studied in this paper. The novelty of the present study is to analyze the effect of chemical reaction, time dependant fluctuative suction and permeability of the medium on a visco-elastic fluid flow. It is interesting to note that presence of sink contributes to oscillatory motion leading to flow instability. Further it is remarked that presence of heat source and low rate of thermal diffusion counteract each other in the presence of reacting species.

Keywords MHD flow, Visco-elastic, mass and heat transfer, porous medium, chemical reaction.

MSC(2000) 76D10, 76S05, 76W05.

1. Introduction

MHD flow with heat and mass transfer has been a subject of interest of many researchers because of its varied applications in science and technology. Such phenomenon is observed in buoyancy induced motions in the atmosphere, water bodies,quasi -solid bodies such as earth, etc. An important class of two dimensional time-dependant flow problem dealing with the response of boundary layer to external unsteady fluctuations of the free stream velocity about a mean value attracted the attention of many researchers. In natural processes and industrial applications many transportation processes exist where transfer of heat and mass takes place simultaneously as a result of the thermal diffusion and diffusion of chemical species. There are many transport processes that are governed by the combined action of buoyancy forces due to both thermal and mass diffusion in the presence of chemical reaction. These processes are observed in the nuclear reactor safety and combustion systems, solar collectors, metallurgical and chemical engineering.

MHD convection flow problems are very significant in the fields of steller and planetary magnetospheres, aeronautics, chemical engineering and electronics. Model studies of the above phenomena of MHD convection have been made by many

[†]the corresponding author.Email address:mnkec1973@gmail.com(M.K. Nayak)
[1]Department of Physics, Radhakrishna Institute of Technology and Engineering, Biju Patnaik University of Technology, Odisha, India
[2]Department of Mathematics, I.T.E.R, Siksha 'O' Anusandhan University, Odisha, India
[3]Department of Physics, Utkal University, Odisha, India

researchers namely Bejan & Khair [6], Trevisian & Bejan [32], Acharya et al. [1], Raptis & Kafousias [25] and Ahmed et al. [2]. Singh & Singh [27] investigated the effect of mass transfer on MHD flow considering constant heat flux and induced magnetic field. Ahmed [3] studied MHD free and forced convection with mass transfer from an infinite vertical porous plate.

Several researchers have analyzed the free convective and mass transfer flow of a viscous fluid through porous medium. The permeability of the porous medium is assumed to be constant while the porosity of the medium may not be necessarily being constant. Kim [17] studied the unsteady MHD convective heat past a semi-infinite vertical porous moving plate with variable suction. The problem of three dimensional free convective flow and heat transfer through porous medium with periodic permeability has been discussed by Singh & Sharma [28]. Singh & Singh. [29] have analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Bathul [7] discussed the heat transfer in a three dimensional viscous flow over a porous plate moving with harmonic disturbance. Postelnicu [24] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

Nomenclature

C'	Species concentration	C	Non-dimensional species conc.
D	Molecular diffusivity	G_c	Grashof number for mass transfer
G_r	Grashof number for heat transfer	g	Acceleration due to gravity
K'_p	Permeability of the medium	K_p	Permeability parameter
k	Thermal diffusivity	M	Magnetic parameter
R_c	Elastic parameter	B_0	Magnetic field of uniform strength
N_u	Nusselt number	P_r	Prandtl number
S	Heat source parameter	S_c	Schmidt number
S_h	Sherwood number	T'	Temperature of the fluid
T	Non-dimensional temperature	t'	Time
t	Non-dimensional time	u'	Velocity component along x-axis
u	Non- dimensional velocity	$v(t')$	Suction velocity
V_0	Constant suction velocity	y'	Distance along y-axis
y	Non-dimensional distance along y-axis	K_c	Chemical reaction parameter
ε	A small positive constant	ρ	Density of the fluid
β^*	Vol. coef. of exp. for heat transfer	μ	Viscosity
β'	Vol. coef. of exp. with species conc.	τ	Skin friction
ν	Kinematic coefficient of viscosity	ω'	Frequency of oscillation
σ	Electrical conductivity	w	Condition on porous plate
ω	Non-dimensional freq. of oscillation	k_0	viscoelastic parameter

Singh & Gupta [30] have investigated the MHD free convective flow of a viscous fluid through a porous medium bounded by oscillatory porous plate in slip flow regime with mass transfer. Ogulu & Prakash [23] considered heat transfer to unsteady magneto hydrodynamic flow past an infinite vertical moving plate with variable suction. Das et al. [12] analyzed the mass transfer effect on unsteady flow past an accelerated vertical porous plate with suction employing numerical methods.

The basic equations of incompressible MHD flow are non-linear. No fluid is incompressible but all may be treated as such whenever the pressure changes are small in comparison with the bulk modulus. Ferdows et al. [15] analyzed free convection flow with variable suction in presence of thermal radiation. Alam et al. [4] studied Dufour and Soret effect with variable suction on unsteady MHD free convective flow along a porous plate.

Mishra et al. [19] have studied fluctuating flow of a non-Newtonian fluid past a porous flat plate with time varying suction. Majumdar & Deka [20] gave an exact solution for MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Muthucumaraswamy et al. [21] studied unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Asghar et al. [5] have reported the flow of a non-Newtonian fluid induced due to oscillations of a porous plate. Moreover, Dash et al. [13] have studied free convective MHD flow of a visco-elastic fluid past an infinite vertical porous plate in a rotating frame of reference in the presence of chemical reaction. The specific area of application of heat and mass transfer phenomena is common in chemical process industries such as food processing and polymer production. Recently, Gireesh Kumar & Satyanarayana [18], Sivraj & Rushi Kumar [31] have considered MHD flow of visco-elastic fluid with short memory (Walters B' model).

An important aspect of the present study is the oscillating visco-elastic fluid flow. There are many natural phenomena such that when a fluid is set into oscillation, as in the presence of an accoustic wave or an oscillating boundary, steady streaming boundary motions are created. These steady streaming belong to a class of secondary flows. For a purely viscous liquid this kind of phenomenon has been reported over a century ago. Further, the steady streaming has been considered both theoretically and experimentally by many authors. An extensive list of more significant contribution has been given by Riley [26].

Inspite of the numerous investigations of the steady streaming phenomenon in purely viscous flow, there are comparatively a few investigations of this kind for elastico-viscous flows. Frater [16] has considered the effect of the elasticity of the fluid on the steady streaming produced by an oscillating cylinder. Chang et al. [8–10] have experimentally confirmed that the addition of small amount of polymer solution to a Newtonian fluid can significantly reverse the secondary flow. Chang [11] also made a theoretical study of the problem using Walters liquid B' as model for visco-elastic liquid. However, he has erroneously omitted the term $-\frac{\partial^3 u}{\partial t \partial y^2}$ in the boundary layer equation. The study of oscillatory flow of visco-elastic fluid with chemical reaction is still less in number.

Recently, Das et al. [14] and Mishra et al. [22] have studied the effect of heat and mass transfer on free convective flow of viscous and visco-elastic incompressible electrically conducting fluid past a vertical porous plate. They have not considered the chemical reaction in their studies.

Therefore, the objective of the present study is to analyze the effects of variable permeability and oscillatory suction velocity on free convective mass transfer flow of a visco-elastic fluid (Walters B' model) past an infinite vertical porous plate through a porous medium in presence of a uniform transverse magnetic field, heat source and chemical reaction.

The present study is likely to have bearing on the geothermal problem of ground water flowing through porous media, earth's surface being treated with chemical fertilizers, problem of salt water encroachment of coastal aquifers or in the process of oil extrusion. Also porous media are very widely used for a heated body to maintain its temperature.

2. Formulation of the problem

The unsteady free convective flow of a visco-elastic fluid (Walters B' model) past an infinite vertical porous plate in a porous medium with time dependant oscillatory suction as well as permeability in presence of a transverse magnetic field is considered. Let x'-axis be along the plate in the direction of the flow and y'-axis normal to it. Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t' \leq 0$, the plate as well as fluid are at the same temperature and also concentration of the species is very low so that the Soret and Dufour effects are neglected. When $t' > 0$, the temperature of the plate is instantaneously raised to T'_w and the concentration of the species is set to C'_w.

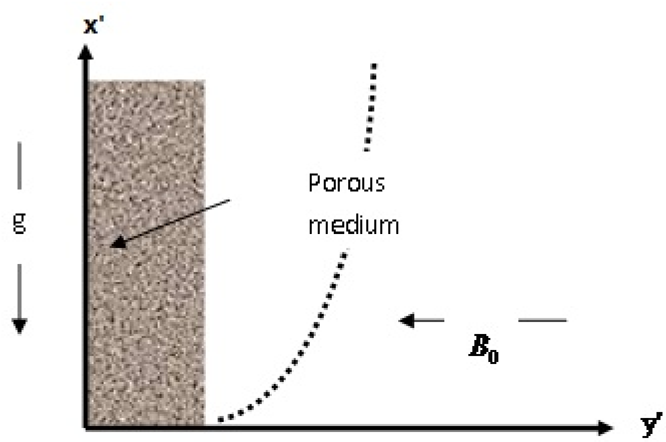

Figure 1. Flow geometry

Let the permeability of the porous medium and the suction velocity be of the form

$$K_p(t') = K'_p(1 + \varepsilon e^{i\omega' t'}) \tag{2.1}$$

and

$$v(t') = -v_0(1 + \varepsilon e^{i\omega' t'}), \tag{2.2}$$

where $v_0 > 0$ and $\varepsilon << 1$ are positive constants. Under the above assumption with usual Boussinesq's approximation, the governing equations and boundary conditions are given by

$$\rho\left(\frac{\partial u'}{\partial t'}+v\frac{\partial u'}{\partial y'}\right)=\mu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta(T'-T_\infty)+\rho g\beta^*(C'-C_\infty)-\sigma B_0^2 u' \\ -\frac{\mu u'}{K_p'(1+\varepsilon e^{i\omega' t'})}-k_0\left(\frac{\partial^3 u'}{\partial t\partial y'^2}+v\frac{\partial^3 u'}{\partial y'^3}\right), \tag{2.3}$$

$$\frac{\partial T'}{\partial t'}+v\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+S'(T'-T_\infty), \tag{2.4}$$

$$\frac{\partial C'}{\partial t'}+v\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K_c'(C'-C_\infty), \tag{2.5}$$

$$u=0,\ T'=T_\infty+\varepsilon(T_w-T_\infty)e^{i\omega' t'},\ C=C_\infty+\varepsilon(C_w-C_\infty)e^{i\omega' t'}\ \text{at } y=0, \\ u\to 0,\ T'\to T_\infty,\ C'\to C_\infty,\ \text{as } y\to\infty. \tag{2.6}$$

In boundary layer approximation we have assumed μ and k_0 to be of same order of magnitude. This will ensure that the viscosity and elasticity effects are of equal importance in determining the flow characteristics.

Introducing the non-dimensional quantities,

$$y=\frac{v_0 y'}{\nu},\ t=\frac{v_0^2 t'}{4\nu},\ \omega=\frac{4\nu\omega'}{v_0^2},u=\frac{u'}{v_0},\ T=\frac{T'-T_\infty}{T_w-T_\infty},\ C=\frac{C'-C_\infty}{C_w-T_\infty}, \\ S=\frac{\nu S'}{v_0^2},\ K_p=\frac{v_0^2 K_p'}{\nu^2},\ M^2=\frac{\sigma B_0^2}{\rho}\frac{\nu}{v_0^2},\ P_r=\frac{\nu}{k},\ K_c=\frac{\nu K_c'}{v_0^2}, \\ G_c=\frac{\nu g\beta'(C_w-C_\infty)}{v_0^3},\ G_r=\frac{\nu g\beta'(T_w-T_\infty)}{v_0^3},\ S_c=\frac{\nu}{D},\ R_c=\frac{k_0 v_0^2}{4\rho\nu^2}. \tag{2.7}$$

The equations (2.3), (2.4), (2.5) and (2.6) reduce to following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+G_r T+G_c C-\frac{u}{K_p(1+\varepsilon e^{i\omega t})}-M^2 u \\ -R_c\left(\frac{\partial^3 u}{\partial t\partial y^2}-(1+\varepsilon e^{i\omega t})\frac{\partial^3 u}{\partial y^3}\right), \tag{2.8}$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}+ST, \tag{2.9}$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_c C, \tag{2.10}$$

$$u=0,\ T=1+\varepsilon e^{i\omega t},\ C=1+\varepsilon e^{i\omega t}\qquad \text{at } y=0, \\ u\to 0,\ T\to 0,\ C\to 0,\qquad \text{as } y\to\infty. \tag{2.11}$$

The reduced equations in dimensionless form involve ε and R_c. We shall consider ε to be small i.e. we restrict our discussion to the case of small amplitude of oscillation and to be order of one that is, elastic effects are of same order of importance as the viscous and unsteady inertia effects.

3. Method of solution

In view of periodic suction, temperature and concentration at the plate, let the velocity, temperature, concentration in the neighbourhood of the plate be

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t}, \tag{3.1}$$
$$T(y,t) = T_0(y) + \varepsilon T_1(y)e^{i\omega t}, \tag{3.2}$$
$$C(y,t) = C_0(y) + \varepsilon C_1(y)e^{i\omega t}. \tag{3.3}$$

The above method of solution has been adopted by Mishra et al. [22], Das et al. [14] and Gireesh Kumar and Satyanarayana [18] to solve the periodically fluctuating flow problems.

Substituting equations (3.1)-(3.3) into (2.8)-(2.10) and equating the non-harmonic (coefficient of ε^0) and harmonic (coefficient of ε) terms, we get

$$R_c u_0''' + u_0'' + u_0' - \left(M^2 + \frac{1}{K_p}\right) u_0 = -G_r T_0 - G_c C_0, \tag{3.4}$$

$$R_c u_1''' + (1 + R_c i\omega) u_1'' + u_1' - \left(M^2 + \frac{1}{K_p} + \frac{i\omega}{4}\right) u_1$$
$$= -R_c u_0''' - u_0' - G_r T_1 - G_c C_1 - \frac{u_0}{K_p}, \tag{3.5}$$

$$T_0'' + P_r T_0' + P_r S T_0 = 0, \tag{3.6}$$

$$T_1'' + P_r T_1' + P_r \left(S - \frac{i\omega}{4}\right) T_1 = -P_r T_0', \tag{3.7}$$

$$C_0'' + S_c C_0' - S_c K_c C_0 = 0, \tag{3.8}$$

$$C_1'' + S_c C_1' - \left(\frac{i\omega}{4} + K_c\right) S_c C_1 = -S_c C_0'. \tag{3.9}$$

The boundary conditions now reduce to

$$\begin{aligned} &u_0 = u_1 = 0,\ T_0 = T_1 = 0,\ C_0 = C_1 = 0, && \text{at } y = 0,\\ &u_0 = u_1 \to 0,\ T_0 = T_1 = 0,\ C_0 = C_1 \to 0, && \text{as } y \to \infty. \end{aligned} \tag{3.10}$$

The equations (3.4) and (3.5) are of third order but two boundary conditions are available. Therefore the perturbation method has been applied using $R_c (R_c << 1)$, the elastic parameter as the perturbation parameter.

$$\begin{aligned} u_0 &= u_{00}(y) + R_c u_{01}(y) + o(R_c^2),\\ u_1 &= u_{10}(y) + R_c u_{11}(y) + o(R_c^2). \end{aligned} \tag{3.11}$$

Inserting equation (3.11) into (3.4) and (3.5) and equating the coefficients of R_c^0 and R_c to zero we have following sets of ordinary differential equations:

Zeroth order equations

$$u_{00}'' + u_{00}' - \left(M^2 + \frac{1}{K_p}\right) u_{00} = -G_r T_0 - G_c C_0, \tag{3.12}$$

$$u_{01}'' + u_{01}' - \left(M^2 + \frac{1}{K_p}\right) u_{01} = -u_{00}'''. \tag{3.13}$$

First order equations

$$u''_{10} + u'_{10} - \left(M^2 + \frac{1}{K_p} + \frac{i\omega}{4}\right) u_{10} = -u'_{00} - G_r T_1 - G_c C_1 - \frac{u_{00}}{K_p}, \tag{3.14}$$

$$u''_{11} + u'_{11} - \left(M^2 + \frac{1}{K_p} + \frac{i\omega}{4}\right) u_{11} = -u'''_{10} - i\omega u''_{10} - u'''_{00} - \frac{u_{01}}{K_p}. \tag{3.15}$$

The corresponding boundary conditions are:

$$\begin{aligned} &u_{00} = u_{01} = 0, \quad \text{as} \ \ y = 0, \\ &u_{10} = u_{11} \to 0, \quad \text{as} \ y \to \infty. \end{aligned} \tag{3.16}$$

Solving these differential equations with the help of boundary conditions we get,

$$\begin{aligned} u(y,t) = &\left\{ A_3 e^{-m_4 y} + A_1 e^{-m_1 y} + A_2 e^{-\alpha_1 y} \right. \\ &+ R_c \left(A_7 e^{-m_4 y} + A_4 y e^{-m_4 y} + A_5 e^{-m_1 y} + A_6 e^{-\alpha_1 y} \right) \big\} \\ &+ \varepsilon \left\{ \left(A_{13} e^{-m_5 y} + A_8 e^{-m_1 y} + A_9 e^{-m_2 y} + A_{10} e^{-m_3 y} \right. \right. \\ &\left. + A_{11} e^{-m_4 y} + A_{12} e^{-\alpha_1 y} \right) \\ &+ R_c \left(A_{21} e^{-m_5 y} + A_{14} y e^{-m_5 y} + A_{15} e^{-m_1 y} + A_{16} e^{-m_2 y} \right. \\ &\left. \left. + A_{17} e^{-m_3 y} + A_{18} e^{-m_4 y} + A_{19} e^{-\alpha_1 y} + A_{20} y e^{-m_4 y} \right) \right\}, \end{aligned} \tag{3.17}$$

$$T(y,t) = e^{-m_1 y} + \varepsilon \left(e^{-m_3 y} + \frac{4 i m_1}{\omega} \left(e^{-m_1 y} - e^{-m_3 y} \right) \right) e^{i\omega t}, \tag{3.18}$$

$$C(y,t) = e^{-\alpha_1 y} + \varepsilon \beta_1 \left(e^{-\alpha_1 y} - e^{-m_2 y} \right) e^{i\omega t}. \tag{3.19}$$

The skin friction at the plate is given by

$$\tau_{xy}\big|_{y=0} = \left.\frac{\partial u}{\partial y}\right|_{y=0} - R_c \left[\frac{\partial^2 u}{\partial t \partial y} - (1 + \varepsilon e^{i\omega t}) \frac{\partial^2 u}{\partial y^2} \right]\Bigg|_{y=0}, \tag{3.20}$$

where amplitude, $N = N_r + iN_i$; N_r and N_i are real and imaginary parts of $\tau_{xy}|_{y=0}$ and phase angle is given by

$$\tan \alpha = \frac{N_i}{N_r}. \tag{3.21}$$

The rate of heat transfer, (N_u) at the plate in terms of amplitude and phase is given by

$$N_u = -\left[\left.\frac{\partial T_0}{\partial y}\right|_{y=0} + \varepsilon e^{i\omega t} \left.\frac{\partial T_1}{\partial y}\right|_{y=0} \right] = -\left[\left.\frac{\partial T_0}{\partial y}\right|_{y=0} + \varepsilon |R| \cos(\omega t + \delta) \right], \tag{3.22}$$

where

$$R = R_r + iR_i, \qquad \tan \delta = \frac{R_i}{R_r}. \tag{3.23}$$

The mass transfer coefficient, i.e, the Sherwood number (S_h) at the plate in terms of amplitude and phase is given by

$$S_h = -\left[\left.\frac{\partial C_0}{\partial y}\right|_{y=0} + \varepsilon e^{i\omega t} \left.\frac{\partial C_1}{\partial y}\right|_{y=0} \right] = -\left[\left.\frac{\partial C_0}{\partial y}\right|_{y=0} + \varepsilon |Q| \cos(\omega t + \gamma) \right], \tag{3.24}$$

where

$$Q = Q_r + iQ_i, \qquad \tan \gamma = \frac{Q_i}{Q_r}. \tag{3.25}$$

4. Results and discussion

The flow scenario is exhibited in Fig. 1. It shows the unsteady two dimensional laminar heat and mass transfer flow of an incompressible electrically conducting visco-elastic fluid in a porous medium past a vertical porous plate with time dependant oscillatory permeability and suction in presence of heat source and chemical reaction.

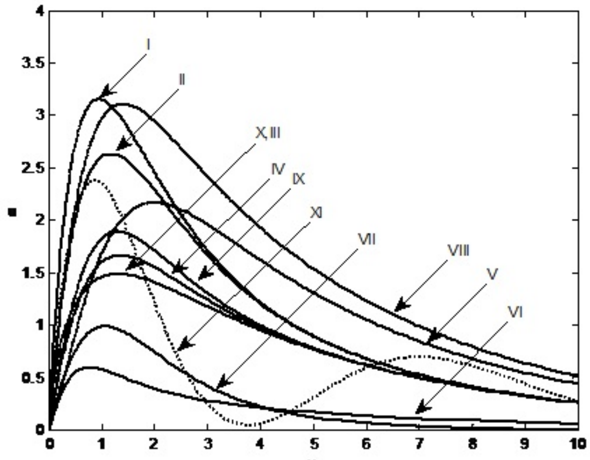

Figure 2. Velocity profile

curve	G_r	G_c	S_c	M	K_p	S	R_c	K_c	P_r
I	10	5	0.22	1	1	0	0	0	0.71
II	10	5	0.22	1	1	0	0.2	0	0.71
III	10	5	0.22	1	1	0	0.2	1	0.71
IV	10	5	0.22	1	1	1	0.2	1	0.71
V	10	5	0.22	1	10	1	0.2	1	0.71
VI	10	5	0.22	3	1	1	0.2	1	0.71
VII	10	5	0.60	1	1	1	0.2	1	0.71
VIII	10	10	0.22	1	1	1	0.2	1	0.71
IX	20	5	0.22	1	1	1	0.2	1	0.71
X	10	5	0.22	1	1	1	0.2	1	7.0
XI	10	5	0.22	1	1	-1	0.2	1	0.71

The significance of the following discussion is to bring out the effect of destructive chemical reaction ($K_c > 0$) coupled with other pertinent parameters on the flow characteristics.

Chemical reaction involves the breaking of bonds in the reactive substances and the formation of bonds to form product species. So it is evident that the rate of reaction depends upon number and nature of the bonds involved. According to the intermolecular concepts reactants are made up of molecules or ions and reactions take place due to intermolecular collision. The rate at which a reaction proceeds, depends largely upon the frequency with which the reacting molecules collide. Effects of concentration, temperature, nature of reactants, catalyst and radiation are all the factors affecting the rate of reaction.

The concentration of solute is very important in studying chemical reactions because it determines how often molecules collide in solution and thus indirectly the rates of reactions and the conditions at equilibrium.

On careful observation of Fig. 2 it is observed that the elastic parameter causes a significant decrease in velocity near the plate. Moreover, curves II and III show a further decrease due to the presence of chemical reaction ($K_c \neq 0$).

It is important to note that elasticity in conjunction with destructive chemical reaction ($K_c > 0$) causes a significant reduction in velocity field resulting a thinner boundary layer. The present study restricts the value of K_c to be positive otherwise it renders the expression to be imaginary.

Further it is seen that the heat source and porosity parameter contribute to enhance the velocity throughout the flow field. In particular, comparing curves IV ($K_p = 1.0$) and V ($K_p = 10.0$) it is seen that a significant increase is marked when $K_p = 10.0$. This increase is due to reduction of resistive force offered by porous matrix. This is quite compatible to the physical behaviour of the flow in the presence of porous medium and clear the flow without it.

The careful observation further reveals that the effect of elasticity is confined to a few layers near the plate because when the visco-elastic fluid is in flow, a certain amount of energy is stored up in the material as strain energy in addition to the viscous dissipation. This property of the fluid is significant only in a few layers near the plate in the present study.

Another striking feature of velocity profile is displayed by the curve XI which is due to the presence of sink ($S = -1$). An oscillatory motion sets in after a few layers of the porous plate. This may be attributed to the loss of heat energy due to presence of sink which prevents steady streaming before its final decay.

The role of destructive chemical reaction is to reduce the velocity on a visco-elastic fluid flow which is exhibited by the curves II and III in the absence of source.

Another interesting observation is the coincidence of curves III and X. The curve X exhibits the case of high Prandtl number flow i.e, fluid with slow rate of thermal diffusion and the presence of heat source. These two phenomena compensate each other as a result of which the curves X ($S = 1.0,\ P_r = 7.0$) and III ($S = 0,\ P_r = 0.71$) coincide.

Thus in the present study presence of heat source and slow rate of thermal diffusion counter- balance each other in the presence of chemical reaction.

The effects of magnetic parameter (curves IV and VI) and Schmidt number (curve IV: $S_c = 0.22$, Hydrogen; curve VII: $S_c = 0.60$, water vapour) are to reduce the velocity distribution at all points of the flow domain. Further it is seen that the decrease in magnitude in case of higher value of M is significant than the higher value of S_c. Thus it may be concluded that the resistive forces due to magnetic field as well as heavier species yield significant reduction in velocity distribution at all points resulting in a thinner boundary layer.

Curve I Das et al. [14] for viscous flow and curve II Mishra et al [22] for visco-elastic flow, both representing without chemical reaction are shown in Fig. 2. The curves I and II coincide with the curves of previous authors. Hence the present findings are well supported.

The objective of Fig. 3 is to study exclusively the case of visco-elastic liquid in a water solution with high Schmidt number, $S_c = 900$ (arbitrary), $617(Cl_2)$,

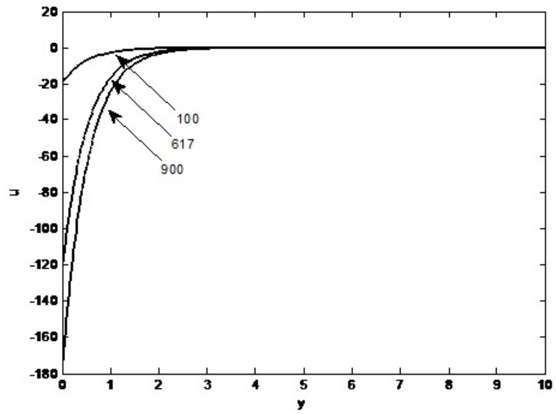

Figure 3. Velocity profile(showing effect of heat source/sink on a dilute water solution of visco-elastic fluids) for $S_c = 900,\ 617,\ 100$, $K_p = 1$, $S = 1,\ -1$, $M = 1$, $G_r = 10$, $G_c = 5$, $R_c = 0.2$, $P_r = 7.0$

100(arbitrary). The velocity profile takes high negative values near the plate and increases sharply to attain ambient state within a few layers near the plate setting aside the effect of source and sink. Thus it is concluded that visco-elastic liquid with heavier species ($S_c = 900, 617$ and 100) with low diffusivity ($P_r = 7.0$) reduces the velocity profile significantly near the plate causing backflow irrespective of source and sink.

Fig. 4 displays the temperature distribution for various values of P_r and S. It is seen that an increase in P_r either in the presence of source/sink leads to decrease the thermal boundary layer thickness. Then it is evident that sharp fall of temperature is marked in the fluid layers close to the surface with low diffusion and in the presence of the sink.

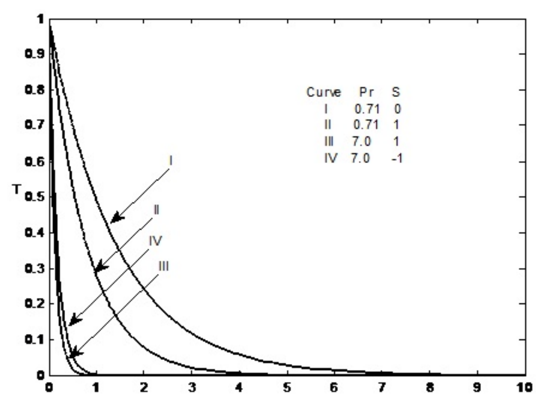

Figure 4. Temperature profile with $\varepsilon = 0.002$, $\omega t = \frac{\pi}{2}$

In Fig. 5, concentration profiles are displayed for various values of Schmidt number (S_c) corresponding to diffusing chemical species of common interest in air, $S_c = 0.22$ and $S_c = 0.60$ correspond to hydrogen and water vapour respectively.

It is seen that fall of concentration occurs for heavier species in the presence of chemical reaction.

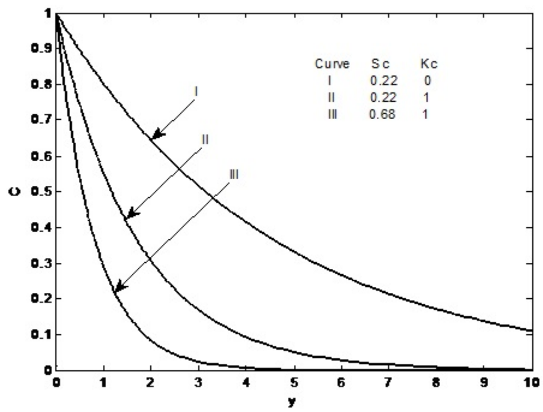

Figure 5. Concentration profile with $\varepsilon = 0.002$, $\omega_t = \frac{\pi}{2}$

The numerical values of skin friction coefficient τ, amplitude ($|N|$), and phase angle ($\tan\alpha$) are entered in Table 1. It is interesting to note that skin friction is positive for Newtonian viscous fluid irrespective of the presence or absence of heat source/chemical reaction, but the presence of elasticity gives rise to negative values. Further it is seen that chemical reaction as well as heat source enhance the amplitude of oscillations and skin friction in magnitude but decrease the phase angle. Therefore it may be concluded that heat source and chemical reaction are found to be counterproductive in reducing the skin friction.

Table 1. Values of skin friction τ , amplitude $|N|$ and phase angle $tan\alpha$.

G_r	G_c	S_c	M	K_p	S	R_c	P_r	K_c	$\|N\|$	$tan\alpha$	τ
10	5	0.22	1	1	0	0	0.71	0	9.948668	0.002547	9.9486356
10	5	0.22	1	1	0	0.2	0.71	0	2.560174	1.526927	-1.40265
10	5	0.22	1	1	0	0.2	0.71	1	3.008137	0.986783	-2.141177
10	5	0.22	1	1	1	0.2	0.71	1	5.955483	0.65919	-4.972353
10	5	0.22	1	10	1	0.2	0.71	1	8.694537	-0.73339	-8.00498
10	5	0.22	3	1	1	0.2	0.71	1	6.25E+00	0.423913	-2.323564
10	5	0.60	1	1	1	0.2	0.71	1	15.63255	2.497326	-7.469841
10	10	0.22	1	1	1	0.2	0.71	1	6.220314	1.838376	-5.170786
20	5	0.22	1	1	1	0.2	0.71	1	11.62756	0.668688	-9.705928
10	5	0.22	1	1	1	0.2	7.0	1	1.42E+03	0.659673	-4.49E+02
10	5	0.22	1	1	0.05	0.2	0.71	1	3.198047	0.942192	-2.327637
10	5	0.22	1	1	1	0.05	7.0	1	1.25E+02	-3.00094	-1.19E+02
10	5	0.22	1	1	-1	0.2	0.71	1	4.230415	15.48379	-0.272648

Rates of heat and mass transfer coefficients at the plate are displayed in Table 2 when $S = 1.0$ and phase angle $\omega t = \frac{\pi}{2}$. It is observed that an increase in P_r leads to increase the amplitude and Nusselt number but reduces the phase angle in case of chemically reacting species. Thus it is possible to control the rate of heat transfer by introducing fluid with high thermal diffusivity. Further, it is also seen

Table 2. Values of amplitude $|R|$ and phase angle $tan\alpha$ for heat and mass transfer coefficient with $S = 1$, $\varepsilon = 0.002$ and $\omega = \frac{\pi}{2}$.

P_r	ω	S_c	K_c	$\|R\|$	N_u	$tan\delta$	$\|Q\|$	S_h	$tan\gamma$
0.71	5	0.22	0	2.215852	1.266666	0.75853	0.714568	0.219095	0.818722
0.025	5	0.30	1	0.226676	0.170728	1.530945	0.251256	0.717751	0.289606
7.0	5	0.66	1	20.00294	7.864045	0.722882	0.609164	1.206501	0.317481
0.71	10	0.78	1	2.076165	1.265907	0.677664	0.736887	1.354996	0.326361

that chemical reaction enhances the rate of mass transfer (S_h) but decreases both amplitude $|Q|$ and phase angle $tan\gamma$ but an increase in frequency parameter, ω has the reverse effect on $|Q|$ and $tan\gamma$ but having the same effect on their counterparts of heat transfer.

5. Conclusion

- The elasticity of the fluid in the presence of destructive chemical reaction causes thinning of the velocity boundary layer.
- Presence of heat source and the absence of porous matrix enhances the flow.
- Presence of sink contributes to oscillatory motion giving rise to flow instability.
- One striking result of the present study is that presence of heat source and slow rate of thermal diffusion counteract each other in presence of chemical reaction.
- Another interesting remark is that flow of reacting species in aqueous solution that is with higher S_c and P_r produces flow reversal.
- Flow of visco-elastic fluid giving rise to negative values of skin friction whereas viscous liquid gives positive values
- Presence of chemical reaction enhances the rate of mass transfer which is a desired consequence of the flow of reacting species.

Appendix

$$m_1 = \frac{P_r + \sqrt{P_r^2 + 4P_rS}}{2},\ m_2 = \frac{S_c + \sqrt{S_c^2 + (i\omega + 4K_c)S_c}}{2},$$
$$m_3 = \frac{P_r + \sqrt{P_r^2 + 4S + i\omega P_r}}{2},$$
$$\alpha_1 = \frac{S_c + \sqrt{S_c^2 + 4S_cK_c}}{2},\ a_1 = M^2 + \frac{1}{K_p},\ a_2 = M^2 + \frac{1}{K_p} + \frac{i\omega}{4},$$
$$m_4 = \frac{1 + \sqrt{1 + 4a_1}}{2},\ m_5 = \frac{1 + \sqrt{1 + 4a_2}}{2},\ \beta_1 = \frac{S_c\alpha_1}{\alpha_1^2 - \alpha_1 S_c - \left(\frac{i\omega}{4} + K_c\right)S_c},$$

$$A_1 = \frac{G_r}{m_1^2 - m_1 - a_1},\ A_2 = -\frac{G_c}{\alpha_1^2 - \alpha_1 - a_1},\ A_3 = -A_1 - A_2,\ A_4 = \frac{m_4^3 A_3}{1 - 2m_4},$$

$$A_5 = \frac{m_1^3 A_1}{m_1^2 - m_1 - a_1},\ A_6 = \frac{\alpha_1^3 A_2}{\alpha_1^2 - \alpha_1 - a_1},\ A_7 = -A_5 - A_6,$$

$$A_8 = \frac{m_1 A_1 - 4im_1 G_r/\omega - A_1/K_p}{m_1^2 - m_1 - a_2},\ A_9 = \frac{\beta_1}{m_2^2 - m_2 - a_2}, A_{10} = \frac{\beta_1}{m_2^2 - m_2 - a_2},$$

$$A_{11} = \frac{A_3 m_4 - A_3/K_p}{m_3^2 - m_3 - a_2},\ A_{12} = \frac{m_1 A_1 - 4im_1 G_r/\omega - A_1/K_p}{m_4^2 - m_4 - a_2},$$

$$A_{13} = -A_8 - A_9 - A_{10} - A_{11} - A_{12},\ A_{14} = \frac{A_{13} m_5^3 - i\omega m_5 A_{13}}{1 - 2m_5},$$

$$A_{15} = \frac{(m_1^3 - m_1^2 i\omega)A_8 + m_1^3 A_1 - A_1/K_p}{m_1^2 - m_1 - a_1},\ A_{16} = \frac{A_{14} m_2^3 - i\omega m_2 A_{14}}{m_2^2 - m_2 - a_2},$$

$$A_{17} = \frac{A_{15} m_3^3 - i\omega m_3 A_{15}}{m_3^2 - m_3 - a_2},\ A_{18} = \frac{(m_4^3 - m_4^2 i\omega)A_{16} + m_4^3 A_3 - \frac{A_7}{K_p} + \frac{A_4(2m_4 - 1)}{K_p(m_4^2 - m_4 - a_2)}}{m_4^2 - m_4 - a_2},$$

$$A_{19} = \frac{(\alpha_1^3 - \alpha_1^2 i\omega)A_{17} + \alpha_1^3 A_2 - A_6/K_p}{\alpha_1^2 - \alpha_1 - a_2},\ A_{20} = \frac{A_4/K_p}{m_4^2 - m_4 - a_2},$$

$$A_{21} = -(A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19})$$

References

[1] M. Acharya, G.C. Dash and L.P. Singh , *Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux,* Indian J. pure Appl. Math, 31(1)(2000),1-18.

[2] N. Ahmed, D. Sarma and D.P. Baruah, *Three dimensional free convective flow and mass transfer along a porous vertical plate,* Bulletin of the Allahabad Mathematical Society, 21(2006),128-141.

[3] N. Ahmed, *MHD free and forced convection with mass transfer from an infinite vertical porous plate*, Journal of Energy, Heat and Mass Transfer, 32(2010), 55-70.

[4] M.S. Alam, M.M. Rahman and M.A. Samad, *Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical plate in a porous medium.* Nonlinear Analysis: Modeling and Control, 11(2006), 211-226.

[5] S. Asghar, M.R.Mohyuddin, T.Hayat and A.M. Siddiqui, *The flow of a non-Newtonian fluid induced due to the oscillation of a porous plate,* Math.Probl.Eng., 2(2004), 133-143.

[6] A. Bejan and K.R. Khair, *Heat and mass transfer in a porous medium*, Int. J. Heat and mass transfer, 28(1985), 902-918.

[7] S. Bathul, *Heat transfer in three dimensional viscous flow over a porous plate moving with harmonic disturbances,* Bull.Pure Appl.Sci.E, 24(1) (2005), 69-97.

[8] C.F. Chang and W.R. Schowalter, *Flow near on oscillating cylinder in dilute viscoelastic fluid,* Nature, 252(1974), 686-688.

[9] C.F. Chang, S.K. Das and D. Raybould, *Method of making high strength, high stiffness, magnesium base metal alloy composites,* US Patent 5149496 A, 1992.

[10] C.F. Chang and W.R. Schowalter, *Secondary flow in the neighborhood of a cylinder oscillating in a viscoelastic fluid*, J. Non-Newtonian Fluid Mech., 6(1979), 47-67.

[11] C.F. Chang, *Boundary-layer analysis of oscillating cylinder flows in a viscoelastic liquid,* Z. Angew. Math. Mech., 28(1977), 283-288.

[12] S.S. Das, S.K. Sahoo and G.C. Dash, *Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction*, Bull.Malays.Math.Sci.Soc., 29(1)(2006), 33-42.

[13] G.C. Dash, P.K. Rath, N. Mohapatra and P.K. Dash, *Free convective MHD flow through porous media of a rotating visco-elastic fluid past an infinite vertical porous plate with heat and mass transfer in the presence of a chemical reaction,* AMSE Modeling B, 78(4)(2009), 21-37.

[14] S.S. Das, A. Satapathy, J.K. Das and J.P. Panda, *Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source,* Int. J. Heat and Mass Transfer, 52(2009), 5962-5969.

[15] M. Ferdows, M.A. Sattar and M.N.A. Siddiqui, *Numerical approach on parameters of the thermal radiation interaction with convection in boundary layer flow at a vertical plate with variable suction*, Thammasat Int. J. of Science and Technology, 9(2004), 19-28.

[16] K.R. Frater, *Acoustic streaming in an elastic-viscous fluid*, J. Fluid Mech., 30(4)(1967), 689-697.

[17] Y.J. Kim, *Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction*, Int.J.Eng.Sci., 38(8) (2000), 833-845.

[18] J.G. Kumar and P.V. Satyanarayana, *Mass transfer effect on MHD unsteady free convective Walters memory flow with constant suction and heat sink,* Int. J. Appl. Math and Mech., 7(19)(2011), 97-109.

[19] S.P. Mishra, G.C. Dash and J.S. Roy, *Fluctuating flow of a non-Newtonian fluid past a porous flat plate with time varying suction*, Indian. J. Phys., 48(1974), 23-36.

[20] M.K. Majumder and R.K. Deka, *MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation*, Rom. Jour. Phys., 52(5-7)(2007), 565-573.

[21] R. Muthucumaraswamy, M. Sundarraj and V. Subhramanian, *Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion,* Int. J. Appl. Math. and Mech., 5(6)(2009), 51-56.

[22] S.R. Mishra, G.C. Dash and M. Acharya, *Mass and heat transfer effect on MHD flow of a visco-elastic fluid through porous medium with oscillatory suction and heat source,* Int.J. Heat and Mass transfer, 57(2)(2013), 433-438.

[23] A. Ogulu and J. Prakash, *Heat transfer to unsteady magneto hydrodynamics flow past an infinite moving vertical plate with variable suction,* Phys. Scr., 74(2006), 232-239.

[24] A. Postelnicu, *Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects,* Int. J. Heat and Mass Transfer, 47(2004), 1467-1472.

[25] A. Raptis and N. Kafousias, *Magneto hydrodynamics free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux*, Can. J. Phy., 60(1982), 1725-1729.

[26] N. Riley, *Oscillating viscous flow. Review and extension*, J. Inst., Maths. Applics., 03(1967), 419-434.

[27] N.P. Singh and Atul K. Singh, *MHD effects on heat and mass transfer in flow of a viscous fluid with induced magnetic field*, Ind. J. Pure Appl. Phys, 38(2000), 182-189.

[28] K.D. Singh and R. Sharma, *Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability*, Indian J. Pure Appl.Math., 33(6)(2002), 941-949.

[29] A.K. Singh and N.P. Singh, *Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity*, Indian J. Pure Appl.Math., 34(2.3)(2003), 429-442.

[30] P. Singh and C.B. Gupta, *MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer,* Indian J.Thear.Phys., 53(2)(2005), 111-120.

[31] R. Sivraj and B. Rushi Kumar, *Unsteady MHD dusty visco-elastic fluid Couette flow in an irregular channel with varying mass diffusion*, Int. J. Heat and Mass Transfer, 55(2012), 3076-3089.

[32] O.V. Trevisan and A. Bejan, *Mass and heat transfer by natural convection in a vertical slot filled with porous medium,* Int. J. Heat and Mass Transfer, 29(1986), 1513-1519.