Journal of Applied Analysis and Computation
Volume 1, Number 2, May 2011

Website:http://jaac-online.com/
pp. 267–277

EXACT EXPLICIT TRAVELING WAVE SOLUTIONS FOR A NEW COUPLED ZK SYSTEM

Minzhi Wei[a] and Shengqiang Tang[a,†]

Abstract The extended tanh-coth method and sech method are used to construct exact solutions of a new coupled ZK system. Traveling wave solutions are determined, which include solitary wave and periodic wave solutions.

Keywords the extended tanh-coth method, the sech method, solitary wave, periodic wave, coupled ZK equation.

MSC(2000) 74J35, 35Q51.

1. Introduction

The KdV equation is a model that governs the one-dimensional propagation of small-amplitude, weakly dispersive waves [5, 6]. The nonlinear term uu_x in the KdV equation

$$u_t + auu_x + u_{xxx} = 0, \tag{1.1}$$

causes the steepening of wave form, whereas the dispersion effect term u_{xxx} in the same equation makes the wave form spread. The balance between this weak nonlinear steepening and dispersion gives rise to solitons. The KdV equation is therefore incapable of shock waves [11]. The KdV equation plays an important role in the development of the soliton theory, where nonlinearity and dispersion dominate, while dissipation effects are small enough to be neglected in the lowest order approximation [1, 2].

The KdV equation is considered a spatially one-dimensional model. An extensive research work has been done in developing higher dimensional models, particularly those in the $(2+1)$, two spatial and one time, dimensions [4]. The best known two-dimensional generalizations of the KdV equations are the Kadomtsev-Petviashivilli (KP) equation, and the Zakharov-Kuznetsov (ZK) equation. The ZK equation given by

$$u_t + auu_x + b(u_{xx} + u_{yy})_x = 0, \tag{1.2}$$

is investigated in [5, 6, 10, 11, 13, 14, 22] by many distinct approaches.

The ZK equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of

[†]the corresponding author. Email addresses: weiminzhi@21cn.com(M. Wei), tangsq@guet.edu.cn(S. Tang)
[a]School of Mathematics and Computing Science,Guilin University of Electronic Technology, Guilin, Guangxi, 541004, P. R. China
[*]The authors were supported by National Natural Science Foundation of China (10961011,11061010) and the Foundation of Guangxi Key Lab of Trusted Software.

a uniform magnetic field [5, 6]. The ZK equation, which is a more isotropic two-dimensional, was first derived for describing weakly nonlinear ion-acoustic waves in a strongly magnetized lossless plasma in two-dimensions [22]. It was found that the solitary-wave solutions of the ZK equation are inelastic.

Recently, a new hierarchy of nonlinear evolution equations was derived by Qin [9] by using a finite-dimensional integrable system. An interesting equation in this hierarchy is a new coupled KdV equation

$$\begin{cases} u_t = \beta u_{xxx} + \alpha(uv)_x + \gamma(vw)_x, \\ v_t = \beta v_{xxx} + \lambda(wu)_x, \\ w_t = \beta w_{xxx} + \lambda(uv)_x, \end{cases} \tag{1.3}$$

where $\alpha,\ \beta,\ \gamma,\ \lambda$ are arbitrary constants. Later, this new coupled equation was investigated by Wu [21], by using matrix transformation and Lax pair. Most recently, in the sense of the KP equation, Wazwaz [15] has extend the new coupled KdV equation to the new coupled KP equation and studied the new coupled KdV equation and the new coupled KP equation, by using the Hirota's bilinear method. The physical phenomena for this system was investigated thoroughly in [9, 15, 21].

Following the sense of the ZK Eq. (1.2) we can extend the coupled KdV system (1.3) to the new coupled ZK system in the form

$$\begin{cases} u_t - \alpha(uv)_x - \gamma(vw)_x - \beta(u_{xx} + u_{yy})_x = 0, \\ v_t - \lambda(wu)_x - \beta(v_{xx} + v_{yy})_x = 0, \\ w_t - \lambda(uv)_x - \beta(w_{xx} + w_{yy})_x = 0. \end{cases} \tag{1.4}$$

The derivation of this system is simply made by following the sense of the ZK equation.

Many reliable direct methods are presented to deal with equations arising from physical problems, such as the further improved F-expansion method [18], the multi-auxiliary equations expansion method [19], the Riemann-Hilbert method [20], the extended tanh-function method [3], and so on.

In this work, we aim to study the new coupled ZK system. The extended tanh-coth method and the sech method will be mainly used to back up our analysis. The extended tanh-coth method and the sech method are direct and effective algebraic method for handling many nonlinear equations, where solitary wave solutions and triangular periodic solutions are generated.

2. The methods

In what follows, the methods will be reviewed briefly. Full details can be found in [16, 17, 12, 7, 8] and the references therein.

For both methods, we first use the wave variable $\xi = x + y - ct$ to carry a PDE in three independent variables

$$P(u, u_t, u_x, u_y, u_{xx}, u_{xy}, u_{yy}, u_{xxx}, ...) = 0, \tag{2.1}$$

into an ODE

$$Q(u, u', u'', u''', ...) = 0. \tag{2.2}$$

Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

2.1. The extended tanh-coth method

The standard tanh-coth method is developed by Malfliet where the tanh is used as a new variable, since all derivatives of a tanh are represented by a tanh itself. Introducing a new independent variable

$$Y = \tanh(\mu\xi), \tag{2.3}$$

that leads to the change of derivatives:

$$\frac{d}{d\xi} = \mu(1-Y^2)\frac{d}{dY},\ \frac{d^2}{d\xi^2} = \mu^2(1-Y^2)\left(-2Y\frac{d}{dY} + (1-Y^2)\frac{d^2}{dY^2}\right). \tag{2.4}$$

The extended tanh method admits the use of the finite expansion

$$\begin{aligned} u(\mu\xi) = S(Y) &= \sum_{k=0}^{M} A_k Y^k + \sum_{k=1}^{M} A_{k+M} Y^{-k}, \\ v(\mu\xi) = P(Y) &= \sum_{k=0}^{M_1} B_k Y^k + \sum_{k=1}^{M_1} B_{k+M_1} Y^{-k}, \\ w(\mu\xi) = Q(Y) &= \sum_{k=0}^{M_2} C_k Y^k + \sum_{k=1}^{M_2} C_{k+M_2} Y^{-k}, \end{aligned} \tag{2.5}$$

where M, M_1, M_2 are positive integer that will be determined to derive a closed form analytic solution. Substituting (2.5) into the simplified ODE (2.2) results in an algebraic equation in powers of Y. To determine the parameter M, M_1, M_2, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. With M, M_1, M_2 determined, we collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $A_k,\ B_k,\ C_k,\ \mu$ and c. Having determined these parameters, knowing that M is a positive integer in most cases, and using (2.5) we obtain an analytic solution $u(x, y, t),\ v(x, y, t),\ w(x, y, t)$ in a closed form.

2.2. The sech method

In a manner parallel to the discussion presented above, we use

$$u(\mu\xi) = S(Z) = \sum_{k=0}^{M} A_k Z^k, \tag{2.6}$$

where $Z = sech\mu(x - ct)$. The algorithms described above certainly works well for a large class of nonlinear equations. The main advantage of the methods is that it is capable of greatly reducing the size of computational work compared to existing techniques such as the pseudo spectral method, the inverse scattering method, Hirota's bilinear method, and the truncated Painlevé expansion.

3. Using the tanh-coth method

In this section we employ the extended tanh-coth method to the Eq. (1.4).

Let $u(x,y,t)=u(\xi),\ v(x,y,t)=v(\xi),\ w(x,y,t)=w(\xi),\ \xi=x+y-ct$. Then (1.4) becomes to

$$\begin{cases} cu'+\alpha(uv)'+\gamma(vw)'+2\beta u'''=0, \\ cv'+\lambda(wu)'+2\beta v'''=0, \\ cw'+\lambda(uv)'+2\beta w'''=0. \end{cases} \tag{3.1}$$

Integrating it with ξ and neglecting constants of integration we find

$$\begin{cases} cu+\alpha uv+\gamma vw+2\beta u''=0, \\ cv+\lambda wu+2\beta v''=0, \\ cw+\lambda uv+2\beta w''=0. \end{cases} \tag{3.2}$$

Balancing u'' with uv in the first equation, v'' with wu in the second equation, and w'' with uv in the third equation gives

$$M+2=M_2+M_1,\ M_1+2=M+M_2,\ M_2+2=M+M_1, \tag{3.3}$$

so that

$$M=M_1=M_2=2. \tag{3.4}$$

The extended tanh-coth method admits the use of the substitution

$$\begin{aligned} u(\mu\xi)=S(Y)&=\sum_{k=0}^{2}A_kY^k+\sum_{k=1}^{2}A_{k+2}Y^{-k}, \\ v(\mu\xi)=P(Y)&=\sum_{k=0}^{2}B_kY^k+\sum_{k=1}^{2}B_{k+2}Y^{-k}, \\ w(\mu\xi)=Q(Y)&=\sum_{k=0}^{2}C_kY^k+\sum_{k=1}^{2}C_{k+2}Y^{-k}. \end{aligned} \tag{3.5}$$

Substituting (3.5) into (3.2), collecting the coefficients of each power of Y, and solve the resulting system of algebraic equations with the help of Maple to find the sets of solutions:

Case (1): $A_0=-\frac{3c}{2\lambda}=-A_2,\ B_0=C_0=-B_2=-C_2=\frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma}$,

$$A_1=A_3=A_4=C_1=C_3=C_4=B_1=B_3=B_4=0,\ c^2=c^2,\ \mu^2=-\frac{c}{8\beta}.$$

Case (2): $A_0=\frac{3c}{2\lambda}=-A_2,\ B_0=-C_0=C_2=-B_2=\frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma}$,

$$A_1=A_3=A_4=C_1=C_3=C_4=B_1=B_3=B_4=0,\ c^2=c^2,\ \mu^2=-\frac{c}{8\beta}.$$

Case (3): $A_0=-\frac{c}{2\lambda}=-\frac{1}{3}A_2,\ B_0=-C_0=-\frac{1}{3}B_2=\frac{1}{3}C_2=\frac{c(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma}$,

$$A_1=A_3=A_4=C_1=C_3=C_4=B_1=B_3=B_4=0,\ c^2=c^2,\ \mu^2=\frac{c}{8\beta}.$$

Case (4): $A_0=\frac{c}{2\lambda}=-\frac{1}{3}A_2,\ B_0=C_0=-\frac{1}{3}C_2=-\frac{1}{3}B_2=\frac{c(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma},$

$$A_1=A_3=A_4=C_1=C_3=C_4=B_1=B_3=B_4=0,\ c^2=c^2,\ \mu^2=\frac{c}{8\beta}.$$

Case (5): $A_0=-\frac{3c}{2\lambda}=-A_4,\ B_0=C_0=-C_4=-B_4=\frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma},$

$$A_1=A_2=A_3=C_1=C_2=C_3=B_1=B_2=B_3=0,\ c^2=c^2,\ \mu^2=-\frac{c}{8\beta}.$$

Case (6): $A_0=\frac{c}{2\lambda}=-\frac{1}{3}A_4,\ B_0=C_0=-\frac{1}{3}C_4=-\frac{1}{3}B_4=\frac{c(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{4\lambda\gamma},$

$$A_1=A_2=A_3=C_1=C_2=C_3=B_1=B_2=B_3=0,\ c^2=c^2,\ \mu^2=\frac{c}{8\beta}.$$

Case (7): $A_0=\frac{3c}{4\lambda}=-2A_2=-2A_4,\ B_0=-2C_4=-2B_4=\frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{8\lambda\gamma},$

$$-2C_0=C_2=B_2=\frac{-\frac{3c}{8}\left[\alpha(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^3}{2\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})-\lambda\gamma-\alpha^2\right]}{\frac{\lambda\gamma}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^2}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})-\alpha\lambda\gamma},$$

$$A_1=A_3=C_1=C_3=B_1=B_3=0,\ c^2=c^2,\ \mu^2=-\frac{c}{32\beta}.$$

Case (8): $A_0=-\frac{c}{4\lambda}=\frac{2}{3}A_2=\frac{2}{3}A_4,\ B_4=C_4=\frac{3}{2}B_0=\frac{3c}{8\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}),$

$$C_0=\frac{2}{3}C_2=\frac{2}{3}B_2=\frac{-\frac{c}{4}\left[\alpha(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^3}{2\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\lambda\gamma+\alpha^2\right]}{\frac{\lambda\gamma}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^2}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\alpha\lambda\gamma},$$

$$A_1=A_3=C_1=C_3=B_1=B_3=0,\ c^2=c^2,\ \mu^2=\frac{c}{32\beta}.$$

Case (9): $A_0=\frac{3c}{2\lambda}=-A_4,\ B_0=-C_0=C_4=-B_4=\frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}),$

$$A_1=A_2=A_3=C_1=C_2=C_3=B_1=B_2=B_3=0,\ c^2=c^2,\ \mu^2=-\frac{c}{8\beta}.$$

Case (10): $A_4=\frac{3c}{2\lambda}=-3A_0,\ C_0=-B_0=-\frac{1}{3}C_4=\frac{1}{3}B_4=\frac{c}{4\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}),$

$$A_1=A_2=A_3=C_1=C_2=C_3=B_1=B_2=B_3=0,\ c^2=c^2,\ \mu^2=\frac{c}{8\beta}.$$

Case (11): $A_4=-\frac{3c}{8\lambda}=-\frac{1}{2}A_0=A_2,\ C_4=\frac{1}{2}B_0=-B_4=\frac{3c}{16\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}),$

$$C_0=-2C_2=-2B_2=\frac{-\frac{3c}{4}\left[\alpha(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^3}{2\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\lambda\gamma+\alpha^2\right]}{\frac{\lambda\gamma}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^2}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\alpha\lambda\gamma},$$

$$A_1=A_3=C_1=C_3=B_1=B_3=0,\ c^2=c^2,\ \mu^2=-\frac{c}{32\beta}.$$

Case (12): $A_2 = \frac{3c}{8\lambda} = \frac{3}{2}A_0 = A_4,\ C_4 = -\frac{3}{2}B_0 = -B_4 = \frac{3c}{16\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})$,

$$B_2 = -C_2 = -\tfrac{3}{2}C_0 = \frac{\frac{3c}{8}\left[\alpha(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^3}{2\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})-\lambda\gamma-\alpha^2\right]}{\frac{\lambda\gamma}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})+\frac{\alpha^2}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})-\alpha\lambda\gamma},$$

$$A_1 = A_3 = C_1 = C_3 = B_1 = B_3 = 0,\ c^2 = c^2,\ \mu^2 = \frac{c}{32\beta}.$$

Based on these results, we obtain the following solitary solutions for (1.4):

$$\begin{cases} u_1 = -\frac{3c}{2\lambda}(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ v_1 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ w_1 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}<0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.6}$$

$$\begin{cases} u_2 = \frac{3c}{2\lambda}(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ v_2 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ w_2 = -\frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\tanh^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}<0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.7}$$

$$\begin{cases} u_3 = -\frac{c}{2\lambda}(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ v_3 = \frac{c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ w_3 = -\frac{c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}>0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.8}$$

$$\begin{cases} u_4 = \frac{c}{2\lambda}(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ v_4 = \frac{c}{4\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ w_4 = \frac{c}{4\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-3\tanh^2\sqrt{\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}>0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.9}$$

and

$$\begin{cases} u_5 = -\frac{3c}{2\lambda}(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ v_5 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ w_5 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}<0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.10}$$

$$\begin{cases} u_6 = \frac{3c}{2\lambda}(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ v_6 = \frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ w_6 = -\frac{3c}{4\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})(1-\coth^2\sqrt{-\frac{c}{8\beta}}(x+y-ct)),\\ \qquad \frac{c}{8\beta}<0,\ \alpha^2+4\lambda\gamma\ge 0, \end{cases} \tag{3.11}$$

$$\begin{cases} u_7 = -\frac{c}{2\lambda}(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ v_7 = \frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ w_7 = -\frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.12}$$

$$\begin{cases} u_8 = \frac{c}{2\lambda}(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ v_8 = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ w_8 = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 - 3\coth^2\sqrt{\frac{c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.13}$$

$$\begin{cases} u_9 = \frac{3c}{4\lambda}\left(1 - \frac{1}{2}\tanh^2\sqrt{\frac{-c}{32\beta}}(x+y-ct) - \frac{1}{2}\coth^2\sqrt{\frac{-c}{32\beta}}(x+y-ct)\right), \\ v_9 = B_0 + B_2\tanh^2\sqrt{\frac{-c}{32\beta}}(x+y-ct) - \frac{1}{2}B_0\coth^2\sqrt{\frac{-c}{32\beta}}(x+y-ct), \\ w_9 = -2B_2(1 - 2\tanh^2\sqrt{\frac{-c}{32\beta}}(x+y-ct)) + \frac{1}{2}B_0\coth^2\sqrt{\frac{-c}{32\beta}}(x+y-ct), \\ B_0 = \frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{8\lambda\gamma},\ B_2 = \frac{\frac{3c}{8}\left[\alpha(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^3}{2\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \lambda\gamma + \alpha^2\right]}{\frac{\lambda\gamma}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^2}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \alpha\lambda\gamma}, \\ \qquad \frac{c}{32\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0. \end{cases} \tag{3.14}$$

$$\begin{cases} u_{10} = \frac{c}{4\lambda}\left(1 + \frac{3}{2}\tanh^2\sqrt{\frac{c}{32\beta}}(x+y-ct) + \frac{3}{2}\coth^2\sqrt{\frac{c}{32\beta}}(x+y-ct)\right), \\ v_{10} = B_0 + B_2\tanh^2\sqrt{\frac{c}{32\beta}}(x+y-ct) + \frac{3}{2}B_0\coth^2\sqrt{\frac{c}{32\beta}}(x+y-ct), \\ w_{10} = -B_2(\frac{2}{3} + \tanh^2\sqrt{\frac{c}{32\beta}}(x+y-ct)) - \frac{3}{2}B_0\coth^2\sqrt{\frac{c}{32\beta}}(x+y-ct), \\ B_0 = \frac{c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{8\lambda\gamma},\ B_2 = \frac{\frac{3c}{8}\left[\alpha(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^3}{2\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) - \lambda\gamma - \alpha^2\right]}{\frac{\lambda\gamma}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^2}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) - \alpha\lambda\gamma}, \\ \qquad \frac{c}{32\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0. \end{cases} \tag{3.15}$$

We can obtain the following triangular periodic solutions for (1.4):

$$\begin{cases} u_{11} = -\frac{3c}{2\lambda}\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ v_{11} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ w_{11} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.16}$$

$$\begin{cases} u_{12} = \frac{3c}{2\lambda}\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ v_{12} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ w_{12} = -\frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\sec^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.17}$$

$$\begin{cases} u_{13} = -\frac{c}{2\lambda}(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ v_{13} = \frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ w_{13} = -\frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.18}$$

$$\begin{cases} u_{14} = \frac{c}{2\lambda}(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ v_{14} = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ w_{14} = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\tan^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0 \end{cases} \tag{3.19}$$

and

$$\begin{cases} u_{15} = -\frac{3c}{2\lambda}\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ v_{15} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ w_{15} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.20}$$

$$\begin{cases} u_{16} = \frac{3c}{2\lambda}\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ v_{16} = \frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ w_{16} = -\frac{3c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\csc^2\sqrt{\frac{c}{8\beta}}(x+y-ct), \\ \qquad \frac{c}{8\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.21}$$

$$\begin{cases} u_{17} = -\frac{c}{2\lambda}(1 + 3\cot^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ v_{17} = \frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\cot^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ w_{17} = -\frac{c}{4\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\cot^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.22}$$

$$\begin{cases} u_{18} = \frac{c}{2\lambda}(1 + 3\cot^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ v_{18} = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\cot^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ w_{18} = \frac{c}{4\lambda\gamma}(-\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})(1 + 3\coth^2\sqrt{\frac{-c}{8\beta}}(x+y-ct)), \\ \qquad \frac{c}{8\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.23}$$

$$\begin{cases} u_{19} = \frac{3c}{4\lambda}\left(1 + \frac{1}{2}\tan^2\sqrt{\frac{c}{32\beta}}(x+y-ct) + \frac{1}{2}\cot^2\sqrt{\frac{c}{32\beta}}(x+y-ct)\right), \\ v_{19} = B_0 - B_2\tan^2\sqrt{\frac{c}{32\beta}}(x+y-ct) + \frac{1}{2}B_0\cot^2\sqrt{\frac{c}{32\beta}}(x+y-ct), \\ w_{19} = -2B_2(1 + 2\tan^2\sqrt{\frac{c}{32\beta}}(x+y-ct)) - \frac{1}{2}B_0\cot^2\sqrt{\frac{c}{32\beta}}(x+y-ct), \\ B_0 = \frac{3c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{8\lambda\gamma},\ B_2 = \frac{\frac{3c}{8}\left[\alpha(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^3}{2\lambda\gamma}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \lambda\gamma + \alpha^2\right]}{\frac{\lambda\gamma}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^2}{2}(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \alpha\lambda\gamma}, \\ \qquad \frac{c}{32\beta} > 0,\ \alpha^2+4\lambda\gamma \geq 0, \end{cases} \tag{3.24}$$

$$\begin{cases} u_{20} = \frac{c}{4\lambda}\left(1 - \frac{3}{2}\tan^2\sqrt{\frac{-c}{32\beta}}(x+y-ct) - \frac{3}{2}\cot^2\sqrt{\frac{-c}{32\beta}}(x+y-ct)\right), \\ v_{20} = B_0 - B_2\tan^2\sqrt{\frac{-c}{32\beta}}(x+y-ct) - \frac{3}{2}B_0\cot^2\sqrt{\frac{-c}{32\beta}}(x+y-ct), \\ w_{20} = -B_2(\frac{2}{3} - \tan^2\sqrt{\frac{-c}{32\beta}}(x+y-ct)) + \frac{3}{2}B_0\cot^2\sqrt{\frac{-c}{32\beta}}(x+y-ct), \\ B_0 = \frac{c(\alpha\pm\sqrt{\alpha^2+4\lambda\gamma})}{8\lambda\gamma},\ B_2 = \frac{\frac{3c}{8}\left[\alpha(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^3}{2\lambda\gamma}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) - \lambda\gamma - \alpha^2\right]}{\frac{\lambda\gamma}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) + \frac{\alpha^2}{2}(-\alpha\pm\sqrt{\alpha^2+4\lambda\gamma}) - \alpha\lambda\gamma}, \\ \qquad \frac{c}{32\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0. \end{cases} \tag{3.25}$$

4. Using the sech method

The sech method (2.6) admits the use of the finite expansion

$$
\begin{aligned}
u(\mu\xi) &= S(Z) = \sum_{k=0}^{2} A_k Z^k + \sum_{k=1}^{2} A_{k+2} Z^{-k},\\
v(\mu\xi) &= P(Z) = \sum_{k=0}^{2} B_k Z^k + \sum_{k=1}^{2} B_{k+2} Z^{-k},\\
w(\mu\xi) &= Q(Z) = \sum_{k=0}^{2} C_k Z^k + \sum_{k=1}^{2} C_{k+2} Z^{-k}.
\end{aligned} \tag{4.1}
$$

Substituting (4.1) into (3.2), collecting the coefficients of each power of Z, and solve the resulting system of algebraic equations with the help of Maple to find the sets of solutions:

Case (1): $A_0 = B_0 = C_0 = A_1 = B_1 = C_1 = A_3 = B_3 = C_3 = A_4 = B_4 = C_4 = 0$,

$$c^2 = c^2,\ \mu^2 = \frac{c}{16\beta},\ A_2 = \frac{3c}{4\lambda},\ B_2 = -C_2 = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}).$$

Case (2): $A_0 = B_0 = C_0 = A_1 = B_1 = C_1 = A_3 = B_3 = C_3 = A_4 = B_4 = C_4 = 0$,

$$c^2 = c^2,\ \mu^2 = \frac{c}{16\beta},\ A_2 = -\frac{3c}{4\lambda},\ B_2 = C_2 = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}).$$

Based on these results, we obtain the following solitary wave solutions for (1.4):

$$
\begin{cases}
u_{21} = \frac{3c}{4\lambda} sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
v_{21} = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
w_{21} = -\frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
\qquad \frac{c}{16\beta} > 0,\ \alpha^2 + 4\lambda\gamma \geq 0,
\end{cases} \tag{4.2}
$$

$$
\begin{cases}
u_{22} = -\frac{3c}{4\lambda} sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
v_{22} = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
w_{22} = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) sech^2 \sqrt{\frac{c}{16\beta}}(x + y - ct),\\
\qquad \frac{c}{16\beta} > 0,\ \alpha^2 + 4\lambda\gamma \geq 0.
\end{cases} \tag{4.3}
$$

However, for $\frac{c}{16\beta} < 0$, we obtain the following periodic wave solutions for (1.4):

$$
\begin{cases}
u_{23} = -\frac{3c}{4\lambda} \tan^2 \sqrt{\frac{-c}{16\beta}}(x + y - ct),\\
v_{23} = -\frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) \tan^2 \sqrt{\frac{-c}{16\beta}}(x + y - ct),\\
w_{23} = \frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2 + 4\lambda\gamma}) \tan^2 \sqrt{\frac{-c}{16\beta}}(x + y - ct),\\
\qquad \frac{c}{16\beta} < 0,\ \alpha^2 + 4\lambda\gamma \geq 0,
\end{cases} \tag{4.4}
$$

$$\begin{cases} u_{24} = \frac{3c}{4\lambda}\tan^2\sqrt{\frac{-c}{16\beta}}(x+y-ct), \\ v_{24} = -\frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\tan^2\sqrt{\frac{-c}{16\beta}}(x+y-ct), \\ w_{24} = -\frac{3c}{8\lambda\gamma}(\alpha \pm \sqrt{\alpha^2+4\lambda\gamma})\tan^2\sqrt{\frac{-c}{16\beta}}(x+y-ct), \\ \qquad \frac{c}{16\beta} < 0,\ \alpha^2+4\lambda\gamma \geq 0. \end{cases} \tag{4.5}$$

5. Discussion

In this paper, we used the extended tanh-coth method and the sech method to study a new coupled ZK equation. As a result, we obtained twenty and four kinds of exact solutions including solitary wave solutions and periodic wave solutions. The methods provided solitary wave solutions and triangular periodic solutions. Moreover, the obtained results in this work clearly demonstrate the reliability of the methods that were used.

References

[1] MJ. Ablowitz and PA. Clarkson, *Solitons, nonlinear evolution equations and inverse scattering,* Cambridge: Cambridge University Press, 1991.

[2] B. Feng, B. Malomed and T. Kawahara, *Cylindrical solitary pulses in a two-dimensional stabilized Kuramoto-Sivashinsky system*, Phys. D., 175 (2003), 127-38.

[3] E. Fan, *Extended tanh-function method and its applications to nonlinear equations*, Phys. Lett. A, 277(2000), 212-218.

[4] B. Li, Y. Chen and H. Zhang, *Exact travelling wave solutions for a generalized Zakharov-Kuznetsov equation*, Appl. Math. Comput., 146(2003), 653-666.

[5] S. Munro and EJ. Parkes, *The derivation of a modified Zakharov-Kuznetsov equation and the stability of its solutions*, J. Plasma. Phys., 62(1999), 305-317.

[6] S. Munro and EJ. Parkes, *Stability of solitary-wave solutions to a modified ZakharovCKuznetsov equation*, J. Plasma. Phys., 64(2000), 411-426.

[7] W. Malfliet, *The tanh method: I. Exact solutions of nonlinear evolution and wave equations*, Phys. Scr., 54 (1996), 563-568.

[8] W. Malfliet and W. Hereman, *The tanh method: II. Perturbation technique for conservative systems*, Phys. Scr., 54(1996), 569-575.

[9] ZY. Qin, *A finite-dimensional integrable system related to a new coupled KdV hierarchy*, Phys. Lett. A, 355(2006), 452-459.

[10] BK. Shivamoggi, *The Painlev analysis of the Zakharov-Kuznetsov equation*, Phys. Scr., 42(1990), 641-642.

[11] H. Schamel, *A modified Kortweg-de Vries equation for ion acoustic waves due to resonant electrons*, J. Plasma. Phys., 9(1973), 377-387.

[12] S. Tang, Y. Xiao and Z. Wang, *Travelling wave solutions for a class of nonlinear fourth order variant of a generalized Camassa-Holm equation*, Appl. Math. Comput., 210(2009), 39-47.

[13] A.M. Wazwaz, *Exact solutions with solitons and periodic structures for the Zakharov-Kuznetsov(ZK) equation and its modified form*, Commun. Nonlinear. Sci. Numer. Simul., 10(2005), 597-606.

[14] A.M. Wazwaz, *The extended tanh method for the Zakharov-Kuznetsov (ZK) equation, the modified ZK equation, and its generalized forms*, Commun. Nonlinear. Sci. Numer. Simul., 13(2008), 1039-1047.

[15] A.M. Wazwaz, *Completely integrable coupled KdV and coupled KP systems*, Commun. Nonlinear. Sci. Numer. Simulat., 15(2010), 2828-2835.

[16] A.M. Wazwaz, *Solutions of compact and noncompact structures for nonlinear Klein-Gordon-type equation*, Appl. Math. Comput., 134(2003), 487-500.

[17] A.M. Wazwaz, *A sine-cosine method for handling nonlinear wave equations*, Math. Comput. Model., 40(2004), 499-508.

[18] D. S. Wang and H.Q. Zhang, *Further improved F-expansion method and new exact solutions of Konopelchenko-Dubrovsky equation*, Chaos, Solitons and Fractals, 25(2005), 601-610.

[19] D. S. Wang and H. Li, *Single and multi-solitary wave solutions to a class of nonlinear evolution equations*, J. Math. Anal. Appl., 343(2008), 273-298.

[20] D. S. Wang, D. J. Zhang and J. Yang, *Integrable properties of the general coupled nonlinear Schrödinger equations*, J. Math. Phys., 51(2010), 023510.

[21] J. Wu, *New explicit travelling wave solutions for three nonlinear evolution equations*, Appl. Math. Comput., 217(2010), 1764-1770.

[22] VE. Zakharov and EA. Kuznetsov, *On three-dimensional solitons*, Sov. Phys., 39(1974), 285-288.